

DESIGN OPTIMIZATION OF SPECIAL PURPOSE TRANSFORMERS

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Overview of the Project

This report is based on the investigations on optimal design of Special purpose transformers viz., semi-conductor rectifier transformers, high-frequency transformers and furnace transformers.

With the growth of static converting equipment supplying high current, low voltage loads semi-conductor rectifier transformers are subjected to harmonic loss and thermal problems. The converting equipment and the connection of the transformer normally influence the primary and secondary currents of a transformer. Due to the non-sinusoidal nature of the current in the windings, eddy current losses in the conductors, temperature rise of the equipment and mechanical stresses in windings are to be derived on the basis of actual current waveform. The design optimization procedure of a rectifier transformer must take many characteristics into consideration based on the specification of the harmonic spectrum, service conditions and transformer type.

Transformers supplying direct arc furnace loads must conform to a very special technique that results from the very particular operating conditions of the furnace itself. A transformer for a direct arc furnace must be designed to supply a very large current by its secondary winding at a very low voltage. Also it is to be designed to withstand repeated electrical and mechanical stresses. In developing the furnace transformer rating, arc power, secondary circuit impedance, arc resistance and regulation of the furnace voltage over a wide range are important factors.

High-frequency power transformer is an important component in the switched mode power supplies and other power electronic controllers. For design optimization, considerations regarding core geometry, switching frequency, current harmonics, temperature rise are relevant.

With this aim research is carried out to develop design optimization procedures for each of the above transformers, by means of Genetic Algorithms. Non-linear mathematical models and programs in C++ are developed for the optimal designs and results are successfully obtained and published.

The investigations are divided into three parts. In Part I studies on three-phase and single-phase Rectifier transformers are presented. Part II is devoted for investigations on Furnace power transformers. In Part III, studies on High Frequency transformer are presented.

PART I

1. Three-phase Semi-conductor Rectifier Power Transformer

Design optimization of a semi-conductor rectifier power transformer operating with non-sinusoidal current need important considerations with the standards imposed by IEEE and IEC. The harmonic currents also contribute additional I^2R loss in the windings. Important characteristics and different configurations of rectifier transformers have appeared in references [1,2]. The KVA rating of a transformer is to be decided by the r.m.s. current or fundamental component of current drawn from the lines by the primary winding, as recommended by IEEE or IEC respectively [3]. The static converting equipment and the type of 3-phase connection of the transformer normally influence the transformer primary and secondary currents as well as the input line current. In addition, the source impedance also has some effect on the current waveform. In a two winding transformer, if the primary voltage is not unduly high, the HV winding can be inner most, nearest to the iron core, while the heavy current secondary can be on the outside and subdivided into a number of parallel coils. The windings are usually wound with rectangular conductors of large cross-section and are so arranged to possess high mechanical strength in the direction of the greatest force. Due to non-sinusoidal nature of current in the windings and because these transformers supply large currents at low voltage, the mechanical stresses arising from short-circuit currents are different from a conventional power transformer.

A rectifier power transformer connection influence the duty of the converting equipment and it should be coordinated with the available rectifying devices. The phase current waveform in the windings for a specific connection of the transformer is necessary before proceeding with the design analysis.

2. Phase current waveform and KVA rating

A 3-phase star-delta connected semi-conductor rectifier power transformer supplying a DC load through a 3-phase, 6-pulse bridge rectifier is considered and represented in Fig.1. The phase current waveform of the secondary winding without and with source impedance is shown in Figs.2.1 and 2.2 respectively.

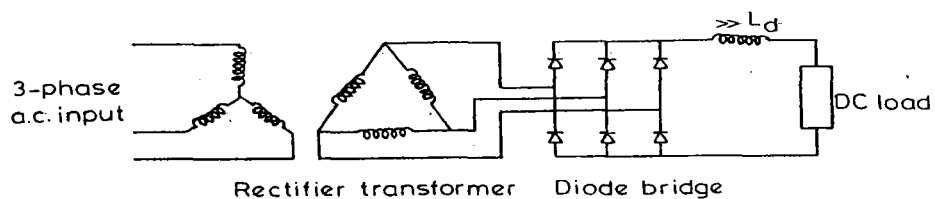


Figure 1

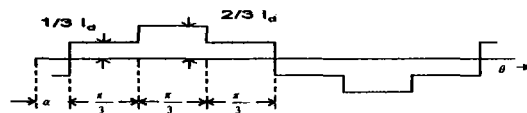


Figure 2.1

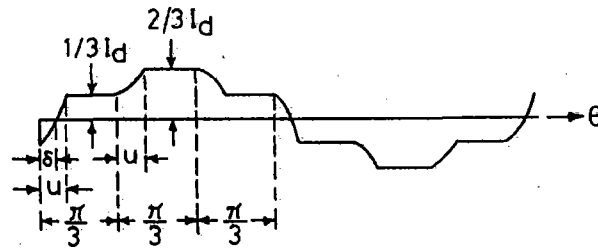


Figure 2.2

The secondary winding r.m.s. phase current, I_2 for a specified dc load is derived by assuming:

- Direct current supplied to the load is without ripple,
- Commutation overlap of the rectifying devices is neglected (Fig.2.1) or commutation loop comprises only inductance (Fig.2.2).

The KVA rating and secondary voltage may be fixed according to r.m.s. or fundamental component of current derived from the actual waveform considering dc rated load and overlap angle for the source inductance. The overlap angle depends upon the leakage reactance of the transformer and current at commutation, and initially it is assumed as 15° . Investigations are carried out and rated KVA is selected from the r.m.s. current of actual waveform as prescribed by IEEE standards.

3. Losses in the transformer [4,5]

3.1 Core loss: While the current waveform is distorted, the voltage impressed on the primary, when the rectifier is fed from a system whose capacity is reasonably large, approximates a sine wave.

3.2 Eddy current loss: There are two approaches to evaluate eddy current loss with pulsating current:

- To derive a formula similar to eddy loss ratio with sinusoidal current.
- To determine the losses as sum of several contributions of harmonic components of actual current.

The first approach is very simple as the expression for r.m.s. current can be derived from Fig. 2.2

4. Design variables, constraints and the objective function

The selection of the following six independent design variables is based on the significant effect of these on the short-circuit reactance, weight of the core and windings, total losses, core dimensions and also on the active material cost of the transformer.

Maximum flux density in the core, Tesla	x_1
Current density in the HV winding, A/mm ²	x_2
Current density in the LV winding, A/mm ²	x_3
Height of the windings, m	x_4
Voltage per turn, V	x_5
Distance between core centers, m	x_6

The degree of utilization of the rectifier transformer is limited by the permissible temperature rise of windings, dc voltage regulation and by the magnetic properties of the active iron.

Based on the above factors, the following constraints are imposed on the design mathematical model:

Temperature rise of windings above ambient, °C	$\leq \theta_{wg}$
Temperature rise of oil above ambient, °C	$\leq \theta_{oa}$
Percentage short circuit impedance	$\leq Z_{sc}$
Permissible flux density in the core, Tesla	$\leq x_1$
Percentage no-load current	$\leq I_0$
Clearance between phase windings, m	$\leq b_{ph}$
Maximum height of the windings, m	$\leq x_4$
Percentage Efficiency	$\geq \eta$
Maximum current density in the HV winding, A/mm ²	$\leq x_2$
Maximum current density in the LV winding, A/mm ²	$\leq x_3$

In addition, some design variables are constrained with upper and lower bounds to satisfy mechanical restrictions.

The objective function to minimize the active material cost of the transformer is formulated with the cost of stampings, windings and capitalized cost of losses. Manufacturing costs and labor costs are not included in the objective function as a wide variation is normal in different shop floors.

$$\text{Objective function, } F(\mathbf{x}) = c_i G_i + c_c G_c + c_1 P_c + c_2 P_i \quad (1)$$

where c_i, c_c = cost of iron, windings Rs./kg

$c_1 \& c_2$ = capitalized cost of losses

G_i, G_c = weight of iron, windings, kg

$P_c \& P_i$ = copper and core losses

All the above expressions are derived in terms of the design variables.

5. Non-linear optimization problem [7,8]

The design optimization mathematical model posed as a nonlinear programming problem is stated as:

Find $\mathbf{x} = (x_1, x_2, \dots, x_n)$ such that $F(\mathbf{x})$ is a minimum

Subject to $g_j(\mathbf{x}) \{ \leq \} 0, j = 1, 2, \dots, m$

With $\mathbf{x} \geq 0$ being a non-negative solution.

$\mathbf{x} = x_1, x_2, \dots, x_n$ = independent design variables

$F(\mathbf{x})$ = nonlinear objective function i.e. cost function

$g_j(\mathbf{x})$ = nonlinear constraint functions i.e. geometry and performance characteristics.

6. Exterior penalty function method [9]

In this method, the augmented function P is formulated as

$$P(\mathbf{x}, r) = F(\mathbf{x}) + r \sum_{j=1}^m [g_j(\mathbf{x})]^q, \quad r \geq 0 \quad (2)$$

where $g_j(\mathbf{x})$ is defined as $\max[g_j(\mathbf{x}), 0]$.

A popular value of q is 2, although other values are possible.

Starting with an initial value x_1 and r_1 , minimize $P(\mathbf{x}, r_1)$ by Powell's method.

Let x_2 be the resulting point. A new function is formed with $r_2 = c r_1$, $c > 1$, such that

$$P(\mathbf{x}, r_2) = F(\mathbf{x}) + r_2 \sum_{j=1}^m [g_j(\mathbf{x})]^q, \quad (3)$$

This process of minimization continues and as $r_k \rightarrow \infty$, it can be proved that

$$\min_{k \rightarrow \infty} P(\mathbf{x}, r_k) = \min F(\mathbf{x}) \quad (4)$$

7. Design optimization

A mathematical model for the non-linear program is formulated with the objective and constraint functions in terms of design variables. The general optimization program in C++ includes a main program together with seven subprograms, viz., HV continuous disc winding, LV helical winding, temperature rise of windings and oil, mechanical forces in windings, Powell-Botm optimization routine etc. The mathematical model is converted into a sequence of unconstrained minimization problems with normalized constraints using Zangwill's exterior penalty function method, as an augmented objective function [9]. With a starting vector of independent variables and a penalty parameter, the minimization process is carried out using Powell's sequential transformation method [8]. Powell's pattern search method using conjugate directions imposes quadratic convergence.

Optimal design of a 4 MVA, 3-phase, star-delta connected, 33 kV / 296 V semi-conductor rectifier power transformer supplying a dc load is computed, minimizing the active material cost and satisfying the desired constraints. The results are shown in Table1.

Table1 Optimized Design Data

Design Variables:

Variable	Description	Optimal value
x_1	Maximum flux density in the core, Tesla	1.67
x_2	Current density in the HV winding, A/mm ²	2.58
x_3	Current density in the LV winding, A/mm ²	2.65
x_4	Height of the windings, m	1.40
x_5	Voltage per turn, V	36.00
x_6	Distance between core centers, m	0.80

Constraints and Performance Data:

Winding temperature rise above ambient, °C	55.0
Top oil temperature rise above ambient, °C	44.0
Percentage short-circuit impedance	2.95
Percentage no-load current	2.42
Percentage efficiency	99.27
Clearance between phase windings, m	0.04
Core circle diameter, m	0.372
Number of HV disc coils	80
Area of cross-section of HV strap conductor, mm ²	14.00
Number of LV disc coils	18
Area of cross-section of HV strap conductor, mm ²	16.80
Number of parallel conductors in LV coil	6
Eddy current loss in conductors, kW	2.533
Total I ² R loss, kW	17.48
Core loss, kW	10.19
Weight of copper, kg	1091.75
Weight of stampings, kg	4562.98
Overlap angle due to source inductance, deg	16.26
Secondary voltages for different tap settings, V	328, 320, 312, 304, 296, 290, 283, 277, 270

8. Conclusions

A new procedure for design optimization of three-phase semi-conductor rectifier transformers based on nonlinear programming technique is described. The mathematical model considers the KVA rating according to IEEE standards and provides a procedure to compute transformer I²R loss and temperature rise from an actual waveform of current. Optimum design values have proved the suitability of Powell's sequential unconstrained minimization technique together with Zangwill's exterior penalty function formulation. A general design procedure presented in this report is suitable for a line of transformers with different specifications.

9. Design Optimization of Single-Phase Rectifier Transformer

A small rectifier transformer supplying a dc load through a diode bridge rectifier is shown in Figure 3. As the rectifier is the source of harmonics, the design of such a transformer need to consider the effect of harmonics and the volt-ampere rating is to be specified as r.m.s. VA or fundamental VA [10,11].

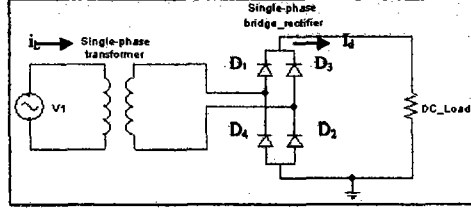


Figure 3 Transformer with DC Load

The input line current of the transformer is non-sinusoidal and is also affected by the source impedance. The source inductance distorts the current waveform and introduces an overlap period. During this period all the diodes conduct together at the same time and is represented by the overlap angle, u . An approximate current waveform as shown in Figure 4(a) and the actual waveform as shown in Figure 4(b) are drawn, neglecting and considering the input source inductance, respectively.

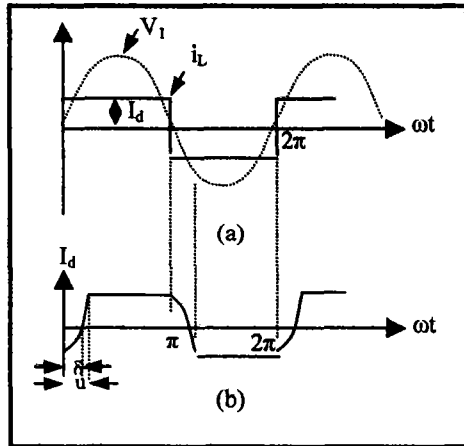


Figure 4 (a) Approximate Waveform (b) Actual Waveform

Based on the nature of commutation of the diodes,

$$\delta = \cos^{-1} \left(\frac{1 + \cos u}{2} \right) \quad (5)$$

Neglecting the source inductance, the transformer secondary voltage and current are expressed as

$$\text{secondary voltage, } V_2 = \frac{V_d \pi}{2\sqrt{2}} \quad (6)$$

$$\text{secondary rms current, } I_{2,rms} = I_d \quad (7)$$

where V_d, I_d are dc load voltage and current, respectively.

It is observed that rms current, $I_{2,rms}$ calculated from the waveform of Figure 4(b) is more effective in the estimation of VA rating, temperature rise and eddy current losses in the conductors. Accordingly, the volt-ampere rating of the transformer is fixed from the rms values of voltage and current for a given dc load as specified in Table 2.

Table 2 Specification of Rectifier Transformer

1-phase 500 VA, 240 V/ 30 V, 50 Hz, core type

10. Losses in The Transformer

Transformer losses consist of core loss, $I^2 R$ loss in windings and additional eddy current loss in conductors due to harmonics.

10.1 Core loss

The core loss depends on the peak value of the sinusoidal core flux. For non-sinusoidal line current the primary voltage waveform is assumed to be sinusoidal. As long as the transformer voltages remain within a few percent of rated value, the core loss is essentially constant, and is estimated as [12]

$$\text{Core loss, } P_i = FF(B_c)G_i + 1.075 FF(B_y)G_y \quad (8)$$

where

B_c, B_y = flux density in the core and yoke, Tesla

G_i, G_y = weight of the limbs and yoke, kg

$FF()$ = a polynomial function of the flux density derived from B-H curve of the material

10.2 Additional Eddy Current Loss

As the current is non-sinusoidal, additional eddy current losses in the windings are estimated as [5]

$$\text{Eddy current loss, } P_e = \sum K_{e,pul} I^2 R \quad (9)$$

$$\text{where } K_{e,pul} = \left(\frac{5m^2 - 1}{45} \frac{h_{ca}^2}{h_w^2} \frac{BO^4}{4} \right) \left(\frac{I'}{I} \right)^2 \left(\frac{\mu_0^2}{\rho^2} \right) \quad (10)$$

$$\left(\frac{I'}{I} \right)^2 = \omega^2 = (2\pi f)^2$$

m = number of parallel conductors in radial direction

BO = radial dimension of the conductor, mm or diameter of conductor, d_c

f = frequency of supply, Hz

μ_0 = magnetic space constant, $4\pi \times 10^{-7}$ H/m

ρ = conductivity of the material, ohm-m/mm²

h_{ca} = height of copper in axial direction, m

h_w = height of winding, m

I = rms current, A

I' = time derivative of rms current, A

Eddy current losses are estimated for primary and secondary windings from Equation (9).

10.3 Total Copper Loss

The total copper loss is the sum of the primary and secondary $I^2 R$ losses and additional eddy current loss.

$$\text{Copper loss, } P_c = I_1^2 R_1 + I_2^2 R_2 + P_e \quad (11)$$

11. Temperature Rise

The temperature rise of the windings is estimated from the total copper loss and exposed surface area of the windings.

$$\text{Temperature rise, } \Delta\theta = \theta_{\max} - \theta_a = P_c R_{\theta a} \quad (12)$$

where $R_{\theta a} = \frac{1}{\lambda_w A_s}$ = winding ambient thermal resistance

λ_w = winding heat transfer coefficient, watts/m²

$A_s = \pi D_s h_w$ = Area of open surface of the winding

$D_s = d + 2b_2 + 2b_1 + k$ = diameter of the outer surface cylinder, m

d = core diameter, m

b_1, b_2 = width of lv and hv windings, m

k = constant

12. Optimization Procedure with GA [13,14]

GA operates on a population of points in the search space and offers a convenient way of handling constraints and single or multi objective functions of a design problem. In GA approach, each design variable is represented as a binary string (chromosome) of fixed length and the search operations are typically selection, crossover, and mutation. An index of merit is assigned to each chromosome and is evaluated by using a fitness function. GA provides solutions by generating a set of chromosomes referred to as a generation. A new generation is selected having largest probability of the fittest individual. Generations are created from pairs of chromosomes as parents with reproduction operators, mutation and crossover. If the search has to continue, the GA creates a new generation from the old one until a decision is made on the convergence. A number of evolutions of the fitness function is necessary to arrive at a final optimal solution. In this paper the selection of parameters to successive generation is achieved by tournament selection strategy. In binary tournament selection two individuals are selected at random from the population, and the better one is duplicated in the next generation. This process is repeated until the individuals reach a specified population size. A crossover operator exchanges information contained in two parent individuals to produce two

offspring and then replace the parents. The number of times the crossover operator is applied to the population is determined by the probability of crossover and the population size. The mutation operator randomly selects an individual from the population and then chooses two elements in this individual to exchange positions [14]. The entire process of evaluation and reproduction then continues until either the population converges to an optimal solution for the problem or the Genetic Algorithm has been run for a specific number of generations. The genetic parameters of Table 3 play an important role in GA to provide a global optimal solution. The design variables and constraints are selected as in Table 4, considering the effect on core dimensions, losses, short-circuit reactance, weight, volume and finally on the cost of the transformer.

Table 3 GA Parameter Settings

Chromosome Size	16 bits
Population size	100
Selection Process	Tournament
Crossover rate	0.8867 or 88.67%
Mutation rate	0.044 or 4.4%
Genome mapping	According to constraints

Table 4 Design Variables

x_1	Maximum flux density in core, T
x_2	Current density in HV winding, A/mm ²
x_3	Current density in LV winding, A/mm ²
x_4	Height of windings, m
x_5	Voltage per turn, V
x_6	Distance between core centers, m
x_7	Clearance between windings, m

The fitness function of GA is the objective function of the design problem. A mathematical model in terms of design variables for objective and constraint functions explained in the following sections is developed for the specification in Table 2.

13. Design Constraints

Two constraints i.e. temperature rise and percentage efficiency are imposed on the design problem.

1. Percentage efficiency

$$eff \geq 95$$

$$eff = \left(1 - \frac{P_c + P_i}{S \cdot pf + P_c + P_i} \right) \times 100 \quad (13)$$

where S = VA rating of the transformer

pf = power factor of the transformer

2. Temperature rise of winding above ambient

$$\Delta\theta \leq 50^\circ c$$

14. Objective Function

The objective function or fitness function is the minimum active material cost of the transformer i.e. cost of stampings and cost of windings. Insulating materials cost, manufacturing cost etc. is not included in the objective function as there may be wide variation with different manufacturers.

$$\text{Objective function, } F = c_1(G_l + G_y) + c_2 G_c \quad (14)$$

where c_1, c_2 = cost per kg of iron and copper, RM/kg

G_l, G_y, G_c = weight of limb, yoke and copper, kg

The objective function together with the constraints is converted into an augmented objective function to suitably adopt GA [15] to a constrained design optimization problem.

15. Results and Discussion

The performance results of rectifier transformer design with GA approach are found to be quite satisfactory. Optimal design parameters, minimizing the objective function together with the constraints are derived for a set of GA parameters and penalty factors. The GA technique with a 16-bit chromosome has been tested for several combinations of GA parameters and penalty factors. A comparison is made on different choices and finally the probability of mutation is set at 0.044 and the probability of crossover at 0.866. For a population size of 100, the cost objective function is minimized over number of generations. It is observed from Figure 5 that the optimal objective function of cost converged to a minimum for a number of generations above 300. In all the cases the design analysis program together with GA has taken about 5 to 8 seconds, to arrive at an optimal solution.

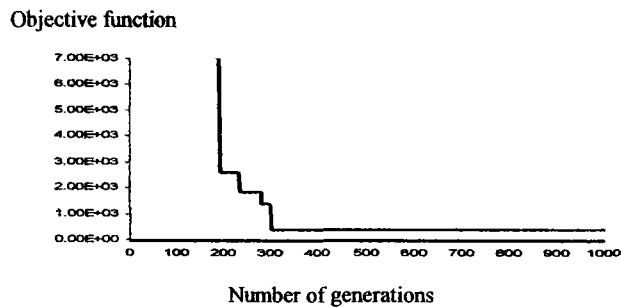


Figure 5 Minimized Cost Objective Function

The program is run with different values of penalty factors. The constraints are observed to be within the specified limits for all optimal solutions. For a set of GA parameters and a penalty factor for constraints, the performance results of optimization of a rectifier transformer as reported in Tables 5 and 6 are satisfactory.

Table 5 Optimal Values

Design Variable		
x_1	Maximum flux density in core, T	1.21792
x_2	Current density in HV winding, A/mm ²	2.35232
x_3	Current density in LV winding, A/mm ²	2.27671
x_4	Height of windings, m	0.13475
x_5	Voltage per turn, V	1.00191
x_6	Distance between core centers, m	0.08233
x_7	Clearance between windings, m	0.01298
<u>Constraints:</u>		
% Efficiency at full load		95.005
Temperature rise, °C		49.999

Table 6 Design Parameters

Core diameter, m	0.08577
Core length, m	0.14279
Core width, m	0.15953
Turns ratio	240/30
Primary current, A	2.08333
Secondary current, A	16.6667
Conductor diameter - primary, mm	1.08667
Conductor diameter - secondary, mm	3.12421

16. Conclusions

The volt-ampere rating of a rectifier transformer and estimation of copper losses is based on non-sinusoidal input current when the transformer is connected to a dc load. This report has proposed a general procedure for design optimization of such a rectifier transformer by Genetic Algorithm. Design parameters and optimal objective function derived by this non-linear programming technique demonstrated the suitability of GA for rectifier transformer design.

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PART II

1. Design Optimization of a Furnace Transformer Using Genetic Algorithm

In a direct-arc furnace, the arcs get short-circuited as the electrodes touch the metal. In order to limit the short-circuit currents, a furnace transformer secondary circuit is designed with a specific reactance. The secondary circuit consists of secondary bus, flexible cables, copper bus tubes and the electrodes. For optimum operating conditions of the furnace, arc resistance and electrical design of the associated power system, including a transformer, plays a major role. A sufficiently large enough arc voltage is to be maintained by the low voltage winding of the transformer to supply the desired power [1, 2]. For a specified effective arc voltage and arc current, a wide range of circuit impedances can be applied that result in the same average arc resistance [3-6]. Genetic Algorithm based design optimization procedure of a furnace transformer is investigated in this project. A mathematical model with a single objective or a multi-objective function together with a number of constraints is developed for the optimization problem. Minimum cost or weight or volume of the transformer forms a single objective function. A multi-objective function is derived from the three single objective functions with proper weighting and penalty factors.

2. Configuration of Power System and Rating of Transformer

In the power system illustrated in Fig. 1, a 3-phase star-delta connected, core-type transformer is considered to supply a 22 feet shell diameter, 150 tons per hour direct-arc melting furnace. Optimum operating point and best furnace operation mainly depends on arc power and arc resistance.

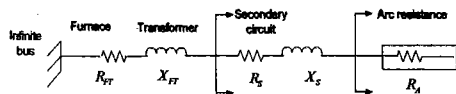


Figure 1 Furnace transformer power system

R_{FT}	Resistance of furnace transformer
R_S	Resistance of secondary circuit
X_{FT}	Reactance of furnace transformer
X_S	Reactance of secondary circuit
R_A	Arc resistance

Based on arc resistance, secondary circuit impedance and secondary current the specification of the transformer is selected as in Table 1.

Table 1. Specification of furnace transformer

3-phase core type 64.5 MVA, 33 kV / 384 – 712 V, star- delta connected transformer with forced oil cooling.

3. Optimization Procedure with GA [7-12]

As explained in Section 12 of Part I, GA is applied to the optimal design of a furnace transformer. The design variables and constraints are selected as in Table 2, considering their effect on core dimensions, losses, short-circuit reactance, weight, volume and finally on the cost of the transformer.

Table 2. Design variables and constraints

Specified design variables	Constraints
x_1 Flux Density in the Core, T	Temperature rise of windings above ambient, °C
x_2 Current density in hv winding, A/mm ²	Temperature rise of oil above ambient, °C
x_3 Current density in lv winding, A/mm ²	% Short circuit reactance
x_4 Height of the windings, m	% No-load current
x_5 Voltage per turn, V	% Efficiency
x_6 Distance between core centers, m	Clearance between different phase windings, m

The fitness function of GA is the objective function of the design problem. A mathematical model in terms of design variables for objective and constraint functions is developed for the specification in Table 1. A complete design analysis procedure is developed in C++ together with function programs to calculate hv and lv winding details, losses, temperature rise, mechanical forces in windings and the GA optimization program. Temperature rise of oil calculations is based on forced oil cooling. In addition to the calculation of I^2R losses; core, eddy current and stray losses are modeled in the design analysis program [9,13].

4. Objective functions

The main aim of optimal design is to obtain minimum cost or weight or volume or a combined multi-objective function of a specified furnace transformer.

Cost as an objective function consists of active material cost of stampings, windings and capitalized cost of losses.

$$F_c(X) = c_i G_i + c_c G_c + c_1 P_c + c_2 P_i \quad (1)$$

where c_i, c_c = cost of iron, cost of copper Rs./kg

c_1, c_2 = loss capitalization factors, Rs./kg

G_i, G_c = weight of iron, weight of copper, kg

Similarly the weight and volume objective functions are defined as

$$F_w(X) = G_i + G_c \quad (2)$$

$$F_v(X) = V_i + V_c \quad (3)$$

where V_i, V_c = volume of iron, volume of copper, m³

5. Selection of GA parameters and Penalty factors

In GA approach, a balance between exploitation of the current population and exploration of the design search space is accomplished through the genetic operators used. An initial population size of 100 is found to be satisfactory after testing the optimization problem with 25, 50, 100, 150 and 200. As crossover and mutation rates are responsible for better solutions in the design space, different combinations of these parameters are examined. The probability of mutation and crossover,

respectively are set to 0.044 and 0.866. A design space is feasible if all the constraints are satisfied. Penalty factors of 2,5,10,15 are examined with augmented objective functions. It has been observed that a penalty factor of 5 was satisfactory.

6. Results and Observations

The design program of the 64.5 MVA transformer together with GA is first tested with the selected parameters of Table 3.

Table 3. GA Parameters and penalty factor

Number of generations	= 100
Population size	= 50
Probability of mutation	= 0.044
Probability of cross-over	= 0.866
Penalty factor for constraints	= 5
Exponent q of constraint function	= 2

The resultant solutions are analyzed by changing the number of generations in GA. Figs. 2 to 4 shows the corresponding graphs for each minimized objective function, satisfying the constraints.

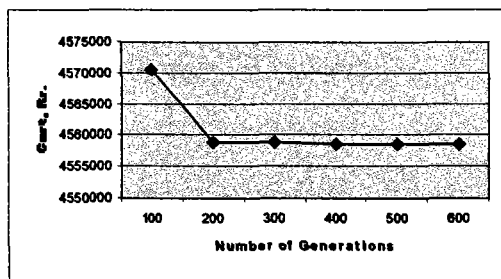


Fig. 2 Cost as objective function

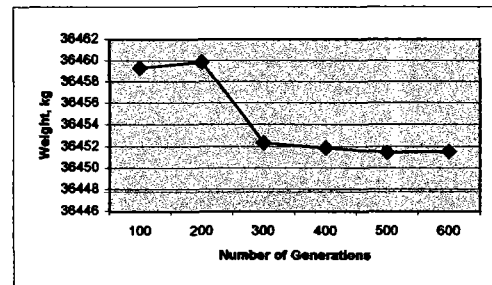


Fig. 3 Weight as objective function

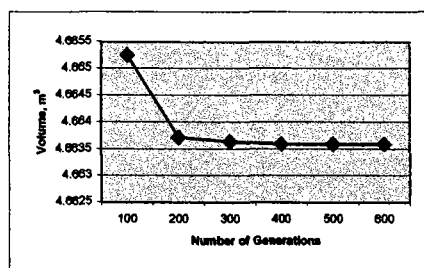


Fig. 4 Volume as objective function

It is observed from the graphs that the single objective functions converged to a minimum for a penalty factor of 5 and number of generations above 400, the other GA parameters being fixed. In all the above cases, the constraints imposed on the design problem are satisfied.

For a comparison, Fig. 5 shows the cost and multi-objective function values for a change in number of generations in GA.

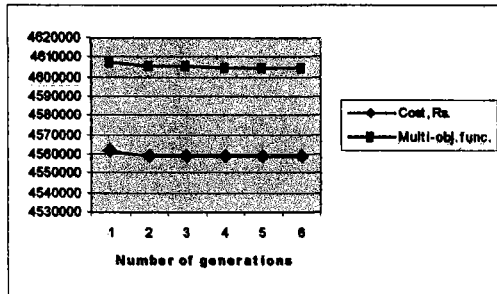


Fig. 5 Cost and multi-objective functions

Tables 4 and 5 show design variables and constraints obtained from the optimal design for both single and multi-objective minimization cases.

Table 4 Design variables

Variable	Single Objective	Multi-Objective
x_1 , Tesla	1.68	1.67999
x_2 , A/mm ²	3.63188	3.64005
x_3 , A/mm ²	2.50003	2.50055
x_4 , m	3.13618	3.13679
x_5 , volts	111.113	111.111
x_6 , m	1.19999	1.2

Table 5 Constraints

Constraint	Single Objective	Multi-Objective
Temperature rise of windings above ambient, °C	57.6317	57.8738
Temperature rise of oil above ambient, °C	48.4405	49.3349
% Short circuit reactance	8.98729	8.97334
% No-load current	1.28311	2.28299
% Efficiency	99.296	99.4407
Clearance between different phase windings, m	0.07088	0.063061

7. Conclusions

Optimal design procedure of a furnace transformer using GA for single and multi-objective function minimizations is proposed in this investigation. The performance of the optimization technique discussed is shown to be superior to conventional techniques. The Genetic Algorithm (GA) has been successfully implemented to the optimal design of a 64.5 MVA furnace transformer. Single and multi-objective function minimizations resulted in satisfactory values of optimal design parameters and design data. An improved non-linear mathematical model for the design problem with more number of design variables, constraints and field analysis by finite element method is under consideration for future investigations.

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PART III

1. High Frequency Power Transformer for a Switching Power Supply

With the rapid development of power electronic technology, the operational switching frequency in power electronic systems, such as the Switched Mode Power Supply (SMPS), have been extended to the mega-hertz region. Transformers are the largest and heaviest components in SMPSs and accounts for about 25% of the overall volume and more than 30% of the overall weight. Minimizing volume, weight and cost of a transformer is emphasized for the majority of power supply designs [1-4]. The design optimization of a single-phase high frequency transformer by using Genetic Algorithms (GAs) [5] approach is investigated next.

2. Design Considerations

Switching power supplies have become popular because of their ability to operate at high frequencies, and hence increasing their efficiency. A switching power supply that satisfies the same performance requirements of a linear power supply can be many times smaller in size. Since the induced voltage in a transformer is dependent upon the changing magnetic flux, the more the flux (higher frequency) is changed, the smaller and more efficient the transformer becomes.

Some design considerations of a high frequency power transformer are actually due to:

- i) Selection of topology
- ii) Selection of transformer core – material, shape/geometry, size
- iii) Consideration of peak flux density
- iv) Core loss determination
- v) Winding coil loss determination
- vi) Temperature rise determination

The first step is to select the power supply specification pertaining to the transformer design as in Table 1.

Table 1: Power supply specification

Parameters	Specification
Vin Range	100 V
Output	5 V, 50A
Circuit Topology	Forward Converter
Switching Frequency	200 kHz
Transformer Frequency	200 kHz
Cooling Method	Natural Convection
Transformer Efficiency	> 99 %
Permissible temperature rise in transformer	< 55°C

Five design variables are selected for the optimization problem and the choice is based on significant effect on the weight, losses, core dimensions and cost of the transformer.

- i) Flux density in core, x_1
- ii) Current density in high voltage winding, x_2
- iii) Current density in low voltage winding, x_3
- iv) Height of winding, x_4
- v) Voltage per turn, x_5

Design optimization is performed satisfying the imposed constraints and minimizing the three objective functions viz., volume, weight and active material cost of transformer independently. The specified objective functions are nonlinear and the mathematical expressions are determined, based on the selection of core material, shape and size. The following core is selected from the manufacturer data sheet.

Core Material: Ferrite, Magnetic Type P

Core Type: ETD

Core size: 34mm- ETD34

3. OBJECTIVE AND CONSTRAINT FUNCTIONS

The mathematical expressions of objective functions for minimum volume, minimum weight and minimum cost and similarly for the constraints functions are derived in terms of the selected design variables. The degree of utilization of a high frequency transformer is generally limited by the permissible temperature rise of the windings and also by the allowable losses [6]. Thus two important constraint functions to be satisfied in this design are:

- 1. Efficiency $\geq 99\%$
- 2. Temperature rise $\leq 55^\circ\text{C}$

The objective function as well as the constraint functions are combined to form an augmented objective function before GAs approach is applied, to determine the optimum design variables. This technique is known as Zangwill's exterior penalty function method [7], as described in Section 6 of Part I.

4. RESULTS AND DISCUSSION

A C++ optimization program has been developed based on the GAs library [5] and all the design equations. The design optimization program is run with Microsoft Visual C++. The results from the program's output are studied remembering that the objective of the program is to find out the optimum values of independent variables, which provide minimum value of the objective functions satisfying the imposed constraints. In order to analyze the results from the program, some of the

parameters in the program have been changed. These include GA parameters, penalty factor, and also the ranges of the independent variables. However, it is not practical to have a wide range on design variables although it really contributes to the minimization of objective functions. First, the variation of parameters of augmented objective function is studied to determine how these parameters affect the results. The selected values of penalty factor are 10, 100, 1000, 10000 and 100000 whereas the z value is 3 in the augmented objective function. This value is selected because when z is 2, the results are inconsistent as GA program is not able to converge the augmented objective function to a minimum value. GA parameters of 100, 100, 0.01, 0.6 for number of generations, population size, probability of mutation and probability of crossover respectively are assumed. The corresponding results of volume, weight and cost functions are shown in Figs.6, 7 and 8.

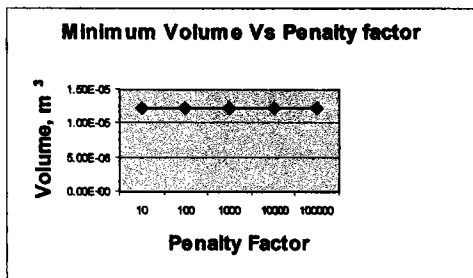


Fig. 6 Minimum Volume Function (m^3) –penalty factor

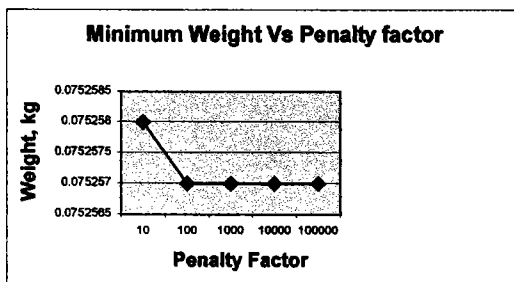


Fig. 7 Minimum Weight Function (kg) - penalty factor.

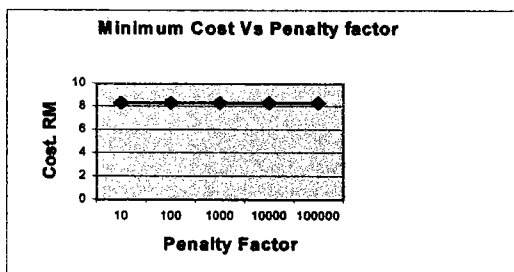


Fig. 8 Minimum Cost Function (RM)- penalty factor

Increasing the number of generations selected as Ngen = 50, 100, 200 and 500 and the size of populations as Npop = 50, 100, 200, 500 and 1000, the corresponding results are shown in the following Figs. 9,10 and 11.

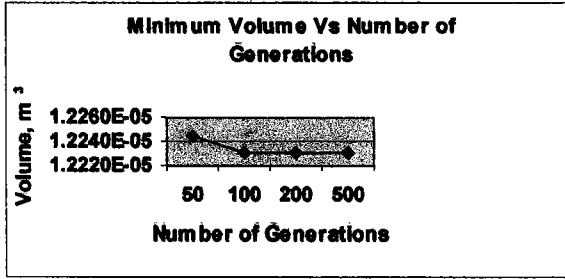


Fig. 9 Minimum Volume Function (m^3) for different Ngen and Npop = 50

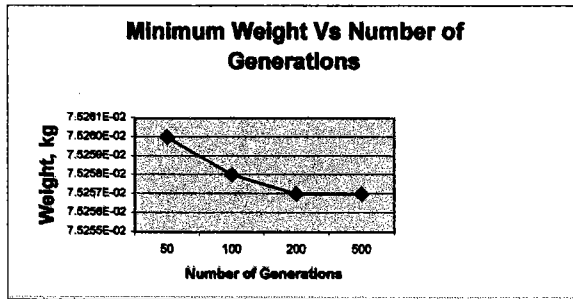


Fig. 10 Minimum Weight Function (kg) for different Ngen and Npop =50

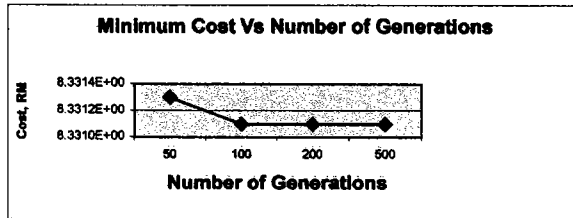


Fig. 11 Minimum Cost Function (RM) for different Ngen and Npop =50

From the above results, the following observations are made:

1. When number of generations and population size are below 100, the minimum value of objective functions cannot occur. It is because GA is not able to converge the objective function to a minimum value in time.
2. When both number of generations and population size exceed 100, a minimum objective function can be obtained. It is because GA is now able to converge the objective function to a minimum value.

Appropriate GA parameters and penalty factor have been selected for three different objective functions and the final results obtained are shown in Table 3. The transformer design data at minimum cost for one optimal solution is shown in Table 4.

Table 3: Optimum design variables

Optimum results	Volume Function	Weight Function	Cost Function
Minimum value	$1.22 \times 10^{-5} \text{ m}^3$	0.075 kg	RM 8.33
Temperature rise	51 °C	51 °C	51 °C
Efficiency	98.78 %	98.78 %	98.78 %
Flux density in core, x1	0.1199 T	0.1022 T	0.1199 T
Current density in high voltage winding, x2	4.00 A/mm ²	4.00 A/mm ²	4.00 A/mm ²
Current density in low voltage winding, x3	4.50 A/mm ²	4.50 A/mm ²	4.50 A/mm ²
Height / length of winding, x4	0.018 m	0.018 m	0.018 m
Voltage per turn, x5	3.00 V	3.00 V	3.00 V

Table 4 Transformer data at minimum cost

Flux density in core, T	x1 = 0.10021
Current density of hv winding, A/mm ²	x2 = 3.2055
Current density of lv winding, A/mm ²	x3 = 4.4799
Height of winding, m	x4 = 0.014125
Voltage per turn, V	x5 = 2.8909
Primary winding turns	T ₁ = 13.837
Secondary winding turns	T ₂ = 1.8679
Primary winding area of c.s., mm ²	a ₁ = 1.9498
Secondary winding area of c.s., mm ²	a ₂ = 10.334
Width of primary winding, m	b ₁ = 0.0046506
Width of secondary winding, m	b ₂ = 0.0034633
Thickness of foil conductor of lv winding, m	B ₂ = .00073164
Diameter of conductor of hv winding, m	dc1 = 0.0015753
Current in primary winding, A	I _p = 6.25
Current in secondary winding, A	I _s = 46.296
Length of center leg / limb, m	L = 0.018125
% Efficiency of transformer	η = 98.722
Temperature rise of winding, °C	ΔT = 54.981

5. CONCLUSIONS

A nonlinear mathematical model with different objective functions and important constraints of a high frequency transformer is developed. Optimization of high frequency power transformer used in switching power supplies has been successful by GA approach. The program has determined a set of optimum design variables that contribute to the minimization of three objective functions viz., (1) Minimum volume, (2) Minimum weight and (3) Minimum cost.

Although the program developed in this investigation is subject to certain changes, it can be adopted to any specification of high frequency transformer with little modifications for material selection, losses and thermal model. It is proved that a set of optimum design variables of a high frequency power transformer can be determined using the GA optimization program. A large number of design variables may be selected and the mathematical model can be improved. Similarly the number of constraints can be increased. Using the optimum design parameters and other design data obtained in this study, a transformer may be constructed and tested to study the performance characteristics.

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