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A Genetic Algorithm Approach For Design Optimization Of Single-Phase Rectifier Transformer

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Abstract

This paper presents a design procedure for optimal design of a semi-conductor rectifier transformer by Genetic Algorithm. Unlike a conventional transformer, the VA rating of a rectifier transformer and estimation of copper losses is modified to account for the effect of non-sinusoidal input current. The transformer design analysis program is combined with the Genetic Algorithm to optimize objective functions such as active materials cost and weight subjected to constraints. The optimal results and design parameters are satisfactory and demonstrated the efficiency of optimization procedure developed in this paper.

Key words: semi-conductor rectifier transformers - non-sinusoidal currents – Genetic Algorithm - design optimization.

Introduction

A small rectifier transformer supplying a dc load through a diode bridge rectifier is shown in Figure 1. As the rectifier is the source of harmonics, the design of such a transformer need to consider the effect of harmonics and the volt-ampere rating is to be specified as rms VA or fundamental VA [1].

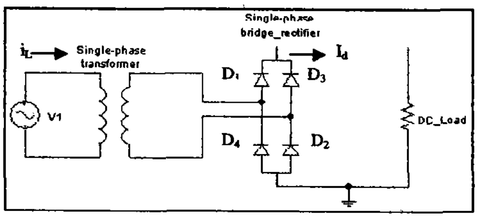

Figure - 1 Transformer with DC Load

The input line current of the transformer is non-sinusoidal and is also affected by the source impedance. The source inductance distorts the current waveform and introduces an overlap period. During this period all the diodes conduct together at the same time and is represented by the overlap angle, u. An approximate current waveform as shown in Figure 2(a) and the actual waveform as shown in Figure 2(b) are drawn, neglecting and considering the input source inductance, respectively. The design of large dry-type and liquid filled transformers for non-sinusoidal load currents, analysis of the eddy loss in windings are reviewed in [1,2]. Optimal design of rectifier power transformer by non-linear programming technique [3] and an improved Genetic Algorithm approach for optimum design of conventional power transformers is presented in [4]. In this paper a design optimization procedure for a small rectifier transformer by Genetic Algorithm is presented.

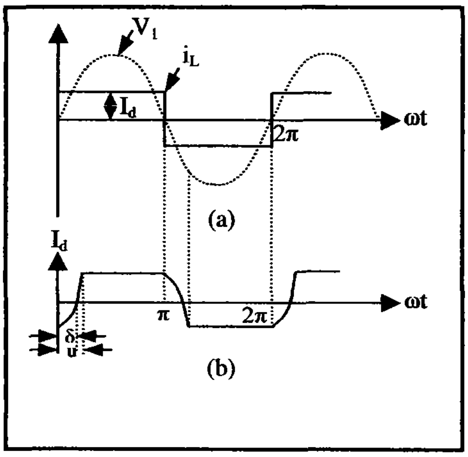

Figure - 2 (a) Approximate Waveform (b) Actual Waveform

Based on the nature of commutation of the diodes,

$$\delta = \cos^{-1}\left(\frac{1+\cos u}{2}\right) \tag{1}$$

Neglecting the source inductance, the transformer secondary voltage and current are expressed as

secondary voltage, $V_2 = \dfrac{V_d \pi}{2\sqrt{2}}$ (2)

secondary rms current, $I_{2,rms} = I_d$ (3)

where V_d, I_d are dc load voltage and current, respectively.

It is observed that rms current, $I_{2,rms}$ calculated from the waveform of Figure 2(b) is more effective in the estimation of VA rating, temperature rise and eddy current losses in the conductors. Accordingly, in this paper the volt-ampere rating of the transformer is fixed from the rms values of voltage and current for a given dc load as in Table - 1.

Table – 1 Specification of Rectifier Transformer

1-phase 500 VA, 240 V/ 30 V, 50 Hz, core type

Losses in The Transformer

Transformer losses consist of core loss, I^2R loss in windings and additional eddy current loss in conductors due to harmonics.

Core loss

The core loss depends on the peak value of the sinusoidal core flux. For non-sinusoidal line current the primary voltage waveform is assumed to be sinusoidal. As long as the transformer voltages remain within a few percent of rated value, the core loss is essentially constant, and is estimated as [3]

Core loss, $P_i = FF(B_c)G_l + 1.075\ FF(B_y)G_y$ (4)

where

B_c, B_y = flux density in the core and yoke, Tesla

G_l, G_y = weight of the limbs and yoke, kg

$FF()$ = a polynomial function of the flux density derived from B-H curve of the material

Additional Eddy Current Loss

As the current is non-sinusoidal, additional eddy current losses in the windings are estimated as [ ]

Eddy current loss, $P_e = \sum K_{e,pul} I^2 R$ (5)

where $K_{e,pul} = (\frac{5m^2-1}{45}\frac{h_{ca}^2}{h_w^2}\frac{BO^4}{4})(\frac{I'}{I})^2(\frac{\mu_0^2}{\rho^2})$ (6)

$$\left(\frac{I'}{I}\right)^2 = \omega^2 = (2\pi f)^2$$

m = number of parallel conductors in radial direction

BO = radial dimension of the conductor, mm

= diameter of conductor, d_c

f = frequency of supply

μ_0 = magnetic space constant, 4π x 10^{-7} H/m

ρ = conductivity of the material, ohm-m/mm^2

h_{ca} = height of copper in axial direction, m

h_w = height of winding, m

I = rms current, A

I' = time derivative of rms current, A

Eddy current losses are estimated for primary and secondary windings from Equation (6).

Total Copper Loss

The total copper loss is the sum of the primary and secondary I^2R losses and additional eddy current loss

Copper loss, $P_c = I_1^2 R_1 + I_2^2 R_2 + P_e$ (7)

Temperature Rise

The temperature rise of the windings is estimated from the total copper loss and exposed surface area of the windings.

Temperature rise, $\Delta\theta = \theta_{max} - \theta_a = P_c R_{\omega a}$ (8)

where

$R_{\omega a} = \frac{1}{\lambda_\omega A_s}$ = winding ambient thermal resistance (9)

λ_ω = winding heat transfer coefficient, watts/m^2

$A_s = \pi D_s h_w$ = Area of open surface of the winding (10)

D_s = diameter of the outer surface cylinder, m

$= d + 2b_2 + 2b_1 + k$ (11)

d = core diameter, m

b_1, b_2 = width of lv and hv windings, m

k = constant

Optimization Procedure with GA

GA operates on a population of points in the search space and offers a convenient way of handling constraints and single or multi objective functions of a design problem. In GA approach, each design variable is represented as a binary string (chromosome) of fixed length and the search operations are typically selection, crossover, and mutation. An index of merit is assigned to each chromosome and is evaluated by using a fitness function. GA provides solutions by generating a set of chromosomes referred to as a generation. A new generation is selected having largest probability of the fittest individual. Generations are created from pairs of chromosomes as parents with reproduction operators, mutation and crossover. If the search has to continue, the GA creates a new generation from the old one until a decision is made on the convergence. A number of evolutions of the fitness function is necessary to arrive at a final optimal solution. In this paper the selection of parameters to successive generation is achieved by tournament selection strategy. In binary tournament selection two individuals are selected at random from the population, and the better one is duplicated in the next generation. This process is repeated until the individuals reach a specified population size. A crossover operator exchanges information contained in two parent individuals to produce two offspring and then replace the parents. The number of times the crossover operator is applied to the population is determined by the probability of crossover and the population size. The mutation operator randomly selects an individual from the population and then chooses two elements in this individual to exchange positions [5]. The entire process of evaluation and reproduction then continues until either the population converges to an optimal solution for the problem or the Genetic Algorithm has been run for a specific number of generations. The genetic parameters of Table - 2 play an important role in GA to provide a global optimal solution.

Table - 2 GA Parameter Settings

Chromosome Size	16 bits
Population size	100
Selection Process	Tournament
Crossover rate	0.8867 or 88.67%
Mutation rate	0.044 or 4.4%
Genome mapping	According to constraints

In this paper design variables and constraints are selected as in Table - 3, considering the effect on core dimensions, losses, short-circuit reactance, weight, volume and finally on the cost of the transformer.

Table – 3 Design Variables

x_1	Maximum flux density in core, T
x_2	Current density in HV winding, A/mm^2
x_3	Current density in LV winding, A/mm^2
x_4	Height of windings, m
x_5	Voltage per turn, V
x_6	Distance between core centers, m
x_7	Clearance between windings, m

The fitness function of GA is the objective function of the design problem. A mathematical model in terms of design variables for objective and constraint functions explained in the following sections is developed for the specification in Table 1.

Design Constraints

Two constraints i.e. temperature rise and percentage efficiency are imposed on the design problem.

1. Percentage efficiency

$$eff \geq 95$$

$$eff = \left(1 - \frac{P_c + P_i}{S.pf + P_c + P_i}\right) \times 100 \tag{12}$$

where S = VA rating of the transformer

pf = power factor of the transformer

2. Temperature rise of winding above ambient

$$\Delta\theta \leq 50^o c$$

Objective Function

The objective function or fitness function is the minimum active material cost of the transformer i.e. cost of stampings and cost of windings. Insulating materials cost, manufacturing cost etc. are not included in the objective function as there may be wide variation with different manufacturers.

Objective function, $F = c_1(G_l + G_y) + c_2 G_c$ (13)

where c_1, c_2 = cost per kg of iron and copper, RM/kg

G_l, G_y, G_c = weight of limb, yoke and copper, kg

The constrained optimum design problem of rectifier transformer is formulated as

Find $X = (x_1, x_2 .... x_n)$, design variables

Minimizing $F(X)$

Subject to $g_j(X) \{\leq\} 0, \quad j = 1, 2, ... m$ (14)

where $F(X)$, the nonlinear objective function

$g_j(X)$, the nonlinear constraint functions

The objective function together with the constraints is converted into an augmented objective function to suitably adopt GA to a constrained design optimization problem as follows:

Augmented objective function,

$$FA = F + \gamma_k \sum_{j=0}^{n} g_j^{\,2} \qquad j = 1,2,...... m \tag{15}$$

where γ_k = penalty parameter and g_j = constraint functions

The design equations, objective and constraint functions are expressed in terms of the selected design variables.

Results and Discussion

The performance results of rectifier transformer design with GA approach are found to be quite satisfactory. Optimal design parameters, minimizing the objective function together with the constraints are derived for a set of GA parameters and penalty factors. The GA technique with a 16-bit chromosome has been tested for several combinations of GA parameters and penalty factors. A comparison is made on different choices and finally the probability of mutation is set at 0.044 and the probability of crossover at 0.866. For a population size of 100, the cost objective function is minimized over number of generations. It is observed from Figure - 3 that the optimal objective function of cost converged to a minimum for a number of generations above 300. In all the cases the design analysis program together with GA has taken about 5 to 8 seconds, to arrive at an optimal solution.

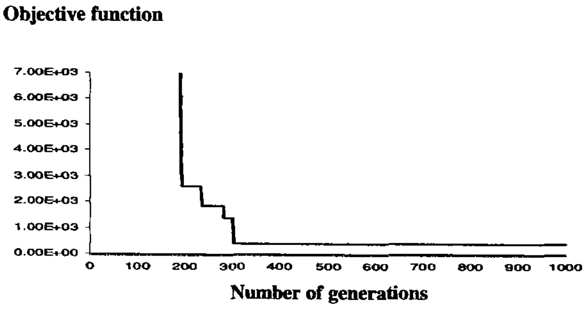

Figure – 3 Minimized Cost Objective Function

The program is run with different values of penalty factors and Table 4 shows the effect on some of the design values. The constraints are observed to be within the specified limits for all optimal solutions.

Table 4 – Effect of Penalty Factor on Design Values

Penalty factor	Cost Objective function	Weight of limbs	Weight of yoke	Weight of copper	Copper loss	Core loss
1.0E+05	405.10	8.26	9.18	3.74	7.05	19.26
1.0E+06	397.73	8.15	8.99	3.65	7.07	19.23
1.0E+07	398.58	8.25	8.90	3.70	6.88	19.43
1.0E+08	410.30	8.54	9.01	3.94	6.72	19.59
1.0E+09	389.20	7.93	8.86	3.54	7.16	19.15
1.0E+10	389.83	7.74	9.18	3.41	7.55	18.76

For a set of GA parameters and a penalty factor for constraints, the performance results of optimization of a rectifier transformer as reported in Tables 5 and 6 are satisfactory.

Table -5 Optimal Values

Design Variable	
x_1 Maximum flux density in core, T	1.21792
x_2 Current density in HV winding, A/mm^2	2.35232
x_3 Current density in LV winding, A/mm^2	2.27671
x_4 Height of windings, m	0.13475
x_5 Voltage per turn, V	1.00191
x_6 Distance between core centers, m	0.08233
x_7 Clearance between windings, m	0.01298
Constraints:	
% Efficiency at full load	95.005
Temperature rise, °C	49.999

Table – 6 Design Parameters

Core diameter, m	0.08577
Core length, m	0.14279
Core width, m	0.15953
Turns ratio	240/30
Primary current, A	2.08333
Secondary current, A	16.6667
Conductor diameter - primary, mm	1.08667
Conductor diameter - secondary, mm	3.12421

Conclusions

The volt-ampere rating of a rectifier transformer and estimation of copper losses is based on non-sinusoidal input current when the transformer is connected to a dc load. This paper has proposed a general procedure for design optimization of such a rectifier transformer by Genetic Algorithm. Design parameters and optimal objective function derived by this non-linear programming technique demonstrated the suitability of GA for rectifier transformer design.

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