

**IMPLEMENTATION AND CHARACTERIZATION OF LOW
NOISE AMPLIFIER AT 4GHz USING RF TEST BOARD**

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ABSTRACT

The purpose of this project is to implement and characterize a single state Pseudomorphic High Electron Mobility Transistor (PHEMT) Low Noise Amplifier (LNA) at 4GHz. The LNA used in this project is Agilent's MGA-72543 which has an operating frequency from 0.1 GHz to 6.0 GHz. This economical, easy-to-use GaAs MMIC Low Noise Amplifier (LNA) is designed for an adaptive CDMA receiver LNA and adaptive CDMA transmit driver amplifier. This LNA features a minimum noise figure of 1.4dB and 14dB associated gain from a single stage, feedback FET amplifier. It is housed in the SOT343 package and is part of the Agilent Technologies CDMA RF chipset. The quantities measured are input reflection coefficient, S_{11} ; gain, S_{21} ; noise figure, NF; P_1 dB compression and input third-order intercept point, IIP3. The LNA has a bypass switch function on chip as well as an adjustable IIP3. The input of the MGA-72543 is internally optimally matched for a low noise figure. However, the input is additionally externally matched for a lower voltage standing wave ratio, VSWR. A simulation has also been performed. However, only the simulation on input reflection coefficient, S_{11} ; gain, S_{21} and noise figure, NF could be done by using Low Noise Amplifier S-parameter model the simulation against the linearity needs the usage of Low Noise Amplifier S-parameter large signal model. This particular model signal is not available in the ADS software. The characterization gives full insight into the performance of the GaAs Low Noise Amplifier scheme.

ABSTRAK

Tujuan projek ini adalah untuk melaksanakan dan mencirikan suatu keadaan tunggal Transistor Kebolehgerakan Elektron Tinggi Pseudomorfik Penguat Hingar Rendah pada 4GHz. Penguat Hingar Rendah yang digunakan untuk projek ini adalah MGA-72543 keluaran Agilent mempunyai frekuensi pengoperasian antara 0.1 GHz sehingga 6.0 GHz. Penguat Hingar Rendah GaAs MMIC yang ekonomik dan mudah digunakan ini direka untuk Penguat Hingar Rendah penerima CDMA suai dan penguat pemacu penghantar CDMA suai. Penguat Hingar Rendah ini mempunyai angka hangar sebanyak 1.4dB dan gandaan sebanyak 14dB daripada satu peringkat, penguat FET suapbalik. Kuantiti yang diukur adalah pekali pemantulan masukan, S_{11} ; gandaan, S_{21} ; angka hangar, NF; pemampatan P1dB dan titik pintasan masukan tertib-tiga, IIP3. Penguat Hingar Rendah tersebut mempunya fungsi suis pirau dalam cip dan juga IIP3 boleh ubah. Masukan MGA-72543 optimum sepadan secara dalaman untuk hingar rendah. Walaubagaimanapun, pepadanan pada masukan boleh ditambah secara luaran untuk nisbah gelombang pegun voltan yang rendah. Simulasi juga telah dilaksanakan. Walaubagaimanapun, hanya simulasi terhadap pekali pemantulan masukan, S_{11} ; gandaan, S_{21} dan hangar rendah, NF sahaja yang dapat dilakukan dengan menggunakan Penguat Hingar Rendah model S-parameter. Simulasi terhadap kelinearan memerlukan Penguat Hingar Rendah model isyarat besar. Model isyarat besar tidak terdapat dalam perisian ADS. Pencirian memberikan pemahaman yang mendalam kepada prestasi skim Penguat Hingar Rendah GaAs.

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LIST OF ABBREVIATION

PHEMT	Pseudomorphic High Electron Mobility Transistor
LNA	Low Noise Amplifier
NF	Noise Figure
IIP3	Input Third-Order Intercept Point
VSWR	Voltage Standing Wave Ratio
RF	Radio Frequency
PCB	Printed Circuit Board
MMIC	Monolithic Microwave Integrated Circuit
MESFET	Metal-Epitaxial-Semiconductor Field-Effect-Transistor
GPS	Global Positioning System
MRI	Magnetic-Resonance-Imaging
IM	Intermodulation
ADS	Advanced Design System
DC	Direct Current
DUT	Device Under Test
DUS	Device Under Simulation
ESD	Electrostatic Discharging

CHAPTER 1

INTRODUCTION

1.0 Introduction

The first stage of a receiver is typically a Low Noise Amplifier (LNA), whose main function is to provide enough gain to overcome the noise of subsequent stages (for example, in the mixer or IF amplifier). Aside from providing enough gain while adding as little noise as possible, an LNA should accommodate large signals without distortion, offer a large dynamic range and present a good matching to its input and output, which is extremely important is a passive band-select and image-reject filter precedes and succeeds the LNA, since the transfer characteristics of many filters are quite sensitive to the quality of the termination.

From the block diagram shown in Figure 1.0, the location of LNA in general communication can be seen. Solid-state microwave amplifiers play an important role in communication where it has different applications, including low noise, high gain, and high power amplifiers. The high gain and low noise amplifiers are small signal low power amplifiers and are mostly used in the receiver side where the signal level is low. The small signal S-parameter can be used in designing these low power amplifiers. The high power amplifier is used in the transmitter side where the signal should be at a high level to cross the desired distance [Al-Shahrani Saad, 2001].

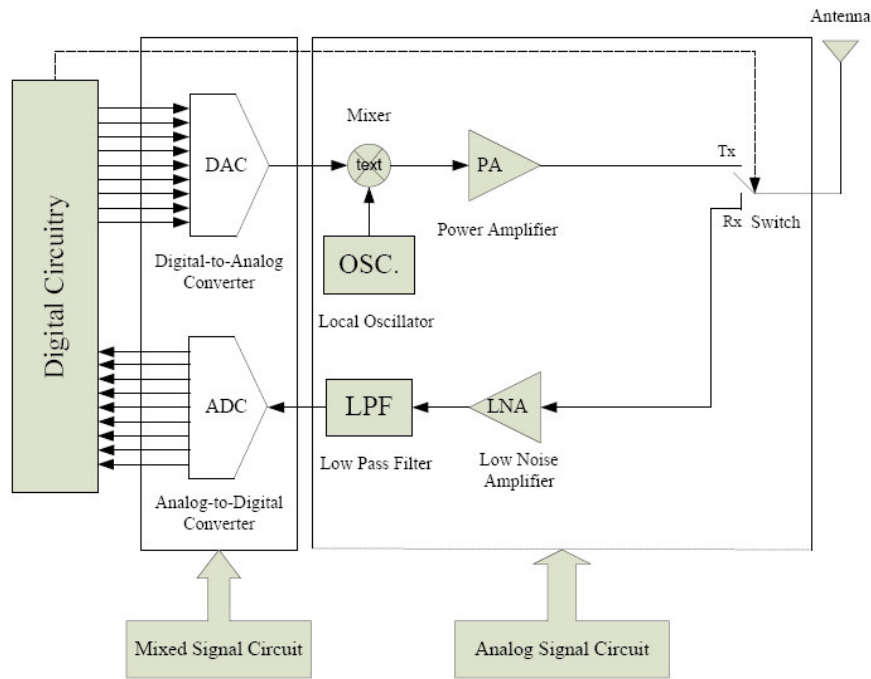


Figure 1.0: Block diagram of general communication system

Based on Figure 1.0, an overview of the operation of general communication system and the importance of a power amplifier can be shown. The following is the explanation on the operation of transmitter where the low noise amplifier is located.

The incoming energy (sound) will be converted to electrical energy in energy conversion stage by a signal conversion device, for example a microphone. Then, the electrical energy will flow into amplification stage. Here, high-gain small signal amplifier causes a higher signal to compensate for losses in the energy conversion stage.

After the amplification stage is the frequency conversion stage where carrier wave produced by local oscillator will be modulated with the signal that has been amplified at amplification stage.

After that, the modulated signal will flow into power amplification stage. This is the stage where the signal power level is boosted greatly so that a higher range of reception is allowed. The purpose of the power amplification stage is to compensate the signal being weakened after going through the different stages. Then, the modulated signal will be transported from 'cause or source point' to the 'effect or receipt point' in transmission link to receiver part.

Stages that include in receiver part are low-noise amplification stage, frequency conversion stage (or demodulation or down-conversion), detector stage, energy conversion stage and finally control stage.

The final stage is the stage where all the decisions with regard to circuit connection/disconnection, routing, switching and so on, take place. A control stage is present at both the source and the receipt points of the communication system [M.Radmanesh Matthew, 2001].

1.1 Background of Problem

The common problem in measuring the test circuit is the measured quantities is not exactly matching the data sheet specification. The circuit board material, passive components, RF bypasses and connectors all introduce losses and parasitic that degrades device performance. Furthermore, inadequate device grounding or poor printed circuit board (PCB) layout techniques could cause the device to be potentially unstable. It is important to capacitively bypass the connection to active bias circuits to ensure stable operation.

Components of insufficient quality for the frequency range of the amplifier can sometimes lead to instability. Component values that are also chosen to be much higher in value than is appropriate for the application can present a problem. In both of these cases, the components may have reactive parasitic that make their impedances very different than expected. Chip capacitors may have excessive inductance, or chip inductor can exhibit resonances at unexpected frequencies.

1.2 Objectives

In general, this research is conducted to measure the low noise amplifier and to seek and survey the mismatch between the actual performance as measured in the test circuit and the datasheet specification hence enabling development of solutions to its problem of instability.

There are several purposes to do this research. The first purpose to consider is to choose the model of the LNA. It is vital to choose the LNA where the fixed frequency

(i.e. 4GHz) is in its frequency range. Therefore, Agilent's MGA-72543 is chosen as the desired frequency, 4GHz is in its range of 0.1 to 6GHz.

The next purpose is to learn and understand the LNA by its characteristics provided in the datasheet as well as by simulate it using ADS2005A. It is important task before the test circuit is build.

The next point in doing this research is to build the test circuit. The test circuit is build base on the study and guide from the datasheet. There are many matters to consider before constructing a test circuit such as biasing, RF bypass, PCB materials, passive components and connectors. The circuit layout is created by using Protel.

The next stage of this project which is the last and the most important stage is to implement and characterize the test circuit. The measurement tools used to carry out this characterization is network analyzer and spectrum analyzer.

1.3 Scope

In this project, the LNA is implemented and characterized using the simulation and design software, ADS2005A and the measurement tools such as network analyzer and spectrum analyzer. The LNA model considered for this project is MGA-72543 provided by Agilent.

The quantities which are needed to be measured are input reflection coefficient, S_{11} ; forward transmission coefficient, S_{21} ; noise figure, NF; P1 dB compression and third order input intermodulation, IIP3. In order to match the input of the device with

the transmission line for low voltage standing wave ratio (VSWR), a single series inductor had been added at the input terminal. To tackle the biasing problem, the source bias method is applied.

1.4 Overview of Final Year Report

This report provides the step to design the test circuit of MGA-72543 and hence, measuring the device using particular equipment to measure particular quantities. In order to design the test circuit, application of biasing, applying the device voltage and RF bypassing is applied. The selection of printed circuit board and soldering technique are also taken into account. The format of this report will therefore follow the goals.

Chapter 2 discusses about semiconductor materials used in microwave devices. It also explains briefly about the GaAs MMIC Low Noise Amplifier. Beside that, the theory of all the quantities measured such as input reflection coefficient, S_{11} ; gain, S_{21} ; noise figure, NF; P_1 dB compression and input inter-order intermodulation intercept point, IIP3, also explained in detail in this chapter.

Chapter 3 contains design and measurement methodology that had been used in this project. Briefly introductory description about MGA-72543 and software simulation is available in this chapter. Mostly, this chapter explain the step by step from designing the test circuit until it was ready to be implemented on the printed circuit board as well as measuring the quantities needed for this project.

Chapter 4 discusses about the simulation and measurement data analysis. It also gives the reason why such measurement results gained.

Last but not least, conclusions from this project are presented in Chapter 5. Result of phenomenon was discussed based on the investigation conducted. Some recommendations for future research and the limitations occurred in this project are also available in this chapter.

CHAPTER 2

LITERATURE REVIEW

2.0 Introduction

In this chapter, the discussion will cover six main topics that need to be clarified as the idea and principles for this project. The topics include the semiconductor materials used in microwave devices, the brief explanation of GaAs MMIC Low Noise Amplifier. The detail explanation of quantities measured also included.

2.1 Semiconductor Materials Used In Microwave Devices

Whereas polymers are highly visible engineering materials with a major impact on contemporary society, semiconductors are relatively invisible but have a comparable social impact. Technology has clearly revolutionized society, but solid-state electronics is revolutionizing technology itself. A relatively small group of elements and compounds has an important electrical property, semi-conduction, in which they are neither good electrical conductors nor good electrical insulators. Instead, their ability to conduct electricity is intermediate. These materials are called semiconductors, and in general, they do not fit into any of the four structural materials categories based on atomic bonding. Metals are inherently good electrical conductors. Ceramics and polymers (non-metals) are generally poor conductors but good insulators. Silicon (Si) and germanium (Ge), widely used elemental semiconductors, are excellent examples of this class of materials. Another important semiconductor material is GaAs, a compound of the group IIIA element Ga; and the group V element, As.

Table 2.1 below shows the list of semiconductor materials using in microwave devices. As a GaAs MMIC LNA is being used in this project, the next chapter will explain about this type of LNA.

Si	- Silicon	Ge	- Germanium
GaP	- Gallium Phosphide	GaAs	- Gallium Arsenide
InAs	- Indium Arsenide	C	- Diamond
GaSb	- Gallium Antimonide	InSb	- Indium Antimonide
InP	- Indium Phosphide	GaAs _{1-x} Sb _x	- Gallium Arsenide Antimonide
Al _x Ga _{1-x} As	- Aluminium Gallium Arsenide	InN	- Indium Nitride
AlN	- Aluminium Nitride	GaN	- Gallium Nitride
BN	- Boron Nitride	Ga _x In _{1-x} P	- Gallium Indium Phosphide
Ga _x In _{1-x} As _y Sb _{1-y}	- Gallium Indium Arsenide Antimonide	Ga _x In _{1-x} Sb	- Gallium Indium Antimonide
Ga _x In _{1-x} As	- Gallium Indium Arsenide	Ga _x In _{1-x} As _y P _{1-y}	- Gallium Indium Arsenide Phosphide
InAs _{1-x} Sb _x	- Indium Arsenide Antimonide	SiC	- Silicon Carbide
Si _{1-x} Ge _x	- Silicon Germanium		

Table 2.1: Semiconductor materials used in microwave devices

2.2 GaAs MMIC Low Noise Amplifiers

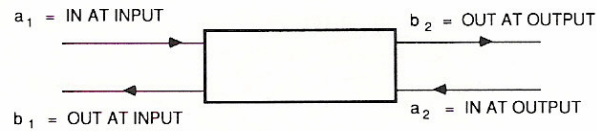
Low Noise Amplifiers (Lanes) for high-frequency applications have been based on Gallium Arsenide (GaAs) metal-epitaxial-semiconductor field-effect-transistor (MESFET) and depletion-mode pseudomorphic-high-electron-mobility-transistor (pHEMT) technologies for some time. It is primarily use in GPS receivers, Personal Communications Service, and other wireless RF applications. These Monolithic Microwave Integrated Circuits (MMICs) are small, broadband gain blocks in Surface

Mount packages; are relatively low in noise, and are intended to be used with 50 ohm input and output impedances.

The pHEMT technology provide a combination of high gain, low noise, and wide dynamic range in high-linearity LNA applications, such as intermediate-frequency (IF) amplifiers for commercial communication systems and preamplifiers for magnetic-resonance-imaging (MRI) systems. These types of applications have been made practical with the availability of low-cost plastic-packaged surface-mount pHEMT devices specifically designed for LNA applications.

2.3 S-Parameters

S-parameters are defined in Figure 2.2, which shows microwave signals entering and exiting a microwave component in both directions. If a microwave signals is incident on the input side of the component, some of the signal is reflected and some is transmitted through the component. The ratio of the reflected electric field to the incident electric field is the reflection coefficient. The ratio of the transmitted electric field to the incident electric field is the transmission coefficient.



$$S_{11} = \frac{b_1}{a_1} \quad \text{Input reflection coefficient}$$

$$S_{21} = \frac{b_2}{a_1} \quad \text{Gain/Loss}$$

$$S_{12} = \frac{b_1}{a_2} \quad \text{Isolation}$$

$$S_{22} = \frac{b_2}{a_2} \quad \text{Output reflection coefficient}$$

S-Parameters have amplitude and phase.

Figure 2.2: S-Parameter

The signals entering the input and leaving the input and output easily understood in terms of reflection and transmission coefficient. But how can there be an input signal at the output? The answer is that as the microwave signal travels down the output transmission line some of it is reflected at some point in the line and comes back into the component input.

To characterize the component completely, the reflection and transmission coefficients must be specified in both directions. In words, such expressions as the “input going into the output”, the “input going into the input”, or “the output coming out of the input”, must be used. S-parameter terminology is designed to avoid these cumbersome descriptions.

S-parameters are defined as follows. Microwave signals going into or coming out of the input port are labeled by a subscript 1. Signals going into or coming out of the output port are labeled by a subscript 2. The electric field of the microwave signal

going into the component ports are designated a; that leaving the ports are designated b.

Therefore,

a_1 is the electric field of the microwave signal entering the component input.

b_1 is the electric field of the microwave signal leaving the component input.

b_2 is the electric field of the microwave signal leaving the component output.

a_2 is the electric field of the microwave signal entering the component output.

By definition, then,

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}$$

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$

Therefore, S_{11} is the electric field leaving the input divided by the electric field entering the input, under the condition that no signal enters the output. Since b_1 and a_1 are the electric fields, their ratio is a reflection coefficient.

Similarly, S_{12} is the electric field leaving the output divided by the electric field entering the input, when no signal enters the output. Therefore, S_{21} is a transmission coefficient and is related to the insertion loss or the gain of the component.

In like manner, S_{21} is a transmission coefficient related to the isolation of the component and specifies how much power leaks back through the component in the

wrong direction. S_{22} is similar to S_{11} , but looks in the other direction into the component [W.S. Allen, 1993].

2.4 Basic Noise Theory

As a result of thermal agitation, the electrons have an inherent random motion which results in a random voltage appearing across the resistor terminals. The random voltage is referred to as noise. A noisy resistor can be modeled as a noise-free resistor in series with a noise voltage generator $e_n(t)$ or in shunt with a noise current source $i_n(t)$ as shown in Figure 2.3(a) and Figure 2.3(b).

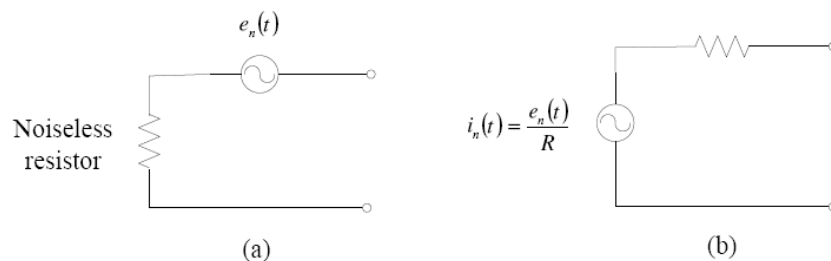


Figure 2.3: (a) Thevenin equivalent circuit which uses a noise voltage generator
(b) equivalent circuit for a noisy resistor in which a noise current source used

Noise is a random process and its effects in a linear system are analyzed using statistical methods. For this purpose an ensemble of macroscopically identical systems, e.g., an infinite number of resistors is constructed with each one producing its own noise voltage. Averages of various products of the noise voltages at different times, such as $e_n(t_1)$, $e_n(t_1)e_n(t_2)$, etc, are obtained by averaging over the ensemble of noise waveforms.

Thermal noise is generally regarded as a stationary ergodic noise process which is a random process for which ensemble averages can be replaced by time averages. Thus, the time-average value of the noise voltage is given by

$$\langle e_n(t) \rangle = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T e_n(t) dt = 0 \quad (2.1)$$

is zero. The correlation function for the noise voltage is the average value of the product of the noise voltage at the time t and that at a later time $t + \tau$; thus

$$\begin{aligned} C(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T e_n(t) e_n(t + \tau) dt \\ &= \langle e_n(t) e_n(t + \tau) \rangle \end{aligned} \quad (2.2)$$

where $C(\tau)$ is the correlation function. If $\tau = 0$, the average power $\langle e_n^2 \rangle$ associated with the noise is obtained. The average power in noise is distributed over a broad band of frequencies because noise voltage waveforms contain a broad spectrum of frequencies. The power spectral density $S_n(\omega)$ of noise is given by the Fourier transform of the correlation function. Thus,

$$S_n(\omega) = \int_{-\infty}^{\infty} C(\tau) e^{-j\omega\tau} d\tau \quad (2.3)$$

The inverse transform relationship is

$$C(\tau) = \int_{-\infty}^{\infty} S_n(\omega) e^{-j\omega\tau} \frac{d\omega}{2\pi} \quad (2.4)$$

The power spectral density represents the noise power in the spectral domain; so $S_n(\omega)\Delta f$ is the noise power in a frequency increment Δf .

At room temperature, the power spectral density of thermal noise is constant up to frequencies of the order of 1000 GHz and decreases at higher frequencies. Thus, at microwave frequencies and below, the spectral density is assumed as a constant or flat. This is equivalent to having a correlation function that is a constant multiplying the delta function $\delta(\tau)$, that is,

$$C(\tau) = C_0 \delta(\tau)$$

since the Fourier transform of $\delta(\tau)$ is a constant equal to unity.

The power spectral density is an even function of ω ; so, the spectral density can be chosen such that only positive values of ω need to be considered. For thermal noise in a resistor, the power spectral density for the noise voltage is given by Nyquist's formula

$$S_e(\omega) = 4kTR \quad \omega = 0 \quad (2.5)$$

where $k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant and T is the absolute temperature of the resistor R. Thus, the amount of noise power P_n in a frequency interval Δf is given by

$$P_n = 4kTR\Delta f \quad (2.6)$$

If the equivalent current source model shown in Figure 2.3(b) is used, then the average power if the current $i_n(t)$ flows in a 1Ω resistor is given by $\langle i_n^2(t) \rangle$ and has a power spectral density given by

$$S_i(\omega) = \frac{4kT}{R} \quad \omega \geq 0 \quad (2.7)$$

for thermal noise [E.C. Robert., 1992].

2.4.1 Filtered Noise

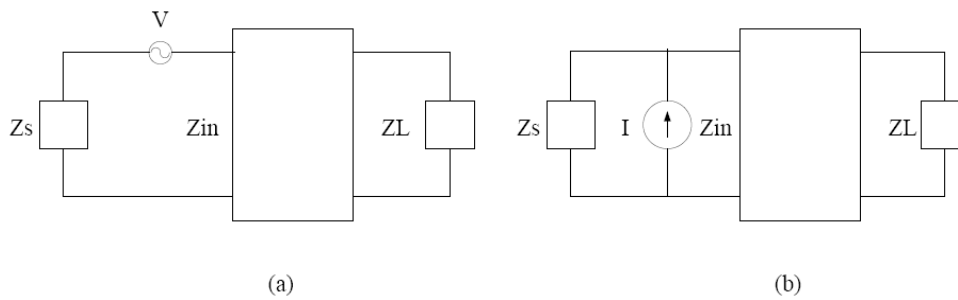


Figure 2.4: A two-port network connected to (a) a voltage source (b) a current source

Consider a sinusoidal voltage generator with a complex rms voltage V that is connected in series with a source impedance Z_s and a two-port network as shown in Figure 2.4(a). The input current produced by V is $V/(Z_s + Z_{in})$, where Z_{in} is the input impedance to the network. The input power produced by V is given by

$$P_{in,1} = \left| \frac{V}{Z_s + Z_{in}} \right|^2 R_{in} = \frac{|V|^2}{4R_s} \frac{4R_s R_{in}}{|Z_s + Z_{in}|^2}$$

$$= \frac{|V|^2}{4R_s} M \quad (2.8)$$

where $|V|^2 / 4R_s$ is the available power and M is the impedance-mismatch factor. The power transfer function is $M / 4R_s$ and is a function of ω since Z_s and Z_{in} are functions of ω . If the voltage generator is replaced by a noise voltage source $e_n(t)$ with a power spectral density $S_e(\omega)$, then the input noise power in a frequency band Δf centred on ω is given by the source power spectral density by the power transfer function and the factor Δf ; thus

$$P_{in,e}(\omega) = S_e(\omega) \frac{M(\omega)}{4R_s} \Delta f \quad (2.9)$$

The total input noise power is obtained by integrating over all frequencies. Thus,

$$P_{nT} = \int_0^\infty S_e(\omega) \frac{M(\omega)}{4R_s} \frac{d\omega}{2\pi} \quad (2.10)$$

Consider next the circuit shown in Figure 2.4(b). The input current from the current source is $Z_s I / (Z_s + Z_{in})$ and the input power is

$$\begin{aligned} P_{in,2} &= \left| \frac{IZ_s}{Z_s + Z_{in}} \right|^2 R_{in} = \frac{|I|^2 |Z_s|^2}{4R_s} \frac{4R_s R_{in}}{|Z_s + Z_{in}|^2} \\ &= \frac{|I|^2}{4G_s} M \end{aligned} \quad (2.11)$$

where $G_s = \text{Re}(1/Z_s) = R_s / |Z_s|^2$. If the sinusoidal current source is replaced by a noise current source $i_n(t)$ with power spectral density $S_i(\omega)$, then the input noise power in a frequency band Δf is given by

$$P_{in,i}(\omega) = S_i(\omega) \frac{M(\omega)}{4G_s} \Delta f \quad (2.12)$$

The power spectral density of the noise power delivered to Z_L would, by analogy, be given by the product of the power spectral density of the source and the power transfer function from the source to the output load impedance. Since the power delivered to Z_L can be described by the product of P_{in} with the power gain $G_p(\omega)$, the noise current source will produce an output power in Z_L with a spectral density given by $G_p(\omega)M(\omega)S_i(\omega)/4G_s$. A similar expression holds for the output power spectral density produced by the noise voltage source $e_n(t)$ acting alone.

When both sources V and I are acting, the input current will be

$$I_{in} = \frac{V + IZ_s}{Z_s + Z_{in}}$$

and the input power will be given by

$$\begin{aligned} P_{in,3} &= \frac{(V + IZ_s)(V^* + I^*Z_s^*)}{|Z_s + Z_{in}|^2} R_{in} \\ &= \frac{|V|^2}{4R_s} M + \frac{|I|^2}{4G_s} M + 2\text{Re} \frac{VI^*Z_s^* R_{in}}{|Z_s + Z_{in}|^2} \end{aligned} \quad (2.13)$$

Because of interaction between the two sources, the input power is not simply the sum of that from each source acting independently.

When two noise sources $e_n(t)$ and $i_n(t)$ acting simultaneously, there is no input power caused by the interaction between $e_n(t)$ and $i_n(t)$ when $e_n(t)$ and $i_n(t)$ are uncorrelated or statistically independent noise sources. For this case the input noise power in a frequency band Δf is the sum that given by (2.9) and (2.12). When there is a degree of correlation between $e_n(t)$ and $i_n(t)$, there will be some input noise power due to the source interaction.

The cross-correlation between the current source $i_n(t)$ and voltage source $e_n(t)$ is given by

$$C_x(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T i_n(t) e_n(t + \tau) dt \quad (2.14a)$$

The Fourier transform of $C_x(\tau)$ gives the cross-power spectral density $S_x(\omega)$, that is,

$$S_x(\omega) = S_{xv}(\omega) + jS_{xi}(\omega) = \int_{-\infty}^{\infty} C_x(\tau) e^{-j\omega\tau} d\tau \quad (2.14b)$$

If ω is replaced by $-\omega$, $S_x(-\omega) = S_x^*(\omega)$ since $C_x(\tau)$ is real. From this result, $S_{xv}(\omega)$ is an even function of ω and $S_{xi}(\omega)$ is an odd function of ω . For input noise power calculation, $|V|^2$ is replaced by the noise voltage source power spectral density $S_e(\omega)$, replaced $|I|^2$ by the power spectral density $S_i(\omega)$ of $i_n(t)$, and replaced VI^* by

the cross-power spectral density $S_x(\omega)$ in equation (2.13). Thus, for partially correlated noise sources, the total input noise in a frequency band Δf is given by

$$P_n = \Delta f \left\{ S_e(\omega) \frac{M}{4R_s} + S_i(\omega) \frac{M}{4G_s} + \frac{4[S_{xr}(\omega)R_s + S_{xi}(\omega)X_s]}{|Z_s + Z_{in}|^2} R_{in} \right\} \quad (2.15)$$

The extra factor of 2 in the last term is due to the fact that the contribution has been combined from negative values of ω with those from positive values of ω using the fact that $S_{xr}R_s$ and $S_{xi}X_s$ are even function of ω . The spectral densities $S_e(\omega)$ and $S_i(\omega)$ has been defined so that only positive values of ω are to be integrated over to get the total input noise power.

The power spectral density of the noise produced in a network differs from that of the noise source because the network respond depends on frequency. The source noise spectrum is filtered by the network. If the source noise spectrum is flat (white noise), the noise spectrum produced at the same point in the network is not flat. Noise with a nonconstant power spectral density is called colored noise.

From (2.15), the power spectral density of the noise power delivered to Z_L is

$$S(\omega) = \left\{ \frac{M}{4R_s} S_e(\omega) + \frac{M}{4G_s} S_i(\omega) + \left[S_{xr}(\omega) + \frac{X_s}{R_s} S_{xi}(\omega) \right] M \right\} G_p(\omega) \quad (2.16)$$

after multiplying by the power gain G_p of the two-port network. The total output noise power delivered to Z_L is

$$P_{n,out} = \int_0^\infty S(\omega) \frac{d\omega}{2\pi} \quad (2.17)$$

[Collin Robert E., 1992]

2.4.2 Noisy Two-Port Network

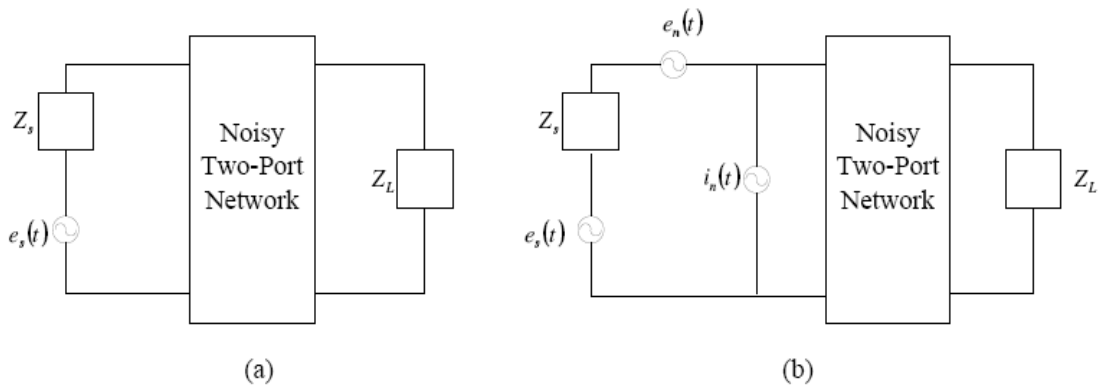


Figure 2.5: Equivalent input noise sources for a noisy linear two-port network

In analyzing the noise produced at the output of a linear two-port network due to the internal noise sources, all the internal noise sources can be replaced by a series noise voltage generator $e_n(t)$ and a shunt series noise current $i_n(t)$ at the input as shown Figure 2.5. The total noise power at the output can be found by evaluating the noise output produced by $e_n(t)$, $i_n(t)$ and the thermal noise in the resistive component R_s of the source impedance. Two equivalent noise sources are needed at the input because if the input is short-circuited, that is $Z_s = 0$, the source $i_n(t)$ does not produce any output noise, yet the noisy two-port does have a noise output under short-circuit conditions at the input so the noise voltage source $e_n(t)$ is required. Similarly, under open-circuit conditions, $e_n(t)$ does not produce any output noise, so a noise source $i_n(t)$ is needed to represent the equivalent input noise source under open-circuit conditions. The two noise sources $e_n(t)$ and $i_n(t)$ are not completely independent since a part of $e_n(t)$ and

$i_n(t)$ may arise from the same basic noise-producing mechanism within the two-port network. Thus, in general, there is some cross-correlation between $e_n(t)$ and $i_n(t)$ with a resultant nonzero cross-power spectral density.

It is common practice to express the power spectral density associated with the two noise sources $e_n(t)$ and $i_n(t)$ in a form similar to that given by (2.5) and (2.7) for thermal noise. When this is done, flicker noise which is low frequency noise with a $1/f$ spectrum, is not included. Since linear microwave amplifiers do not produce an output for low frequency inputs signals, the neglect of the flicker noise. Spectral densities will be specified as follows;

$$\text{For } e_n(t), \quad S_e(\omega) = 4kTR_e$$

$$\text{For } i_n(t), \quad S_i(\omega) = 4kTG_i$$

and
$$2[S_{x_r}(\omega) + jS_{x_i}(\omega)] = 4kT(\gamma_r + j\gamma_i)$$

where R_e is an equivalent noise reactance, G_i is an equivalent noise conductance and $\gamma_r + j\gamma_i$ is a complex equivalent noise impedance. A total of four parameters, R_e , G_i , γ_r and γ_i needed to describe the noise properties of a noisy two-port network.

In terms of the above spectral densities, the total noise input to the noise-free two-port network can be express in a frequency band Δf as follows by using (2.15);

$$P_{n,in} = kT\Delta f M + kT\Delta f \frac{R_e}{R_s} M + kT\Delta f \frac{G_i}{G_s} M + 2kT\Delta f \frac{R_s\gamma_r + X_s\gamma_i}{R_s} M \quad (2.18)$$

In this equation the first term on the right, $kT\Delta fM$ is the input thermal noise from the source resistance R_s . The output noise in Z_L in a frequency band Δf is obtained by multiplying the power gain $G_p(\omega)$ of the network. The noise produced at the output termination by the equivalent sources $e_n(t)$ and $i_n(t)$ placed at the input of the network is fully equivalent to that produced by the internal noise mechanism in the real noisy two-port network [Collin Robert E., 1992].

2.4.3 Noise Figure

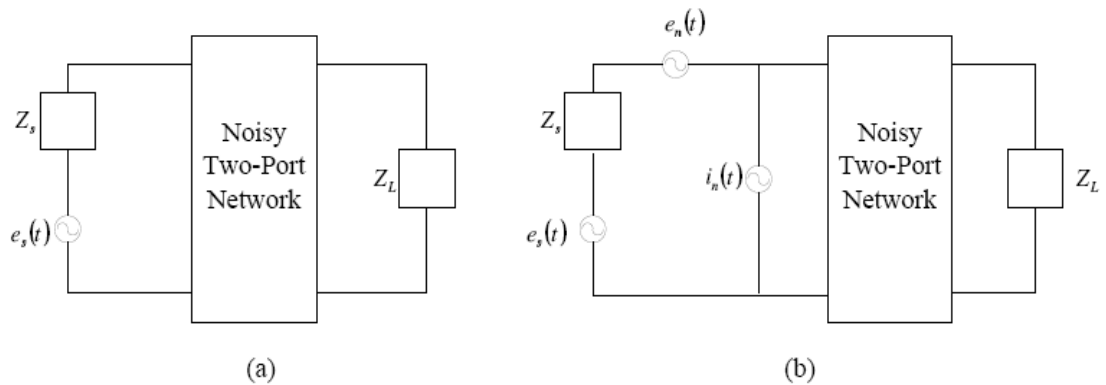


Figure 2.6: Equivalent input noise sources for a noisy linear two-port network

With reference to the circuit shown in Figure 2.6(a), the definition of noise figure (also called noise factor) is

$$F = \frac{SNR_{in}}{SNR_{out}} \quad (2.19)$$

where SNR_{in} is signal-to-noise ratio at input and SNR_{out} is signal-to-noise ratio at output. The output noise is the amplified thermal noise from the source resistance plus the noise produced by the amplifier. The standard definition of F requires the source to

be conjugate impedance matched to the network, that is, $Z_s = Z_{in}^*$ and the source resistance R_s to be at the standard temperature $T_o = 290K$. Very often in practice the source resistance is at a different temperature and the source is not matched to the network. In this case the definition (2.19) gives the operating noise figure. If F is given at a single frequency and is based on the noise power in a small frequency band, Δf then the noise figure is called the spot noise figure. When all of the noise sources are referred to the input as equivalent noise sources, then the spot noise figure can be defined as follows;

$$F = \frac{\text{total input noise power to network}}{\text{thermal noise input power from noise resistance}}$$

$$F = \frac{P_{n,in}}{kT\Delta fM}$$

$$F = \frac{kT\Delta fM + kT\Delta f \frac{R_e}{R_s} M + kT\Delta f \frac{G_i}{G_s} M + 2kT\Delta f \frac{R_s \gamma_r + X_s \gamma_i}{R_s} M}{kT\Delta fM} \quad (2.20)$$

where the noise powers are those in a narrow frequency band Δf . Based on this latter definition, the spot noise figure for the system shown in Figure 2.5 maybe obtained from (2.18) by dividing by $kT\Delta fM$. Thus;

$$F = 1 + \frac{R_e}{R_s} + \frac{G_i}{G_s} + 2 \frac{R_s \gamma_r + X_s \gamma_i}{R_s} \quad (2.21)$$

The noise figure is seen to depend on the source impedance as well as on the noise parameters R_e , G_i , γ_r and γ_i . In (2.21), $G_s = R_s / (R_s^2 + X_s^2)$. The noise figure does