UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2007/2008

April 2008

MSS 211 – Modern Algebra
[Aljabar Moden]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all nine** [9] questions.

*[**Arahan** : Jawab **semua sembilan** [9] soalan.]*

1. Given $\xi = \{x \in \mathbb{Z} \mid -12 \le x \le 12\}$, $A = \{\text{primes}\}$, $B = \{\text{odd integers}\}$ and $C = \{x \mid x^3 - x = 0\}$, obtain the following sets:
   (a) $A \cap B$
   (b) $A - B$
   (c) $A \cup B \cup C$

[6 marks]

2. Prove or disprove the following statement:
   "If A, B and C are sets, then $A \cap B \subseteq C \iff A \subseteq C$ or $B \subseteq C$".

[8 marks]

3. (i) Given that $f : A \to B$ and $g : B \to C$ are functions such that $f \circ g$ is bijective, show that f is one-to-one and g is onto.

   (ii) Given the functions $f : \mathbb{Z} \to \mathbb{Q}$ and $g : \mathbb{Q} \to \mathbb{R}$ where $(x)f = \dfrac{x-3}{x^2+1}$ and $(x)g = \sqrt{x^2+3}$, define $f \circ g$. Hence, determine which of f, g and $f \circ g$ are one-to-one and which are onto. If any of these functions is bijective, find its inverse.

[12 marks]

4. Consider the binary system $\langle S, * \rangle$ in the Cayley table below:

$*$	**e**	**a**	**b**	**c**	**d**
e	e	a	b	c	d
a	a	e	d	b	c
b	b	c	e	d	a
c	c	d	a	e	b
d	d	b	c	a	e

State the definition for each of the binary system below and hence classify whether $\langle S, * \rangle$ is a

(i) quasigroup,
(ii) loop, or
(iii)group.

[12 marks]

1. *Diberikan* $\xi = \{x \in \mathbb{Z} \mid -12 \le x \le 12\}$, $A = \{\textit{nombor perdana}\}$, $B = \{\textit{integer ganjil}\}$ *dan* $C = \{x \mid x^3 - x = 0\}$, *dapatkan set berikut:*
   (a) $A \cap B$
   (b) $A - B$
   (c) $A \cup B \cup C$

   [6 markah]

2. *Buktikan atau sangkalkan pernyataan berikut:*
   "Jika A, B *dan* C *merupakan set, maka* $A \cap B \subseteq C \Leftrightarrow A \subseteq C$ or $B \subseteq C$ *".*

   [8 markah]

3. (i) *Diberikan* $f : A \to B$ *dan* $g : B \to C$ *merupakan fungsi-fungsi supaya* $f \circ g$ *fungsi bijektif, tunjukkan bahawa* f *satu-ke-satu dan* g *keseluruh.*

   (ii) *Diberikan fungsi-fungsi* $f : \mathbb{Z} \to \mathbb{Q}$ *dan* $g : \mathbb{Q} \to \mathbb{R}$ *dengan* $(x)f = \dfrac{x-3}{x^2+1}$ *dan* $(x)g = \sqrt{x^2+3}$, *takrifkan* $f \circ g$. *Dengan demikian, tentukan yang manakah di antara* f, g *dan* $f \circ g$ *merupakan satu-ke-satu dan yang manakah merupakan keseluruh. Sekiranya sebarang dari fungsi-fungsi ini bijektif, cari songsangnya.*

   [12 markah]

4. *Pertimbangkan sistem dedua* $\langle S, * \rangle$ *yang dipaparkan oleh sifir Cayley berikut:*

*	e	a	b	c	d
e	e	a	b	c	d
a	a	e	d	b	c
b	b	c	e	d	a
c	c	d	a	e	b
d	d	b	c	a	e

Nyatakan takrif bagi setiap sistem dedua yang berikut dan dengan itu tentukan sama ada $\langle S, * \rangle$ *merupakan suatu*

(i) *kuasikumpulan,*
(ii) *lup, atau*
(iii) *kumpulan.*

[12 markah]

5. Suppose $\langle S,*\rangle$ is a binary system which contains both a left identity element e_L and a right identity element e_R. Obtain the relationship between e_L and e_R. Is it possible for $\langle S,*\rangle$ to contain another left or right identity element?

[10 marks]

6. (i) State Lagrange's Theorem and its corollary on the order of elements.

   (ii) List all the elements of A_4 and obtain the order of each element.

   (iii) Hence, obtain all the subgroups of A_4, classifying each as normal or not.

[15 marks]

7. Write the definition of a *permutation on a set*. Given that $\langle G,*\rangle$ is a group, g is a fixed element of G, and the function $\lambda_g : G \to G$ which is defined as $(x)\lambda_g = x * g$ for all $x \in G$, show that λ_g is a permutation on G.

   (a) For the case $\langle G,*\rangle = \langle S_3,\circ\rangle$, find $\lambda_{(1\ 2\ 3)}$ and $\lambda_{(1\ 3)}$.

   (b) Show that for the general case, $\lambda_g \circ \lambda_h = \lambda_{g*h}$ for any fixed elements $g, h \in G$.

[12 marks]

8. Show that, up to isomorphism, there exist only two groups of order 6. Obtain a quotient group of order two in each case.

[15 marks]

9. State the definition of a ring. Hence, given the definition of the operations $\oplus$ and $\otimes$ on $\mathbb{R}$ as

$$x \oplus y = x + y - 1 \text{ and } x \otimes y = x + y - xy,$$

prove that $\langle \mathbb{R},\oplus,\otimes\rangle$ is a ring. Determine whether $\langle \mathbb{R},\oplus,\otimes\rangle$ is a commutative ring and if there exists a unity element in it.

[10 marks]

5. *Katakan* $\langle S,*\rangle$ *merupakan suatu sistem dedua yang mempunyai suatu unsur identiti kiri* e_L *serta unsur identiti kanan* e_R. *Dapatkan hubungan antara* e_L *dan* e_R. *Mungkinkah* $\langle S,*\rangle$ *mengandungi satu lagi unsur identiti kiri atau kanan?*

[10 markah]

6. (i) *Nyatakan Teorem Lagrange serta korolarinya berkenaan peringkat unsur.*

   (ii) *Senaraikan semua unsur bagi* A_4 *dan dapatkan peringkat bagi setiap unsur tersebut.*

   (iii) *Dengan demikian, dapatkan semua subkumpulan bagi* A_4, *seterusnya klasifikasikan setiapnya sebagai subkumpulan normal atau tidak.*

[15 markah]

7. *Tuliskan takrif bagi pilihatur atas suatu set. Diberikan* $\langle G,*\rangle$ *suatu kumpulan,* g *suatu unsur tetap dalam* G, *dan fungsi* $\lambda_a : G \to G$ *yang tertakrif sebagai* $(x)\lambda_g = x * g$ *bagi semua* $x \in G$, *tunjukkan bahawa* λ_g *merupakan suatu pilihatur atas* G.

   (a) *Bagi kes* $\langle G,*\rangle = \langle S_3,\circ\rangle$, *cari* $\lambda_{(1\ 2\ 3)}$ *dan* $\lambda_{(1\ 3)}$.

   (b) *Tunjukkan bahawa secara am,* $\lambda_g \circ \lambda_h = \lambda_{g*h}$ *bagi sebarang unsur tetap* $g, h \in G$.

[12 markah]

8. *Tunjukkan bahawa, sehingga isomorfisma, wujud hanya dua kumpulan berperingkat 6. Dapatkan suatu kumpulan hasil bahagi berperingkat dua dalam setiap kes.*

[15 markah]

9. *Nyatakan takrif gelanggang. Dengan itu, diberikan takrif bagi operasi* $\oplus$ *dan* $\otimes$ *atas* $\mathbb{R}$ *sebagi*

$$x \oplus y = x + y - 1 \text{ dan } x \otimes y = x + y - xy,$$

buktikan bahawa $\langle \mathbb{R},\oplus,\otimes\rangle$ *merupakan suatu gelanggang. Tentukan sama ada* $\langle \mathbb{R},\oplus,\otimes\rangle$ *suatu gelanggang kalis tukar tertib serta kewujudan unsur uniti di dalamnya.*

[10 markah]

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