Design Optimization of a High Frequency Power Transformer for a Switching Power Supply by Genetic Algorithms Approach

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Abstract – **The design optimization of a single-phase high frequency power transformer used in switching power supplies is presented in this paper. The objective functions of the transformer are minimum volume, minimum weight and minimum cost with optimum values of several design variables. Non-Linear Programming (NLP) technique together with Genetic Algorithms (GAs) approach has been applied to develop a design optimization program. In addition, the design optimization procedure and optimal values are presented. A C++ program has been successfully developed based on the GAs by using the GAs library.**

I. Nomenclature

B = flux density in core, Tesla
B2 = thickness of foil conductor in secondary winding, m
C_1, C_2 = cost per kg of ferrite and copper, RM/kg
C_{tot} = total cost of materials, RM
d3 = diameter of core center leg, m
dc1 = diameter of round conductor in primary winding, m
f = frequency, Hz
G_{lm}, G_{yo}, G_{wg} = weight of limbs, yoke, windings, kg
G_{core} = weight of core, kg
G_{tot} = total weight of transformer, kg
hc = conductor thickness, m
, I_s = current in primary and secondary winding respectively, A
L = length of center leg / limb, m
L_{mt1}, L_{mt2}= length of mean turn of primary and secondary winding respectively, m
m_1, m_2 = number of parallel conductors
n_c = number of coils in parallel in low voltage winding
P_{ct} = total core loss, W
P_c = corer loss density, KW/m^3
P_{cop} = winding copper loss, W
P_f = power factor
P_{wg} = total winding coil losses, W
R_p = primary resistance, ohms
R_s = secondary resistance, ohms
R_T = thermal resistance, °C/Watt
R_I = Internal thermal resistance, °C/Watt
R_E = External thermal resistance, °C/Watt
r_s = density of ferrite material assumed to be 4850 kg/m^3
S = rating of transformer, VA
T_1, T_2 = number of turns in primary and secondary winding respectively
V_{in} = power supply input voltage, V
V_{core} = core volume, m^3
V_{lm} = limb volume, m^3
V_{yo} = yoke volume, m^3
V_{wg} = winding volume, m^3
V_{pw} = primary winding volume, m^3
V_{sw} = secondary winding volume, m^3
V_{tot} = total volume of transformer, m^3
V_p, V_s = terminal voltage of primary and secondary windings respectively, V
η = % transformer efficiency
σ_p, σ_p = resistivity of primary and secondary winding material respectively, ohm-m
ρ = effective resistance of ferrite
δ = skin depth at 100°C
ΔB = flux density swing, Tesla material respectively, ohm-m

II. Introduction

Evolutionary programming by Genetic Algorithms (GAs) has been successfully applied to optimal design problems [1,2]. With the rapid development of power electronic technology, the operational switching frequency in power electronic systems, such as the Switched Mode Power Supply (SMPS), have been extended to the mega-hertz region. Transformers are the largest and heaviest components in SMPSs and accounts for about 25% of the overall volume and more than 30% of the overall weight. A typical high frequency transformer used in SMPS [3] is shown in Fig. 1.

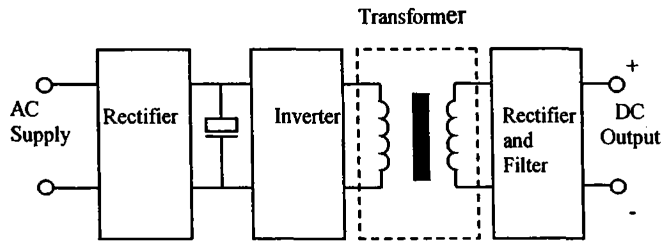

Fig. 1 Block diagram of a typical SMPS

Minimizing volume, weight and cost of a transformer is emphasized for the majority of power supply designs [4]. In this paper, the design optimization of a single-phase high frequency transformer will be discussed by using Genetic Algorithms (GAs) approach. A C++ program has been successfully developed based on the GAs by using the GAs library [5]. This GAs library is a C++ library that contains tools and built-in components for using GAs to minimize the fitness function or objective function. In general, the objective and constraint functions for an engineering optimization problem are highly nonlinear. Therefore NLP technique will be implemented before applying GAs approach. The design procedure includes design considerations, design methodology and discussion on the results.

III. Design Considerations

Switching power supplies have become popular because of their ability to operate at high frequencies, and hence increasing their efficiency. A switching power supply that satisfies the same performance requirements of a linear power supply can be many times smaller in size. Since the induced voltage in a transformer is dependent upon the changing magnetic flux, the more we change the flux (higher frequency), the smaller and more efficient the transformer becomes.

Some design considerations of a high frequency power transformer are actually due to:

i) Selection of topology
ii) Selection of transformer core – material, shape/geometry, size
iii) Consideration of peak flux density
iv) Core loss determination
v) Winding coil loss determination
vi) Temperature rise determination

The selection of topology will depend on the applications and areas of usage. The topology will be selected to minimize the power transistor's off-voltage stress at high input voltage, and peak current stress at maximum output power. Generally, ferrite cores are best suited for high frequency and steel laminations are beat suited for low frequency applications. With higher frequencies, core material selection is guided by core loss considerations. Ferrites are commonly used because of their high electrical resistivity that minimizes eddy current losses. Ferrite cores are manufactured in a relatively small number of geometric shapes and varying dimensions within these shapes. There is a limitation to peak flux density in the ferrite cores. At higher frequencies (> 50 kHz) the peak flux density will have to be reduced to such a value that total core and copper losses result in an acceptable low temperature rise.

Core losses contribute to the temperature rise of a transformer. Hysteresis loss, eddy current loss and residual loss all form to the total core loss. In this paper the manufacturer's data sheet curves are approximated with an analytical polynomial expression and core loss are determined. Winding coil losses contribute to a transformer's total loss. Additional winding coil losses are due to skin effect, proximity effect, effects from fringing flux intersecting windings near the core gap, edge effects and extraneous conductor effects.

The heat dissipation is dependent upon the total exposed surface area of the core and of the total exposed surface area of the windings. Temperature rise also depends upon the thermal resistance, R_T (°C/Watt), from the external ambient to the central hot spot. Thermal resistance has two main components - internal thermal resistance, R_I between the heat sources (core and windings) and the transformer surface, and the external thermal resistance, R_E from the surface to the external ambient. Temperature rise can be estimated by multiplying thermal resistance with total power losses.

IV. GA Optimization Approach

A genetic algorithm (GA) is a problem solving method that uses genetics as a model of problem solving. It applies the rules of reproduction, gene crossover, and mutation so that those organisms can pass beneficial and survival-enhancing traits to new generations. Mainly three items are necessary to adopt a genetic algorithm approach:

1) Define a representation
2) Define the genetic operators
3) Define the objective function

The first two items are provided by GAs library . The built-in representation and operators in this library will be used with no modification. It is only necessary to represent the design variables in genome with upper and lower bounds and define the nonlinear objective function together with the nonlinear constraints functions.

V. Design Methodology

The main objective of this paper is to design an optimum high frequency power transformer used in switching power supply. The design considerations as well as optimization approach with genetic algorithms are presented. A computer program in C++ language that can find optimum solution to the high frequency transformer for switching power supply is developed. The procedure is as follows:

1) Select specification of transformer.
2) Define design variables.
3) Define objective functions in mathematical model.
4) Define constraint functions in mathematical model.
5) Use Nonlinear Programming (NLP) technique to reform objective function.
6) Code all mathematical models in C++ language.
7) Develop the design optimization program.

The first step is to select the power supply specification pertaining to the transformer design as in Table 1.

Table 1: Power supply specification

Parameters	Specification
Vin Range	100 V
Output	5 V, 50A
Circuit Topology	Forward Converter
Switching Frequency	200 kHz
Transformer Frequency	200 kHz
Cooling Method	Natural Convection
Transformer Efficiency	> 99 %
Permissible temperature rise in transformer	< 55°C

The formulation of the design problem as an NLP problem involves the objective and constraints functions in terms of the design variables. Five design variables are selected for the optimization problem and the choice is based on significant effect on the weight, losses, core dimensions and cost of the transformer.

i) Flux density in core, x1
ii) Current density in high voltage winding, x2
iii) Current density in low voltage winding, x3
iv) Height of winding, x4
v) Voltage per turn, x5

Design optimization is performed satisfying the imposed constraints and minimizing the following three objective functions independently:

1. Volume of transformer.
2. Weight of transformer
3. Active material cost of transformer

These objective functions are nonlinear and the mathematical expressions are determined, based on the selection of core material, shape and size. The following core is selected from the manufacturer data sheet.

Core Material: Ferrite, Magnetic Type P
Core Type: ETD
Core size: 34mm- ETD34

ETD cores are in the group of EE cores and similar to other EE cores without a narrow ferrite notch restricting coils leads entering or leaving the bobbin. ETD cores are also called as 'round-center-leg cores'. These have a small advantage in that the mean length of a turn is about 11 percent shorter than that of a square-legged core of equal center-leg area. Coil resistance is thus 11percent less for equal number of turns, and copper loss and temperature rise is less than the square center leg. Figs. 2 and 3 shows the ferrite core ETD34 obtained from the manufacturer 'Ceramic Magnetics Inc' data sheet.

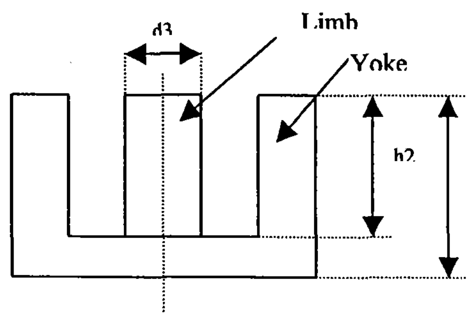

Fig. 2 Core of ETD34 (front-view)

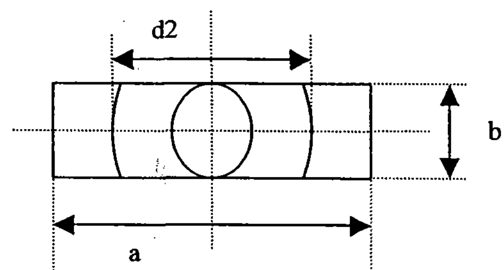

Fig. 3 Core of ETD34 (Top-view)

The dimensions of the core are presented in Table 2.

Type	a	b	d2	d3	h1	h2
	mm	mm	mm	mm	mm	mm
ETD 34	33,4 - 35,0	10,5 - 11,1	25,6 - 27,0	10,5 - 11,1	17,1 - 17,5	11,8 - 12,4

Table 2: Dimensions of core ETD34

Two ETD core sections are used to form one single-phase transformer. The core structure as well as the winding structure of the transformer is shown in Fig. 4.

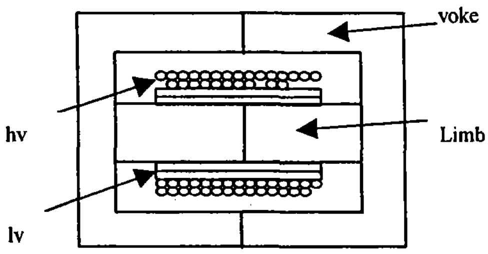

Fig. 4 Transformer core and winding structure

The primary winding, hv is designed with round conductors and the secondary winding, lv with foil conductors.

A. Volume Function

The total volume of the transformer can be express as:

$$V_{tot} = V_{lm} + V_{yo} + V_{wg}$$

where volume of the yoke, Vyo is determined by analyzing the diagram shown in Fig. 5 for the net area.

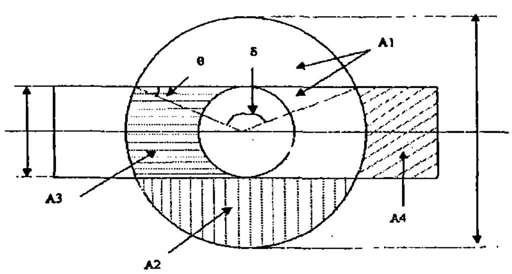

Fig.5 Layout to compute net area of yoke

B. Weight Function

The weight of transformer is the summation of weight of core and weight of the windings. The total weight of the active materials is expressed as:

$$G_{tot} = G_{lm} + G_{yo} + Gwg \quad \text{kg}$$

C. Cost Function

The cost of active materials used is considered as the cost function which is the summation of cost of ferrite core material and cost of copper windings i.e.

$$C_{tot} = (C1 * (G_{lm} + G_{yo})) + (C2 * G_{wg}) \qquad \text{RM}$$

VI. Objective and Constraint Functions

The mathematical expressions of objective functions for minimum volume, minimum weight and minimum cost and similarly for the constraints functions are derived in terms of the selected design variables. The degree of utilization of a high frequency transformer is generally limited by the permissible temperature rise of the windings and also by the allowable losses [6]. Thus two important constraint functions to be satisfied in this design are:

1. Efficiency ≥ 99 %
2. Temperature rise ≤ 55 °C

A. Efficiency

The equation for efficiency of the transformer is:

$$\eta = \left(1 - \left(\frac{P_{ct} + P_{wg}}{S \times pf + P_{ct} + P_{wg}}\right)\right) \times 100 \geq 99$$

Where the core losses,

$$P_{ct} = P_c \times V_{core}$$

The expression for P_c is,

$$Pc = Ka \, . \, f^{Kb} \, . \, B^{Kc}$$

Where
Pc [kW/m3] = core power loss density
f [Hz] = frequency
B [T] = flux density
Ka, Kb, and Kc = curve fitting formula constants

The expression for P_{wg} is,

$$P_{wg} = Kr_1 \cdot I_p^2 \sigma \frac{L_{mt1} T_1^2}{A_{pri}} + Kr_2 \cdot I_s^2 \sigma \frac{L_{mt2} T_2^2}{A_{sec}}$$

Where

$$Kr_1 = 0.5y1\left[M(y1) + (2m-1)^2 D(y1)\right]$$

$$y1 = \frac{hc1}{\delta}$$

$$hc1 = 0.886 \times dc1$$

and $$\delta = \frac{0.071}{\sqrt{f}}$$

$$Kr_2 = 0.5y2\left[M(y2) + (2m-1)^2 D(y2)\right]$$

$$y2 = \frac{hc2}{\delta}$$

Here, hc2 = B2

Power factor is assumed to be 0.85 throughout the design.

B. Temperature Rise

Since the size of the high frequency transformer is quite smaller, only natural convection cooling will be considered to estimate the temperature rise. For the average situation, the following equation can be used.

$$R_E = \frac{800°C - cm^2 / Watt}{A_s in cm^2} \text{ °C/Watt}$$

where A_s is the total surface area of the transformer, excluding the mounting surface. For a given class of cores, such as E-E cores in the ETD or EC series, the usable surface area, A_s is approximately 22 times the winding window area, A_w. Combining this with the equation above, the external thermal resistance:

$$R_E = \frac{36}{A_w \text{ in } cm^2} \text{ °C/Watt}$$

$$R_E = \frac{36 \times 22}{A_s \text{ in } cm^2} \text{ °C/Watt}$$

The total surface area, As is expressed as
A_s = winding surface area + core surface area
$= A_{wg} + A_{su}$

From the Figs. 2,3 and 4, the winding surface area and the core surface area can be defined.

The temperature rise can be expressed as:

$$\Delta T = R_E \times P_L \quad °C$$

Where

$$P_L = P_{ct} + P_{wg} \quad W$$

With $\Delta T \leq 55$ °C

The objective function as well as the constraint functions should be combined to form an augmented objective function before applying the GAs approach to find out the optimum design variables. This technique is known as Zangwill's exterior penalty function method [7], which is one of the NLP techniques. The expression formed is known as augmented objective function and formulated as,

$$P(X, r) = F(X) + r \sum_{j=1}^{m} \left[g_j(X)\right]^z$$

Where

F (X) = normal objective function
r = penalty factor
$g_j(X)$ = constraint functions, j = 1,2, …..n
z = integer value to be determined
$X = x_1, x_2, \ldots, x_n$ design variables

Therefore, the augmented function P can be written as:

i) Volume function
$P(vol) = V_{core} + V_{wg} + r\,(g_1^z + g_2^z)$

ii) Weight function
$P(wgt) = G_{core} + G_{wg} + r\,(g_1^z + g_2^z)$

iii) Cost function
$P(cost) = (C1 * (G_{lm} + G_{yo})) + (C2 \times G_{wg}) + r\,(g_1^z + g_2^z)$

Where

$g1 = 99 - \eta$
$g2 = \Delta T - 55$
r = penalty factor, 10^i, i = 1,2,3,…

The final step of the design is to code all nonlinear mathematical models of the objective function, constraint functions and augmented objective function in C++ language. These expressions together with the built-in component in GA library will be used to develop the design optimization program.

VII. Results

A C++ optimization program has been developed based on the GAs library and all the design equations. The design optimization program is run with Microsoft Visual C++. The results from the program's output are studied remembering that the objective of the program is to find out the optimum values of independent variables, which provide minimum value of the objective functions satisfying the imposed constraints. In order to analyze the results from the program, some of the parameters in the program have been changed. These include GA parameters, penalty factor, and also the ranges of the independent variables. However, it is not practical to have a wide range of design variables although it really contributes to the minimization of objective functions. Analysis of the results with changes of parameters will be explained and the results for three objective functions will be discussed.

First, the variation of parameters of augmented objective function is studied to determine how these parameters affect the results. The selected values of penalty factor are 10, 100, 1000, 10000 and 100000 whereas the z value is 3 in the augmented objective function. This value is selected because when z is 2, the results are inconsistent as GA program is not able to converge the augmented objective function to a minimum value. The following GA parameters are assumed:

Number of generations =100;
Population size = 100
Probability of mutation = 0.01
Probability of crossover = 0.6

The corresponding results of volume, weight and cost functions are shown in Figs.6, 7 and 8.

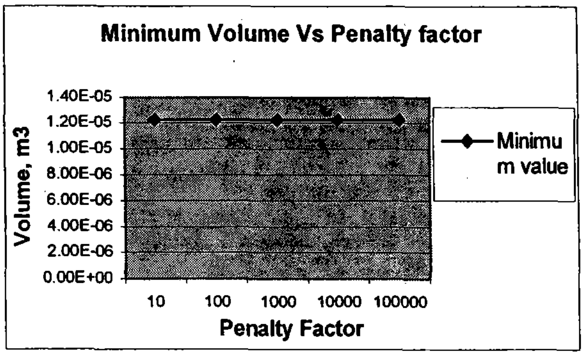

Fig. 6 Minimum Volume Function (m^3) –penalty factor

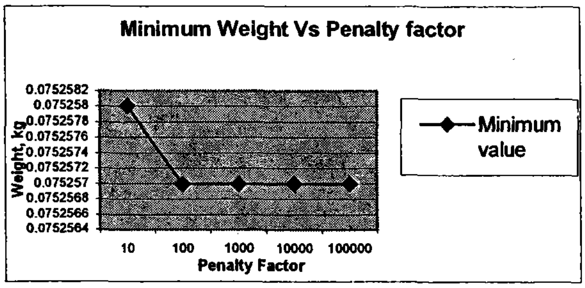

Fig 7: Minimum Weight Function (kg) - penalty factor.

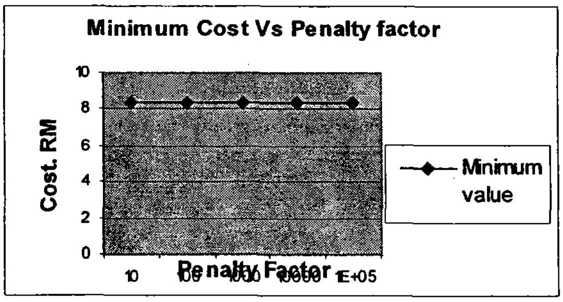

Fig 8: Minimum Cost Function (RM)- penalty factor.

It is seen from the results that a penalty factor above 100 is enough to converge the objective function to a minimum value. Therefore, a penalty factor of 100 is selected for further analysis.

The following GA parameters are used to guide a genetic algorithm during searching process to find out an optimum solution:

1) Number of generations, Ngen
2) Size of populations, Npop
3) Probability of mutation
4) Probability of crossover

Keeping the third and fourth parameters fixed with default values, which are equal to 0.01 and 0.6 respectively, the first two parameters will be changed to determine the corresponding results. The ranges of independent variables are:

X1 = 0.10 - 0.60
X2 = 4.00 - 4.50
X3 = 4.00 - 4.50
X4 = 0.014 - 0.018
X5 = 2.2 - 3.0

The number of generations selected are Ngen = 50, 100, 200 and 500 and the size of populations are Npop = 50, 100, 200, 500 and 1000. The corresponding results are shown in the following Figs. 9,10 and 11.

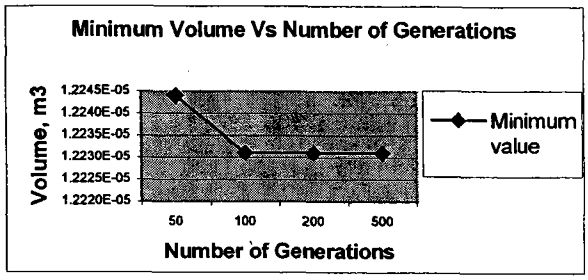

Fig. 9 Minimum Volume Function (m^3) for different Ngen and Npop = 50

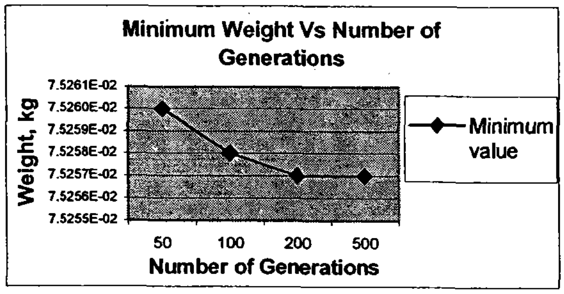

Fig. 10 Minimum Weight Function (kg) for different Ngen and Npop =50

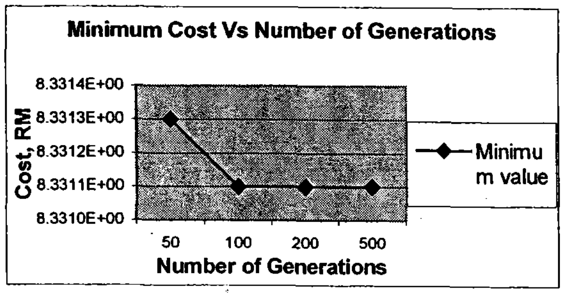

Fig. 11 Minimum Cost Function (RM) for different Ngen and Npop =50

From the above results, the following observations are made:

1. When number of generations and population size are below 100, the minimum value of objective functions cannot occur. It is because GA is not able to converge the objective function to a minimum value in time.

2. When both number of generations and population size exceed 100, a minimum objective function can be obtained. It is because GA is now able to converge the objective function to a minimum value.

Appropriate GA parameters and penalty factor have been selected for three different objective functions and the final results obtained are shown in Table 3. The transformer design data at minimum cost for one optimal solution is shown in Table 4.

Table 3: Optimum design variables

Optimum results	Volume Function	Weight Function	Cost Function
Minimum value	1.22×10^{-5} m^3	0.075 kg	RM 8.33
Temperature rise	51 °C	51 °C	51 °C
Efficiency	98.78 %	98.78 %	98.78 %
Flux density in core, x1	0.1199 T	0.1022 T	0.1199 T
Current density in high voltage winding, x2	4.00 A/mm^2	4.00 A/mm^2	4.00 A/mm^2
Current density in low voltage winding, x3	4.50 A/mm^2	4.50 A/mm^2	4.50 A/mm^2
Height / length of winding, x4	0.018 m	0.018 m	0.018 m
Voltage per turn, x5	3.00 V	3.00 V	3.00 V

Table 4 Typical Transformer data at minimum cost

Flux density in core, T	x1 = 0.10021
Current density of hv winding, A/mm2	x2 = 3.2055
Current density of lv winding, A/mm2	x3 = 4.4799
Height of winding, m	x4 = 0.014125
Voltage per turn, V	x5 = 2.8909
primary winding turns	T_1 = 13.837
secondary winding turns	T_2 = 1.8679
primary winding area of c.s., mm^2	a1 = 1.9498
secondary winding area of c.s., mm^2	a2 = 10.334
width of primary winding , m	b_1 = 0.0046506
width of secondary winding, m	b_2 = 0.0034633
thickness of foil conductor of lv winding, m	B2 = 0.00073164
diameter of conductor of hv winding, m	dc1 = 0.0015753
current in primary winding, A	I_p = 6.25
current in secondary winding, A	I_s = 46.296
length of center leg / limb, m	L = 0.018125
% transformer efficiency	η = 98.722
temperature rise, °C	ΔT = 54.981

VIII. CONCLUSIONS

In this paper, a general procedure for the design optimization of a high frequency power transformer with GA approach in C++ program that include design considerations, design methodology, results and discussions have been presented. A nonlinear mathematical model with different objective functions and important constraints of a high frequency transformer is developed in this paper. Optimization of high frequency power transformer used in switching power supplies has been successfully completed by GA approach in C++ language code. The program has determined a set of optimum design variables that contribute to the minimization of three objective functions viz., (1) Minimum volume, (2) Minimum weight and (3) Minimum cost.

Although the program developed in this paper is subject to certain changes, it can be adopted to any specification of high frequency transformer with little modifications for material selection, losses and thermal model.

It is proved that a set of optimum design variables of a high frequency power transformer can be determined using the GA optimization program. More number of design variables may be selected and the mathematical model can be improved. Similarly the number of constraints can be increased. With the present optimum design parameters and other design data, a transformer may be constructed to study the performance characteristics. To overcome some weaknesses in GA searching approach, a lot of research and studies have to be made to improve its effectiveness and performance.

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