Some Considerations and Design Optimization of 3-phase Semi-conductor Rectifier Power Transformer

Dr K. S. Rama Rao.
School of Electrical & Electronics Engineering , Universiti Sains Malaysia, 14300 Nibong Tebal, Pulau Pinang

Abstract:
Semi-conductor rectifier power transformers are extensively used in railways, mining, chemical industries etc. Static converting equipment supplying a dc load normally influences the primary and secondary currents of an input transformer. Design optimization of such a rectifier power transformer for minimum cost and satisfying important constraints is presented in this paper. A 3-phase, 6-pulse diode bridge rectifier that effect transformer currents is considered. For a specific connection of the transformer, the phase current waveforms in the windings are important before proceeding with the design analysis. An expression for current waveform involving overlap angle of the converter due to the source impedance and dc load current is derived. The importance of proper KVA rating, additional losses due to non-sinusoidal currents are discussed. In the design analysis program eddy current losses in the conductors, temperature rise of the equipment and mechanical stresses in the windings are considered on the basis of actual current waveform. Nonlinear programming technique employing Powell's unconstrained optimization method together with Zangwill's exterior penalty function formulation is applied to a mathematical model of a 4 MVA, 3-phase star-delta connected semi-conductor rectifier power transformer with 6 design variables. Temperature rise of windings and oil, short-circuit impedance, permissible flux density in the core, no-load current, efficiency, dc voltage regulation etc. are imposed as constraints to satisfy the performance characteristics. The optimal solution by this method efficiently proved the design optimization procedure adopted in this paper.

Key words: semi-conductor rectifier power transformers - non-sinusoidal currents – Nonlinear Programming - design optimization.

1. Introduction:

Design optimization of a semi-conductor rectifier power transformer operating with non-sinusoidal current need important considerations with the standards imposed by IEEE and IEC. The harmonic currents also contribute additional I^2R loss in the windings. Important characteristics and different configurations of rectifier transformers have appeared in references [1-3]. The KVA rating of a transformer is to be decided by the r.m.s. current or fundamental component of current drawn from the lines by the primary winding, as recommended by IEEE or IEC respectively [4]. The static converting equipment and the type of 3-phase connection of the transformer normally influence the transformer primary and secondary currents as well as the input line current. In addition, the source impedance also has some effect on the current waveform. In a two winding transformer, if the primary voltage is not unduly high, the HV winding can be inner most, nearest to the iron core, while the heavy current secondary can be on the outside and subdivided into a number of parallel coils. The windings are usually wound with rectangular conductors of large cross-section and are so arranged to possess high mechanical strength in the direction of the greatest force. Due to non-sinusoidal nature of current in the windings and because these transformers supply large currents at low voltage, the mechanical stresses arising from short-circuit currents are different from a conventional power transformer.

A rectifier power transformer connection influence the duty of the converting equipment and it should be coordinated with the available rectifying devices. The phase current waveform in the windings for a specific connection of the transformer is necessary before proceeding with the design analysis.

2. Phase current waveform and KVA rating:

A 3-phase star-delta connected semi-conductor rectifier power transformer supplying a DC load through a 3-phase, 6-pulse bridge rectifier is considered and represented in Fig.1. The phase current waveform of the secondary winding without and with source impedance is shown in Figs.2.1 and 2.2 respectively.

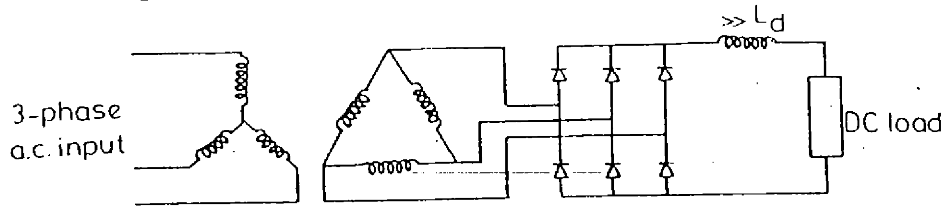

Figure 1

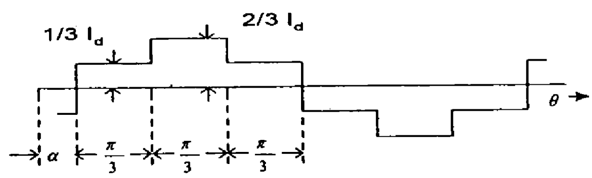

Figure 2.1

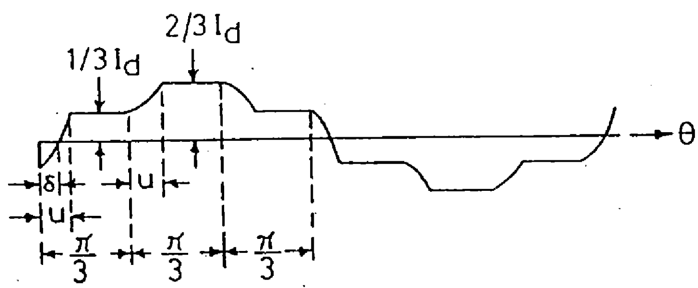

Figure 2.2

The secondary winding r.m.s. phase current, I_2 for a specified dc load is derived by assuming:

i. Direct current supplied to the load is without ripple,

ii. Commutation overlap of the rectifying devices is neglected (Fig.2.1) or commutation loop comprises only inductance (Fig.2.2).

The fundamental component of phase current for a DC load current, I_d is calculated from the Fourier coefficients a_1 and b_1 of Fig. 2.1 as:

$$a_1 = \frac{2}{\pi}\int_{\alpha}^{\alpha+\frac{\pi}{3}} \frac{I_d}{3}\cos\theta d\theta + \int_{\alpha+\frac{\pi}{3}}^{\alpha+\frac{2\pi}{3}} 2\frac{I_d}{3}\cos\theta d\theta + \int_{\alpha+\frac{2\pi}{3}}^{\alpha+\pi} \frac{I_d}{3}\cos\theta d\theta = -\frac{2}{\pi} I_d \sin\alpha \tag{1}$$

$$b_1 = \frac{2}{\pi}\int_{\alpha}^{\alpha+\frac{\pi}{3}} \frac{I_d}{3}\sin\theta d\theta + \int_{\alpha+\frac{\pi}{3}}^{\alpha+\frac{2\pi}{3}} 2\frac{I_d}{3}\sin\theta d\theta + \int_{\alpha+\frac{2\pi}{3}}^{\alpha+\pi} \frac{I_d}{3}\sin\theta d\theta = \frac{2}{\pi} I_d \cos\alpha \tag{2}$$

$$I_{2\max} = \sqrt{a_1^{\,2} + b_1^{\,2}} = \frac{2}{\pi} I_d \tag{3}$$

The secondary r.m.s. phase current, $I_2 = \frac{\sqrt{2}}{\pi} I_d$ (4)

Similarly from Fig.2.2, when source inductance is considered, the secondary r.m.s. phase current is computed for a specified overlap angle, u from the following expression:

$$I_2 = \frac{1}{\sqrt{\pi}}\left[\int_0^{u-\delta}[f_1(\theta)]^2 d\theta + \int_{u-\delta}^{\frac{\pi}{3}-\delta}[f_2(\theta)]^2 d\theta + \int_{\frac{\pi}{3}-\delta}^{\frac{\pi}{3}-\delta+u}[f_3(\theta)]^2 d\theta + \int_{\frac{\pi}{3}-\delta+u}^{\frac{2\pi}{3}-\delta}[f_4(\theta)]^2 d\theta + \int_{\frac{2\pi}{3}-\delta}^{\frac{2\pi}{3}-\delta+u}[f_5(\theta)]^2 d\theta + \int_{\frac{2\pi}{3}-\delta+u}^{\pi-\delta}[f_6(\theta)]^2 d\theta + \int_{\pi-\delta}^{\pi}[f_7(\theta)]^2 d\theta\right]^{\frac{1}{2}} \tag{5}$$

Where $f_1(\theta), f_2(\theta), ...f_7(\theta)$ are the values of instantaneous current in the interval indicated in terms of DC current, I_d [5] and $\delta = \cos^{-1}(\frac{1+\cos u}{2})$.

From the DC load voltage, V_d the secondary r.m.s. voltage, V_2 is specified as

$$V_2 = \frac{V_d \pi}{3\sqrt{2}} \tag{6}$$

The KVA rating, S of the transformer is fixed from Eq. 4 or 5 and Eq. 6 as:

$$S = 3V_2 I_2 10^{-3} \tag{7}$$

As an example for design optimization, select a transformer supplying a DC load at 10,000 A and 400 V. The KVA rating and secondary voltage may be fixed according to r.m.s. current or fundamental component of current.

In this paper, rated KVA is selected from r.m.s. current as prescribed by IEEE standards. The r.m.s. current from actual waveform is used, based on the DC rated load and overlap angle as derived in Eq. 5. The overlap angle depends upon the leakage reactance of the transformer and current at commutation, and initially it is assumed as 15°.

The final specifications of the transformer for design optimization:

A 3-phase, 4 MVA, 33 kV / 296 V, star-delta connected core type rectifier power transformer with ±10 % tapping on primary. HV winding, near the core - continuous disc coils, LV winding - helical coils [3, 7].

3. **Losses in the transformer:**

3.1 **Core loss:** While the current waveform is distorted, the voltage impressed on the primary, when the rectifier is fed from a system whose capacity is reasonably large, approximates a sine wave.

$$Coreloss, P_i = FF(B_c)G_l + 1.075FF(B_y)G_y \quad (8)$$

where B_c = maximum flux density in the core, Tesla

B_y = flux density in the yoke, Tesla

G_l = weight of the limbs, kg

G_y = weight of the yoke, kg

$FF(B)$ = a sixth degree polynomial to represent core loss in watts/kg of 0.35 mm thick c.r.g.o. steel laminations at 50 Hz.

3.2 **Eddy current loss:** There are two approaches to evaluate eddy current loss with pulsating current:

i. To derive a formula similar to eddy loss ratio with sinusoidal current.

ii. To determine the losses as sum of several contributions of harmonic components of actual current.

The first approach is very simple as the expression for r.m.s. current as derived in Eq. 5 is available. The eddy loss ratio for arbitrary wave shape of current is derived as:

$$K_{e,pul} = (\frac{5m^2-1}{45}\frac{h_c^2}{h_w^2}\frac{BO^4}{4})(\frac{I'}{I})^2(\frac{\mu_0^2}{\rho_0^2}) \quad (9)$$

I = rms current, A

I' = time derivative of rms current, A

m = number of parallel conductors in radial direction

BO = radial dimension of the conductor, mm

h_c = height of copper in axial direction, m

h_w = height of winding, m

μ_0 = magnetic space constant, H/m

ρ_0 = conductivity of the material, ohm-m/mm^2

Eq. 9 is used in the design calculations to evaluate eddy current losses in both the windings of the transformer.

4. **Design variables, constraints and the objective function [5]:**

The selection of the following six independent design variables is based on the significant effect of these on the short-circuit reactance, weight of the core and windings, total losses, core dimensions and also on the active material cost of the transformer.

Maximum flux density in the core, Tesla	x_1
Current density in the HV winding, A/mm^2	x_2
Current density in the LV winding, A/mm^2	x_3
Height of the windings, m	x_4
Voltage per turn, V	x_5
Distance between core centers, m	x_6

The degree of utilization of the rectifier transformer is limited by the permissible temperature rise of windings, dc voltage regulation and by the magnetic properties of the active iron.
Based on the above factors, the following constraints are imposed on the design mathematical model:

Temperature rise of windings above ambient, °C	$\leq \theta_{wa}$
Temperature rise of oil above ambient, °C	$\leq \theta_{oa}$
Percentage short circuit impedance	$\leq Z_{sc}$
Permissible flux density in the core, Tesla	$\leq x_1$
Percentage no-load current	$\leq I_o$
Clearance between phase windings, m	$\leq b_{ph}$
Maximum height of the windings, m	$\leq x_4$
Percentage Efficiency	$\geq \eta$
Maximum current density in the HV winding, A/mm^2	$\leq x_2$
Maximum current density in the LV winding, A/mm^2	$\leq x_3$

In addition, some design variables are constrained with upper and lower bounds to satisfy mechanical restrictions.
The objective function [5] to minimize the active material cost of the transformer is formulated with the cost of stampings, windings and capitalized cost of losses. Manufacturing costs and labor costs are not included in the objective function as wide variation is normal in different shop floors.

Objective function, $F(\mathbf{x}) = c_i\,G_i + c_c\,G_c + c_1\,P_c + c_2\,P_i$ (10)

where c_i , c_c = cost of iron, windings Rs./kg

c_1& c_2 = capitalized cost of losses

G_i , G_c = weight of iron, windings, kg

P_c & P_i = copper and core losses

All the above expressions are derived in terms of the design variables [5].

5. Non-linear optimization Problem [8]:

The design optimization mathematical model posed as a nonlinear programming problem is stated as:
Find $\mathbf{x} = (x_1, x_2, \ldots\ldots x_n)$ such that $F(\mathbf{x})$ is a minimum
Subject to $g_j(\mathbf{x})$ {≤ =} 0, j = 1, 2, ... m
With $\mathbf{x} \geq 0$ being a non-negative solution.

$\mathbf{x}$ = $x_1, x_2, \ldots\ldots, x_n$ = independent design variables

$F(\mathbf{x})$ = nonlinear objective function i.e. cost function

$g_j(\mathbf{x})$ = nonlinear constraint functions i.e. geometry and performance characteristics.

6. Exterior Penalty Function Method [10]:

In this method, the augmented function P is formulated as

$$P(x,r) = F(x) + r\sum_{j=1}^{m}[g_j(x)]^q, \qquad r \geq 0 \tag{11}$$

where $g_j(\mathbf{x})$ is defined as $\max[g_j(\mathbf{x}),0]$.
A popular value of q is 2, although other values are possible.
Starting with an initial value $\mathbf{x}_1$ and r_1, minimize $P(\mathbf{x},r_1)$ by Powell's method.
Let $\mathbf{x}_2$ be the resulting point. A new function is formed with $r_2 = c\,r_1$, $c > 1$ such that

$$P(x,r_2) = F(x) + r_2\sum_{j=1}^{m}[g_j(x)]^q, \tag{12}$$

This process of minimization continues and as $r_k \to \infty$, it can be proved that

$$\operatorname*{Min}_{k\to\infty} P(\mathbf{x},r_k) = \operatorname{Min} F(\mathbf{x}) \tag{13}$$

7. Design optimization:

A mathematical model for the non-linear program is formulated with the objective function and constraints in terms of design variables. The general optimization program includes a main program together with seven subroutines [5], viz., HV continuous disc winding, LV helical winding, temperature rise of windings and oil, mechanical forces in windings, Powell-Botm optimization routine etc. The mathematical model is converted into a sequence of unconstrained minimization problems with normalized constraints using Zangwill's exterior penalty function method, as an augmented objective function [10]. With a starting vector of independent variables and a penalty parameter, the minimization process is carried out using Powell's sequential transformation method [9]. Powell's pattern search method using conjugate directions imposes quadratic convergence.

Optimal design of a 4 MVA, 3-phase, star-delta connected, 33 kV / 296 V semi-conductor rectifier power transformer supplying a dc load is computed, minimizing the active material cost and satisfying the desired constraints. The results are shown in Table1. At the optimal solution, it is observed that the mechanical stresses in windings and spacers for maximum short circuit currents are well within the permissible values while satisfying the desired constraints.

8. Conclusions:

A new procedure for design optimization of semi-conductor rectifier transformers based on nonlinear programming technique is described. The mathematical model considers the KVA rating according to IEEE standards and provides a procedure to compute transformer I^2R loss and temperature rise from an actual waveform of current. Optimum design values have proved the suitability of Powell's sequential unconstrained minimization technique together with Zangwill's exterior penalty function formulation. A general design procedure presented in this paper is suitable for a line of transformers with different specifications.

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Table1: Optimized Design Data of a 4 MVA, 3-phase, Star-delta Connected, 33 kV / 296 V Semi-conductor Rectifier Power Transformer:

1.1 Design Variables:

Variable	Description	Optimal value
x_1	Maximum flux density in the core, Tesla	1.67
x_2	Current density in the HV winding, A/mm^2	2.58
x_3	Current density in the LV winding, A/mm^2	2.65
x_4	Height of the windings, m	1.40
x_5	Voltage per turn, V	36.00
x_6	Distance between core centers, m	0.80

1.2 Constraints and Performance Data:

Winding temperature rise above ambient, °C	55.0
Top oil temperature rise above ambient, °C	44.0
Percentage short-circuit impedance	2.95
Percentage no-load current	2.42
Percentage efficiency	99.27
Clearance between phase windings, m	0.04
Core circle diameter, m	0.372
Number of HV disc coils	80
Area of cross-section of HV strap conductor, mm^2	14.00
Number of LV disc coils	18
Area of cross-section of HV strap conductor, mm^2	16.80

Number of parallel conductors in LV coil	6
Eddy current loss in conductors, kW	2.533
Total I^2R loss, kW	17.48
Core loss, kW	10.19
Weight of copper, kg	1091.75
Weight of stampings, kg	4562.98
Overlap angle due to source inductance, deg	16.26
Secondary voltages for different tap settings, V	328, 320, 312, 304, 296, 290, 283, 277, 270

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