

**IMPLEMENTATION AND CHARACTERIZATION OF  
POWER AMPLIFIER ASING RF TEST BOARD**

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## CHAPTER 1 - INTRODUCTION

Power amplifier is a part of RF front-end transmitter block. Usually, it is located at the end of the transmitter. In a wireless system, RF power amplifier is need at the end of the circuit so that the signals transmitted effectively. Power amplifier will amplify the power as high as it could so that it can cross a certain distance specify. Power delivered to the antenna act as a  $50\Omega$  load. The output power of the power amplifier can reach until a few mili-watts for a small network and a few hundreds-watts for a large network.

### 1.1 Objectives

- i. To learn the details about power amplifier application in technology these days.
- ii. To apply the IC design technique via ADS software.
- iii. To understand the IC design levels using simulation and hardware methods.
- iv. To understand the concept of testing and measurement accurately and effectively.

### 1.2 General requirements of Power Amplifier

#### 1.2.1 Bluetooth

- Power Class 1 for long-range device (~100m), Max. output power: 100mW(20dBm); Min. output power: 1mW(0dBm); Power control range: 4dBm to  $P_{max}$  ; power control step: 2(min) to 8(max) dB and must be monotonic. 300 feet (100 meters) transmitting.

- Power Class 2 for ordinary range device (~10m), Max. output power: 2.5mW(4dBm); Min. output power: 0.25mW(-6dBm); Power control: optional down to -30dBm
- Power Class 3 for short range devices (~10cm), Max. output power: 1mW(0dBm); Min. output power: N/A; Power control: Optional down to -30dBm. 30 feet transmitting.

### **1.2.2 Ultra wideband (UWB)**

UWB communication is an emerging wireless technology that promises high data rates over short distance and precise location. The large available bandwidth and the constrain of a maximum power spectral density drives a unique set of system challenges.

UWB is a wireless radio technology designed to transmit data within short ranges (up to 10 meters). It transmits at very high bandwidths (up to 480 Mbps) while using little power. UWB is ideal for exchanging data between consumer electronics (CE), PCs, PC peripherals, and mobile devices at very high speeds over short distances. For instance, it could transfer all the pictures on a digital camera's memory card to a computer in a few seconds.

### **1.2.3 Wireless LAN**

- 802.11b (Wi-Fi) : Direct Sequence Spread Spectrum (DSSS) system, supports 1, 2, 5.5, and 11 Mbps data rates. The maximum transmitted power must be 1000mW or less for the United States, 100mW or less for Europe, and

10mW/MHz or less for Japan.

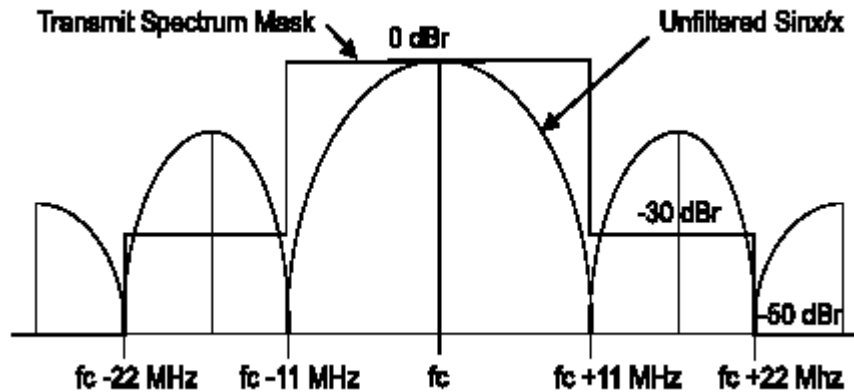


Figure 1 : 802.11b spectral mask

- 802.11g : Frequency Band: 2.4GHz-2.4835GHz; Carries number: 52 (48 for data and 4 for pilot); Channel bandwidth: 16.25 MHz; Data rate: 6-54 Mbps; Modulation: BPSK, QPSK, 16QAM or 64QAM; Maximum output power: 1W (30dBm); EVM: 5.6% or -25dB for 54Mbps; Spectrum mask: -20dBc @ 11MHz offset, -28dBc @ 20MHz offset and -40dBc @ 30MHz offset.
- 802.11a : Spectral-efficient modulation requires high linear PA.

### 1.3 Thesis organization

In this thesis, there are five chapters. Some of the basics of the power amplifier are discussed in detail in chapter 2. Chapter 3 will discuss the design consideration used in designing the circuit. The design is elaborated in this chapter. In the next chapter, Chapter 4, the simulation result and characterization are added in this chapter. While, the conclusion of the thesis is in Chapter 5.

## **CHAPTER 2 - OVERVIEW OF POWER AMPLIFIER**

### **2.1 Transmitter basics**

Proper transmitter design is critical, since it is a device that radiates an electromagnetic signal. This signal can interfere not only with other wireless communication, but also with many different types of non-RF electronic equipment. Usually, the power amplifier is placed in the transmitter area. The PA design is crucial to amplify the signal before it is transmitted to another device. The power amplifier device will be discussed in detail in the next section.

### **2.2 Power Amplifier Specification**

The main characteristics of an amplifier are linearity, efficiency, output power and signal gain. In general, there is a trade off between these characteristics. For example, improving an amplifier's linearity will degrade its efficiency. Therefore knowing the importance degree of each one of these characteristics is an essential step in designing an amplifier. This can be judged based on the application. As an example, a high output power amplifier is used on the transmitter side of a transceiver, whereas a high linear amplifier is used on the receiver side.

An amplifier is said to be linear if it preserves the details of the signal waveform, that is to say,

$$V_o(t)=A.V_i(t)$$

Where  $V_i$  and  $V_o$  are the input and output signals respectively, and  $A$  is a constant gain representing the amplifier gain. But if the relationship between  $V_i$  and  $V_o$  contains the higher power of  $V_i$ , then amplifier produces nonlinear distortion. In this chapter, a few vital aspects about power amplifier will be discussed.

### **2.2.1 Output frequency**

In a narrowband application, a certain range of frequency has to be chosen. It is because all the parameters that want to be measured and tested are at that frequency. In this case, the chosen frequency is 4 GHz. Overall, the simulation and testing are done at this frequency.

### **2.2.2 Output power**

The output power of the RF power amplifier cannot be increased. At a certain state, there will be a state where any power increment in the input will not produce the output power wanted. At this state, the power amplifier is saturated. It means that the output is not proportional to the input signal. At a different case, with a small output

current, the power amplifier will give changes at the input signal and produced a large output signal. The matching network can be added for maximum power out (+22dBm).

The output power level plays an important role in evaluating the power amplifier. The power output capability factor, P<sub>MAX</sub>, is the power output that would be produced with stresses of 1 Volt and 1 Amp on the drain of the field effect transistor (FET). Multiplication of P<sub>MAX</sub> by the drain voltage and current ratings of a real device produces the maximum output power available from the device. The power output capability factor is,

$P_{MAX} = \text{The maximum output power} / \text{the peak drain voltage} \times \text{the peak drain current}$

### **2.2.3 Power gain**

In the IC chip specification, the gain for frequency of 4 GHz is +22dB at dBm input power. Usually, the power amplifier will produce high gain. This gain is than supplied to the later stage usually an adder.

$$\text{Gain (dB)} = 10 \log_{10} G ; \text{ where } G = \text{Gain}$$

Power gain measured using the equation below:-

$$\text{Power gain} = (\text{Voltage} / \text{Output impedance}) / (\text{Voltage} / \text{Input impedance})$$

Power gain is equal to the voltage gain is the input and output impedance is the same.

## 2.2.4 Efficiency

Probability to get a 100% efficiency is impossible which is power delivered to the load is equal to the power produced at the source. The best ways to get high efficiency are by using an E class or an F class amplifier. Switching amplifier from these classes can be used if the measuring and testing is done at high frequency. Well, as a compromise, linearity has to be balance to get a high efficiency.

If power amplifier is compared to the different input stage, PAE (Power Added Efficiency) will be used.

$$PAE = \frac{P_{out} - P_{in}}{P_{DC}}$$

Equation which used gain (G) is as follow:

$$PAE = \frac{P_{in}(G-1)}{P_{DC}}$$

The amplifier's efficiency is a measure of its ability to convert the dc power of the supply into the signal power delivered to the load. The definition of the efficiency can be represented in an equation form as

$$\eta = \text{Signal power delivered to load} / \text{DC power supplied to output circuit}$$

For an ideal amplifier, the efficiency is one. Thus, the power delivered to the load is equal to the power taken from the DC supply. In this case, no power would be consumed in the amplifier. In reality, this is not possible, especially in high frequency realm of RF



circuits. In many high frequency systems, the output stage and driver stage of an amplifier consumed power in the amplification process.

### **2.2.5 Linearity**

Linearity is an important element for the system for all amplifiers. This is because the main objective is to produce a highly linear amplifier. The output power is proportional to the input power. Well, the device usually will enter the saturation state at a certain input power making the harmonic occurred at the output power spectrum. Linearity is something to be considered of because sometimes the device potentially enters the non linear region when producing high output. 1dB compression and IP3 is used to measure the linearity of the device.

The need for linearity is one of the principal drivers in the design of modern power amplifier . Linear amplification is required when the signal contains both amplitude and phase modulation. It can be accomplished either by a chain of linear Pas or a combination of nonlinear Pas. Nonlinearities distort the signal being amplified, resulting in spaltter into adjacent channels and errors in detection.

### **2.2.6 Compression gain**

Gain is a ratio between output power and input power. Referring to Figure 4 Data Sheet, it shows the power gain for the amplifier at 4 GHz frequency is 22 dB which

means any division with the input power must equal to 22. Theoretically, the gain is highly compressed at the saturation region parallel with the increasing input power. Even the input power increased, the output power will not produce the increment that we need. In reality, any adjustment from linear power to saturation power is shown at the curve.

1 dB compression is an input power gain which compressed for 1 dB. Refer to Figure 1. It has relationship with the limit and total output power delivered because faulty IC chip may occurred if the input power is amplify over the limit. The output at the compressed point at 1 dB is given by the sum of the input at the 1 dB compression point and the amplifier gain (dB).

$$P_{1dB}(\text{output}) = [ P_{1dB}(\text{input}) + (\text{Gain}-1) ] \text{ dBm}$$

Many system engineers will choose amplifier power which has higher P1dB required. For instance, if one application needs a maximum output power of 50 W, the system engineer will choose amplifier with P1dB 100 W or more to ensure the amplifier operates at the linear region. The choice is dependent to the linearity needs of the system.

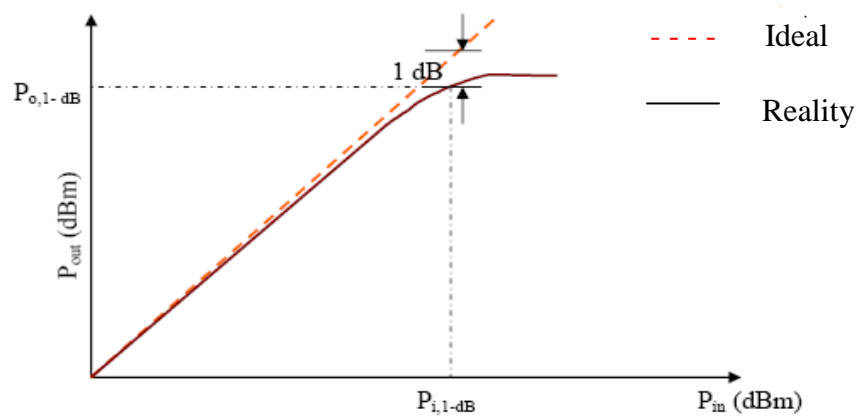


Figure 2 : P1dB characteristic

### 2.2.7 IP3

IP3 referred to the input and output of a system which represent the linearity of the amplifier. An amplifier needs to be linear so that it can pass through a path that cannot be predicted during amplification.

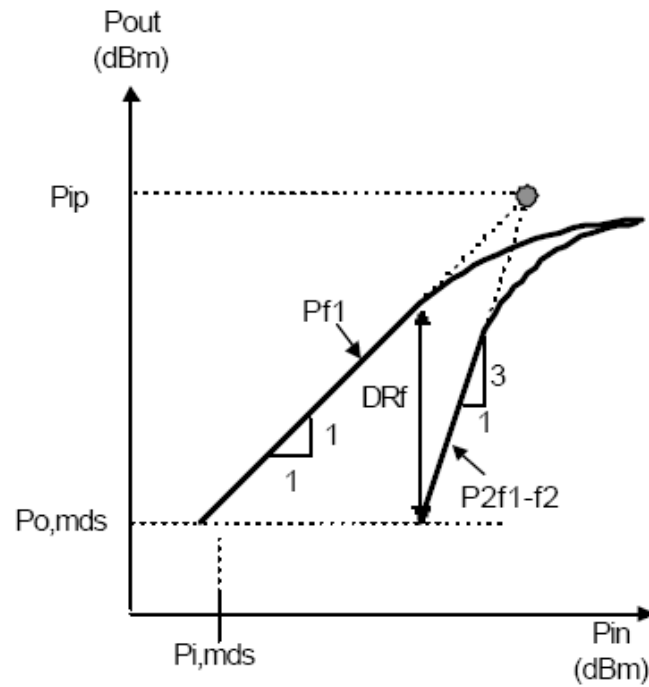


Figure 3 : Third Order Intercept Point

### 2.2.8 Noise factor

Noise factor is a ratio between signals to input noise (SNR<sub>in</sub>) to the ratio of signals to output noise (SNR<sub>out</sub>). The relationship between noise factor (NF) and noise is as below:

$$NF = 10 \log_{10} F ; F = \text{noise}$$

### 2.2.9 Output Return Loss

Output return loss is also known as output VSWR. In a small signal, output return loss is referred to S22 parameter. Parameter S22 is important to determine how much the signal are delivered to the load, depend on stability of the amplifier stage. Often a maximum output return loss is specified for an amplifier. This occurs when amplifier drives the impedance that may vary significantly from the 50 ohm load for which the amplifier is optimized. Output return loss can be written as follow:-

$$RL = -20 \log |\Gamma_{out}|$$

### 2.2.10 Power consumption

In a typical transmitter, the PA consumes the most amount of power from the supply/battery. For portable devices, it is essential that this Parameter be kept to a minimum. This can be achieved with smaller transistor and smaller bias currents.

## 2.3 Classification of Power Amplifier

They are many classifications for power amplifier system. These classes range from entirely linear with low efficiency to entirely non-linear with high efficiency. The most widely used are between linear amplifier and switching amplifier. In a linear amplifier, output signal amplitude is proportional to the input signal amplitude. Class A, B and AB are including in this group. While for switching amplifier, power amplifier is

related to the amplitude for large signal and irrelevant that switch ON and OFF and functionality as a switch. Class C, D, E and F are included to this group. This type of amplifier can achieve the high efficiency.

The active device used in this project is the field effect transistor. The reason for choosing this type of transistor is its superior performance in the microwave range. The characteristics of the FET can be described by:

$$i_D = 0 \quad ; \text{ cut-off region,}$$

$$i_D = g_m \cdot (V_{GS} - V_T) \quad ; \text{ active region}$$

$$i_D = V_D / R_{on} \quad ; \text{ saturation region}$$

The regions of operation are defined by:

$$\text{Cut-off region:} \quad V_{GS} < V_T$$

$$\text{Active region:} \quad V_{GS} \geq V_T \text{ and } i_D < V_D / R_{on}$$

$$\text{Saturation region:} \quad V_{GS} \geq V_T \text{ and } i_D = V_D / R_{on}$$

The term “saturation” is used here to denote the region when further increase in gate voltage produces no increase in drain current, that is to say,  $i_D$  is independent of  $V_{GS}$ .

### **2.3.1 Current source amplifier**

#### **2.3.1.1 Class A**

Class A amplifier is the simplest between the other classes. Operational of the class can be determined via operational point at the load line of the device. I/V characteristics of the transistor are shown as in Figure 2. From the figure, it shows that transistor is biasing in the middle of the load line ; transistor function in the active region at the maximum of output voltage/current. The linearity is the best compare to other amplifier. However, the level of the efficiency is around 50% only.

#### **2.3.1.2 Class B**

Class B amplifier is device at the voltage threshold in order that the conduction angle is  $\pi$ . The sine wave is positif only for one cycle of input voltage. So, the power needed is low compared to the class A amplifier. In theoritical, the effeciency of the class B amplifier is around 78%, but linearity prolem can occur more heavy for this type amplifier. However, class amplifier also undergo crossover distortion at the switching point.

### 2.3.1.3 Class AB

Class a & B amplifier is the two example topology of good power amplifier in a side of efficiency and linearity. However, operation point between these two operation always used and it is known as class AB. Good linearity and efficiency can be achieved at the device in the area. Conduction angle class AB power amplifier is  $\pi < \alpha < 2\pi$ .

### 2.3.1.4 Class C

Class C amplifier is not linear and biasing below the cut-off region. Conduction angle of the power amplifier is below than  $\pi$ . With the minimum utilization of energy, the efficiency can achieved until 100%. But the level of the amplifier power will reduced and not suitable for many application.

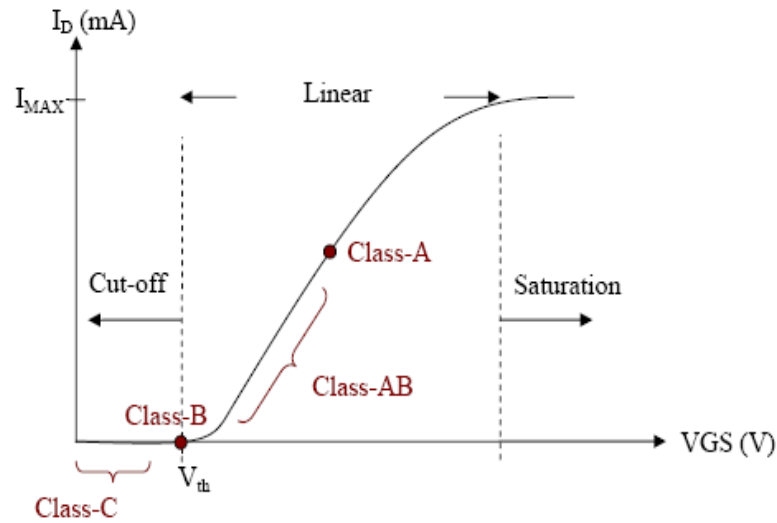


Figure 4 : Q point of class A, B , AB and C type amplifier

### **2.3.2 Switching amplifier**

For the switching amplifier, amplifier power is determined from amplitude of the large signal; ON & OFF device like a switch. This amplifier is operation at the constant power level in the saturation region. The control power can change the gain of the amplifier.

#### **2.3.2.1 Class D**

Class D is used to describe power amplifiers which use switching (or DIGITAL) techniques to reduce the collector dissipation of the output transistors. In Class D amplifiers the output transistor is either saturated or cut and efficiencies of greater than 90% can be achieved. Conduction angle is below than 180 degree. These amplifiers are used mostly in control application but are finding an ever increasing popularity as integrated

#### **2.3.2.2 Class E**

Class E amplifier is the new switch amplifier which the voltage / current wave are derive using inductor and capacitor to achieve the high efficiency of 100%. Class E amplifier needs one transistor only to operate at some GHz frequency. But, the shape of



the component at this type of an amplifier is design for one frequency and cannot be used for UWB applications.

### 2.3.2.3 Class F

Class F amplifier used harmonic resonator at the output to shape drain wave. Harmonic trap is design to make sure the voltage wane is similar to square wave while current wave is similar to half sine wave. Voltage and current wave are not existence simultaneously. Therefore, the high efficiency can be achieved until 100%.

Table 1 : The differences between classes of power amplifier

<b>Type</b>	<b>Maximum Efficiency (%)</b>	<b>Linearity</b>	<b>Conduction Angle</b>	<b>Gain</b>
<b>Class A</b>	<50	Good	100	Large
<b>Class B</b>	78	Moderate	50	Moderate
<b>Class C</b>	100	Small	<50	Small
<b>Class D</b>	100	Poor	50	Large
<b>Class E</b>	100	Poor	50	Large
<b>Class F</b>	100	Poor	50	Large

## CHAPTER 3 - CIRCUIT DESIGN

In an RF power amplifier, an input-side RF terminal is connected to a gate of an FET via an input-side matching line. A source of the FET is grounded. A drain of the FET is connected to an output-side RF terminal via an output-side matching line. To a line connected to the drain of the FET, a circuit for controlling an output impedance for the secondary harmonic wave is connected, including a first line and a first capacitor. To a line connected to the gate of the FET, a circuit for controlling an input impedance for the secondary harmonic wave is connected, including a second line and a second capacitor. The length of the second line is set so that an electric length there of becomes longer than one-fourth of the wavelength for the fundamental wave frequency. Thus, an impedance for the harmonic wave is controlled at the input side of the power transistor. A typical PA output stage consists of a output transistor, a RF choke, and an impedance matching network. In this part, the design on external circuit in detail will discussed in detail.

### 3.1 Stability factor

Before doing the LC matching, the stability factor of the IC must be calculate first. To determine the stability of the system, the value of  $\Delta$  and K have to calculate as follow:-

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|}$$

If  $|\Delta| < 1$  and  $K < 1$ , the device is said to be unstable. Next, one stability circle drawn to differentiate the stable and unstable region of the device. If  $|\Delta| < 1$  and  $K > 1$ , the device is said to be stable. So, no need to draw the stability circle.

The specifications of IC MGA-83563 are as follow:-

$$S_{11} = 0.43 \angle -52$$

$$S_{12} = 0.028 \angle 93$$

$$S_{21} = 9.28 \angle -94$$

$$S_{22} = 0.60 \angle -148$$

$$K = 1.38$$

$$L2 = 0.0nH$$

The stability factor of the power amplifier must be considered by calculating  $K$  and  $\Delta$  at 4GHz.

$$\Delta = S_{11}S_{22} - S_{12}S_{21} \dots\dots\dots (1)$$

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|} \dots\dots\dots (2)$$

From (1) ;

$$\begin{aligned}\Delta &= S_{11}S_{22} - S_{12}S_{21} \\ &= (0.43\angle -52)(0.60\angle -148) \\ &\quad - (0.028\angle 93)(9.28\angle -94) \\ &= (0.258\angle -200) - (0.2598\angle -1) \\ &= (-0.2424 + j0.0882) \\ &\quad - (0.2598 - j0.0045) \\ &= -0.5022 + j0.0927 \\ &= 0.5107\angle 169.54 \approx 0.5107\angle 170\end{aligned}$$

From (2) ;

$$\begin{aligned}K &= \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|} \\ &= \frac{1 + 0.5107^2 - 0.43^2 - 0.60^2}{2(0.028)(9.28)} \\ &= \frac{0.7159}{0.5197} \\ &= 1.3775 \approx 1.38\end{aligned}$$

From the above equation, we can see that  $K=1.38$  ( $K>1$ ) and  $|\Delta|=0.5107\angle 170$  ( $|\Delta|<1$ ).

So, no need to plot stability circle because it is in the stable condition. However the value of  $K=1.38$  is meets with the IC specification.

### 3.2 Interstage Inductor, L2

The basic design procedure on selecting the value of L2 will be done first, followed by design of an output and input matching roughly before optimizing the value to get the actual one. But, in this paper, I will not explain about the optimizing part.

Referring to Table 1 in the datasheet, the value for interstage inductor, L2 is 0.0 nH. The interstage inductor is bypassed to 40pF capacitor, which has reactance of  $1\Omega$  at 4.0 GHz. The supply voltage will be connecting to the bypassed side of the inductor of completing the interstage part of the amplifier. The interstage inductor is bypassed with a 40pF capacitor, which has a reactance of  $1\Omega$  at 4.0GHz. The reactance is calculated as follow:-

$$\begin{aligned}X_c &= \frac{1}{\omega C} \\C &= \frac{1}{\omega X_c} \\&= \frac{1}{2\pi(4G)(1\Omega)} \\&= 39.79 pF \approx 40 pF\end{aligned}$$

### 3.3 Matching network

The purpose of doing the matching is to eliminate the reflected wave at the load. A load may be not only a circuit, but also a generator feeding the line. That is why there are two different direction in the Smith chart which is toward the load and toward the generator. But, normally we used the toward to generator. Figure 4 illustrates this matching using IC MGA-83563.

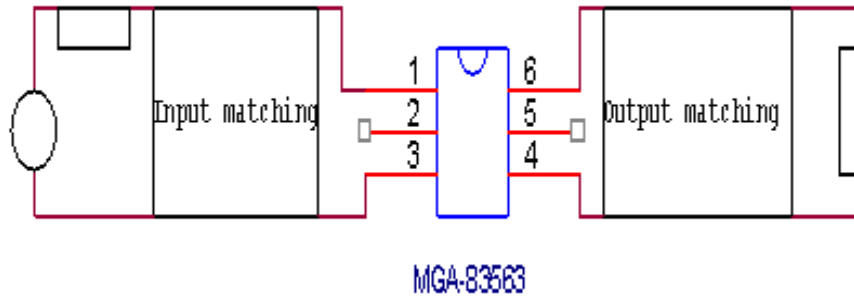


Figure 5 : An IC ( MGA-83563 ) with both input and output matching

The lumped elements (LC) technique was chosen to match the IC. However, there are two ways to describe the technique. I will explain both of the techniques here. The first technique is easily determined using the Smith chart and the other way is by calculating formula calculation. The final result of both of them should be almost the same.

### 3.3.1 Reflection coefficient

The reflection coefficient can be calculated using equation (7) and (8). It consists of several constants and needs to be calculated first. The reflection coefficient needs to be plotted on the Smith Chart. The impedance  $Z_s$ , represented by this reflection coefficient, is looking into the matching section toward the source impedance,  $Z_o$ . Thus, the matching section must transform  $Z_o$  to the impedance  $Z_s$ . The same situation will be done to the load impedance,  $Z_L$ . The reflection coefficient can be calculated as follows:-

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \dots \dots \dots (3)$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \dots\dots\dots(4)$$

$$C_1 = S_{11} - \Delta S_{22}^* \dots\dots\dots(5)$$

$$C_2 = S_{22} - \Delta S_{11}^* \dots\dots\dots(6)$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \dots\dots\dots(7)$$

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \dots\dots\dots(8)$$

From (3) ;

$$\begin{aligned} B_1 &= 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ &= 1 + 0.43^2 - 0.60^2 - 0.5107^2 \\ &= 0.5641 \end{aligned}$$

From (4) ;

$$\begin{aligned} B_2 &= 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ &= 1 + 0.60^2 - 0.43^2 - 0.5107^2 \\ &= 0.9143 \end{aligned}$$

From (5);

$$\begin{aligned} C_1 &= S_{11} - \Delta S_{22}^* \\ &= (0.43 \angle -52) - (0.5107 \angle 170)(0.60 \angle 148) \\ &= (0.2647 - j0.3388) - (0.2277 - j0.205) \\ &= 0.037 - j0.1338 \\ &= 0.1388 \angle -74.54 \end{aligned}$$

From (6);

$$\begin{aligned}
 C_2 &= S_{22} - \Delta S_{11}^* \\
 &= (0.60 \angle -148) - (0.5107 \angle 170)(0.43 \angle 52) \\
 &= (0.60 \angle -148) - (0.2196 \angle 222) \\
 &= -0.3456 - j0.1711 \\
 &= 0.3856 \angle -153.66
 \end{aligned}$$

Source and load reflection coefficient ( $\Gamma_s$  and  $\Gamma_L$ ) can now be determined.

From (7), the source reflection coefficient is calculated as follow:-

Since  $B_2$  equals +, the sign equals - in the  $\Gamma_s$  equation below. The same goes to  $\Gamma_L$ .

$$\begin{aligned}
 \Gamma_s &= \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \\
 &= \frac{0.5641 - \sqrt{0.5641^2 - 4(0.1388)^2}}{2(0.1388) \angle -74.54} \\
 &= \frac{0.5641 - 0.4911}{0.2776 \angle -74.54} \\
 &= 0.2630 \angle 74.54
 \end{aligned}$$

and

From (8), the load reflection coefficient is calculated as follow:-

$$\begin{aligned}
 \Gamma_L &= \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \\
 &= \frac{0.9143 - \sqrt{0.9143^2 - 4(0.3856^2)}}{2(0.3856 \angle -153.66)} \\
 &= \frac{0.9143 \angle 0.4911}{0.7712 \angle -153.66} \\
 &= 0.5488 \angle 153.66
 \end{aligned}$$



The angle of  $\Gamma_L$  the same value as in C2\*. Hence, the load and source impedance can be determined.

From (7),

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_L = \frac{-Z_0 - \Gamma_L Z_0}{\Gamma_L - 1} \dots\dots\dots(9)$$

$$\begin{aligned} &= \frac{-50 - (0.5488 \angle 153.66)}{(0.5488 \angle 153.66) - 1} \\ &= \frac{-50 - (27.44 \angle 153.66)}{(-0.4918 + j0.2435) - 1} \\ &= \frac{-25.4089 - j12.1750}{-1.4918 + j0.2435} \\ &= \frac{28.1752 \angle -154.3980}{1.5115 \angle 170.7296} \\ &= 18.6406 \angle -325.13 \\ &= 15.2937 + j10.6571 \approx 15 + j11 \end{aligned}$$

and

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

$$Z_S = \frac{-Z_0 - \Gamma_S Z_0}{\Gamma_S - 1} \dots\dots\dots(10)$$