PENGOPTIMUMAN PENYAMBUNG BAR DENGAN MENGGUNAKAN ALGORITMA GENETIK

(OPTIMIZATION OF BAR LINKAGE BY USING GENETIC ALGORITHMS)

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Last but not least, I would like to thank all my friends who had helped me more than I could have asked for. I am sorry if your name is not stated here, but the help you had given to me will always be in my heart and memory.

Thank you.

TABLE OF CONTENTS

CAHPTER 3

CHAPTER 4

List Of Tables

List Of Figures

List Of Symbols

$\hat{x}$	= local x axes
$\hat{y}$	= local y axes
$\hat{z}$	= local z axes
$\hat{d}$	= local displacement
d	= global displacement
$\hat{\phi}$	= local nodal degrees of freedom
ϕ	= global nodal degrees of freedom
T	= transformation matrix
$\underline{k}$	= global stiffness matrix
$\hat{f}$	= axial force
$\hat{m}$	= moment
u	= displacement in x-direction
v	= displacement in y-direction
E	= modulus of elasticity
ρ	= density
A	= cross-sectional area
L	= length
σ	= stress
m	= number of constraints
$f(x)$	= fitness function to minimize weight
C	= violation coefficient
φ	= modified objective function
F	= fitness factor
K	= judiciously selected parameter

ABSTRACT

This thesis presents the method of using simple Genetic Algorithms (GAs) in optimizing the size of bar linkage with discrete design variables and continues design variables. Penalty-based transformation method is used in this thesis to change the constrained problems into unconstrained ones. It is well known that GAs is best suited for unconstrained optimization problems. Optimization process will consider, lightest weight of the bar by using GAs without exceeding the allowable stress determined. Two types of problems are chosen to optimize in this thesis. First problem is optimization of four bar linkage under static loading condition and second problem is optimization of four bar linkage under dynamic condition without loading. The result from GA for this problem is compared with some other methods which are presented by V.V Tropov and V.L Markine.It is observed that GA gives better solution compared to other published methods.

ABSTRAK

Projek ini membincangkan Algoritma Genetik sebagai satu kaedah untuk mengoptimumkan sistem penyambung empat bar di bawah daya statik dan keadaan dinamik dengan pembolehubah rekabentuk yang diskrit dan selanjar. Dalam kes statik, daya statik diaplikasi ke atas penyambung bar, manakala dalam kes dinamik, daya luar tak bertindak ke atas bar tersebut.Beberapa kes yang lazim dibincangkan dan diselesaikan dengan kaedah Algoritma Genetik. Algoritma Genetik adalah kaedah yang paling sesuai untuk masalah pengoptimuman tanpa kekangan. Oleh itu adalah penting untuk menukarkan masalah yang melibatkan kekangan kepada masalah yang tidak melibatkan kekangan.Satu kaedah yang berasaskan faktor denda untuk penjelmaan diaplikasikan dalam kajian ini.Konsep dalam pengoptimuman menggunakan Algoritma Genetik diselesaikan secara lebih mendalam menggunakan penyambung empat bar. Jisim minimum dioptimumkan dengan menggunakan GA dalam kajian ini.Diperhatikan bahawa , Algoritma Genetik memberi penyelesaian yang baik jika dibandingkan dengan kaedah-kaedah yang dipersembahkan oleh V.V.Toropov dan V.L Markine (1998).

CHAPTER 1
INTRODUCTION

One of the main concerns in engineering field is the weight of the designed structures. Engineers throughout time have been working on reducing the weight while maintaining the strength and keeping it within allowable stress. The design of structures and machines often consists of finding the best solution among a finite number of feasible choices.

This thesis introduce structural optimization for reducing the weight of selected bar linkage by means of evolutionary algorithm that is "GENETIC ALGORITHM". This thesis presents four bar mechanism under static loading condition and dynamic condition. The dynamic optimization of four bar mechanism has been presented by V.V.Toropov and V.L Markine (1998). They are used simplified numerical models as approximations to optimize dynamic design problem.

In order to reduce the weight, there is only one parameter that can be altered and that is the cross section area of the bar. By using genetic algorithm, various size of area can be tested simultaneously. This method will certainly save much time that could take one human hours to do the same

The approach is known as bar linkage optimization under static loads and dynamics, based on displacement. This thesis shows the feasibility of genetic algorithms for search and optimization problems. This problem was first formulated and mathematical programming problems and then transformed into unconstrained problem using an exterior penalty function approach. This thesis presents a formulation and solution technique using GA for the discrete optimization of four bars under static loading and continues optimization of four bars under dynamic condition without apply load.

1.1 Structural optimization

A structure is a connected system of members designed to support loads safely or to transfer forces effectively. Structures formed by pin-connected members are known as bar linkage. The analysis of structures involves analyzing the equilibrium of the entire structure, part of the structure, or an individual member. Structural optimization is one of the interesting topic in optimization problem. There are several methods in structural optimization such as hill climbing, enumerative, random search algorithms and randomized search techniques.

1.2 Objective

This thesis will focus on optimizing the four bar linkage design by the use of Genetic Algorithm. The main concern is in minimizing the area of each bar in the linkage structure as a measure of reducing the overall weight in order to optimize the design while maintaining the strength of the design within the allowable stress limit.

This thesis also presents solution for static analysis for four bar linkages under static loading condition and solving a dynamic condition for four bar linkages. A few algorithms have been developed to handle the discrete nature of design variables. In other word, a few methods have been reported for optimal design of discrete bar linkage systems and they are found to be useful in solving a number of problems.

CHAPTER 2
GENETIC ALGORITHMS

2.1 Definition of Genetic Algorithms

Genetic algorithms are parameter search procedures based upon the mechanics of natural genetics. Genetic algorithm is a model of machine learning. The machine copied its behavior from a metaphor of the processes of evolution in nature. Individuals represented by chromosomes are created within the machine. The individuals then go through the evolution process, which is, according to Darwin, made up of the principles of mutation and selection.

However, modern evolution theory introduces crossover and isolation mechanisms to improve the adaptive ness of the individuals to their environments. Genetic algorithms allow elements to be swap between individuals as if by sexual combination and reproduction, others are changed by mutation.

New generations are generated from the current population, in proportion to their fitness. A single objective function of the parameters returns a numerical value to distinguish between good and bad solutions. Fitness is then used to apply pressure in selection to the population in a 'Darwin' fashion.

Genetic algorithms need the set of parameter of the optimization problem to be coded as a finite-length string (analogous to chromosomes in biological systems) containing characters, features or detectors. Usually, the binary string that consists of characters 0 and 1 is taken.

The difference between GA and other normal optimization and search procedures are listed below:

- GAs work with a coding of the parameter set and not the parameters themselves.
- GAs search from a population of points and not a single point.
- GAs use payoff (objective function) information and not derivatives or other auxiliary knowledge.
- GAs use probabilistic transition rules and not deterministic rules.

2.2 How Genetic Algorithms Starts?

Research in genetic algorithms was mainly theoretical with few applications. This period is marked by abundant work with fixed length binary representation in function optimization. From this period also, the community of genetic algorithms has experienced plenty of applications, which spread across a large range of fields.

Furthermore, in the process of improving performance of genetic algorithm, new and important findings regarding the generality, robustness and applications of genetic algorithms became available. Active development of genetic algorithms for the last few years in the sciences, engineering and the business world, these algorithms have now been successfully applied to optimization problems, scheduling, data fitting and clustering, trend spotting and path finding.

2.3 Application of Genetic Algorithms

The continuing price and performance improvements of computational systems have made Genetic Algorithms attractive for many types of optimization. In particular, genetic algorithms work very well on mixed (continuous and discrete) and combinatorial problems. They are less at risk to being jammed at local optimum than gradient search methods. However, Genetic Algorithms could be computationally expensive.

Genetic Algorithms (GA's) have gained fame over the past few years with space applications due to their simplicity and power in solving complex problems. GA's have been successful in optimizing low-thrust transfer orbits, deep-space maneuvers, constellation design and dynamics of tethered systems. This chapter will be discussing about the working of GA and explanation of the operators involved in detail.

2.4 Operators of GA

Here are some of GA operators:

- Reproduction
- Crossover
- Mutation
- Dominance
- Inversion
- Intrachromosomal duplication
- Deletion
- Translocation
- Segregation
- Speciation
- Migration
- Sharing

2.5 The GA Cycle

Three basic operation s that characterize GA's are respectively: selection, crossover and mutation. Suppose M *(t)* is the population of chromosomes at generation t, the structure of a simple GA for a particular application consists of the following principle phases (see Goldberg 1989 for more details)

2.5.1 Pseudo Code

An abstract view of an algorithm can be given by pseudo code. The genetic algorithms pseudo code is very simple as shown in table 1 below.

Table 2.1: Pseudo Code of a Genetic Algorithm

i = 0	Set generation number to zero
Initial population P (0)	Initialize a usually random population of individuals
evaluate P (0)	Evaluate fitness of all initial individuals of population
while (not done) do	Test for termination criterion (time, fitness, etc.)
begin	
i = i + 1	Increase the generation number
select P (i) from P(i-1)	Select a sub-population for offspring reproduction
Recombine P (i)	Recombine the genes of selected parents
mutate P (i)	Perturb the mated population stochastically
evaluate P (i)	Evaluate its new fitness
end	

2.5.2 Initial Population

In order to generate successive populations of strings, genetic algorithm will start with an initial or preliminary population of strings. The generation of this population is usually done randomly. This means that if binary strings are used where every allele is set to 0 or 1, each value have a chance of 50 % to occur.

2.5.3 Evaluation

Every individual of the population in a generation must be evaluated to distinguish between the strong and weak individuals. To do this, the objective function is mapped to a 'fitness function' to produce a non-negative figure of merit. According to Goldberg (1989), this can be done in the following ways:

- when the objective is maximization of a utility or profit function u(x) we must overcome the problem of negative u(x) values by transforming fitness according to the equation:

  $f(x) = u(x) + C_{min}$ when $u(x) + C_{min} > 0$,

  $f(x) = 0$ otherwise.

- when the objective is minimization of a cost function g(x) we must transform the minimization problem to a maximization problem and assure that the measure is non-negative by using the following cost-to-fitness transformation:

  $f(x) = C_{max} - g(x)$ when $C_{max} - g(x) > 0$,

  $f(x) = 0$ otherwise.

Input coefficient can be chose between C_{min} and C_{max} , as the absolute value of the worst u-value respectively and the largest g-value in the current or last k generations, or as a function of the population difference.

2.5.4 Reproduction

Genetic algorithms based the selection of parents on string fitness. The selection of parents is an important aspect that needed to be considered intensively for the purpose of procreation. Darwin's "survival of the fittest" principle gives the idea that string A, which is twice as fit as string B would be expected to appear twice as much in the next generation of population. It must be noted that genetic algorithm selects strings directly by their fitness.

Goldberg (1989) suggests that to implement reproduction into a simple genetic algorithm, a biased roulette wheel must be created where each current string in the population has a roulette wheel slot sized in proportion to its fitness. By just simply spinning the weighted roulette wheel a number of times as the number of the population size, would enable GAs to reproduce. Expected count of this individual in the next generation can be calculated by dividing an individual's fitness value with the average of all fitness values.

2.5.5 Recombination

Genetic algorithm combines selected parents to create two new offspring. This is done by using the crossover operator. There are many different types of crossover operators. This study used one-point crossover operator in applying genetic algorithms.

Recombination operator randomly and uniformly selects an integer k between 1 and the string length less than one [1, l-1]. Two new strings are formed by exchanging the bits between positions k and l . Example can be seen in table 2 below.

Table 2.2: The One-point Crossover Monitor

Before Crossover	After Crossover
A B C D | E F G H	A B C D | M N O P
I J K L | M N O P	I J K L | E F G H

The crossover operator allows the advantageous traits of an individual to be spread throughout the population so that the population as a whole may benefit from this.

2.5.6 Mutation

The last operator in genetic algorithm is mutation operator. The importance of this operator is still being highly debated by experts. Mutation is responsible in reintroducing divergence into a converging population. In the latter phases of a genetic algorithm run, it may be converging upon a local maximum. Mutation of individuals may find a way to past this. Parker (1992) suggest that the biological inspiration behind this operator is the way a chance mutation occurred in a natural chromosome can lead to the development of advantageous traits that will give the individual an advantage over other individuals

CAHPTER 3

ANALYSIS OF BAR LINKAGE

The design of the bar linkage that will be optimized is a four-bar linkage under static loading condition and dynamic analysis of four-bar linkage without loading. The reason to choose the selected problems is to show that Genetic Algorithm optimization method can be used for any type of bar linkages design. There are several criteria will be concern in analyzing bar linkage. There are described further in methodology.

Theory and methodology will decide the outcome or the result of the experiment. A slight mistake in this part could cause a major error in the end result. The first subtopic will discuss about developing the equations and method for solutions of static analysis of four bar linkages. This equations and methods are to be used in finding the displacement, axial force and the axial stress of every member in the linkages.

The method applied is from the finite element method, which incorporates the using of stiffness matrix to obtain the solution. The steps for the solution will be use later in the software MATLAB 7.0 to replicate the result and to be use to solve four bar linkage problems

3.1 Static analysis of four bar linkage

The four-bar mechanism shown below has external forces $\mathbf{F_P}$ acting on link 2. The ground is denoted as link 4. The system is in static equilibrium. Joint reaction forces $\mathbf{F_{12}}$, $\mathbf{F_{23}}$, $\mathbf{F_{34}}$, and $\mathbf{F_{14}}$ for static equilibrium at the position shown.

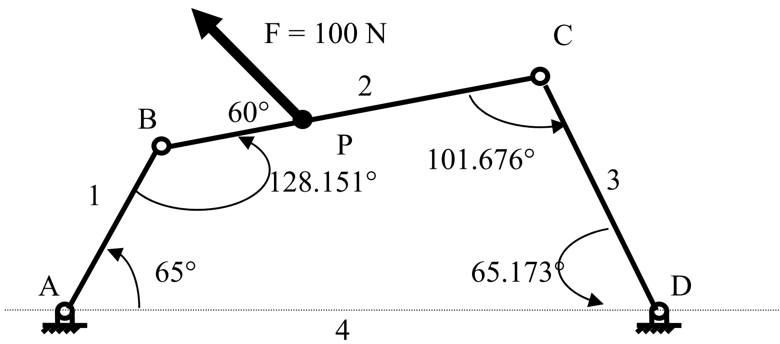

Figure 3.1: Four Bar Mechanism

Material properties for four bar linkage:

Modulus of Elasticity, Aluminum (Alloy 6061-T6), E = 68.95 Gpa

Density of Aluminum (Alloy 6061-T6) , $\rho = 2.757 \times 10^3$ kg/m^3

Maximum allowable stress, $\sigma_{al} = 27.579$ (MPa)

Area of each solid circular bar under consideration, A (cm^2),

S=10.125,11.25,12.4375,13.3125,14.875,16.375,16.4375,18.0,18.3125,19.3125,19.5625,21.125,21.6875,22.1875,22.6875,24.0,24.1875,24.25,26.125,26.375,28.0625,28.6875,30.0,31.0625,32.0,35.875,45.125,49.8125,71.875,84.375,86.875,88.75,96.875,100,105,625,117.5,124.375,137.5,143.125,165.625,187.5,209.375).

This list of discrete values is taken from the American Institute of Construction Manual (Arora)

Table 3.1 Element Characteristics of Four-bar linkage

Bar	Element	Length (m)	Area Type
AB	1	0.3048	A1 = A1
BC	2	0.9144	A2 = A2
CD	3	0.7620	A3 = A3
AD	4	0.9144	-

3.1.1 Free body diagram

Force calculation on each point shown in free body diagram below. External force is applied to link 2 as shown in diagram. Forces are calculated in each linkage part due to external force 100N.Detail calculation method can be found from (Mechanism Design - Analysis and Synthesis, Volume 1, A.G. Erdman and G.N. Sandor)

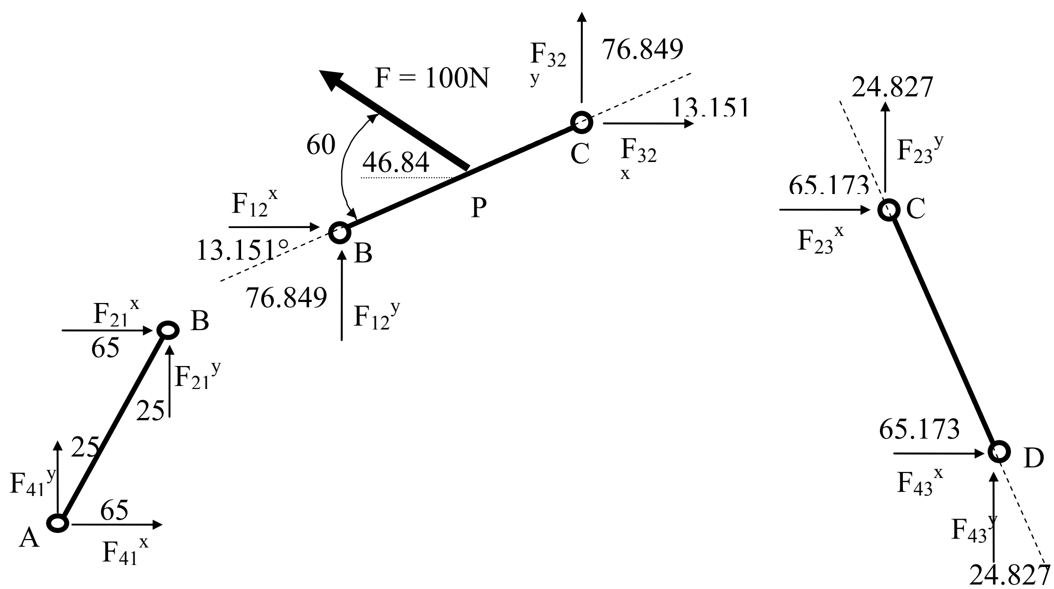

Figure 3.2 Free Body Diagram

3.1.2 Static force analysis

Static equilibrium equation will be used to analysis force and moment at each bar linkage. By using static equation we will obtain matrix form.

ΣF on 2 right + $F_{12}^{x} + F_{32}^{x} = 0$

ΣF on 2 up + $F_{12}^{y} + F_{32}^{y} = 0$

ΣM on 2 about A CCW + $- F_{32}^{x} + F_{32}^{y} = 0$

ΣF on 3 right + $F_{23}^{x} + F_{43}^{x} + F_{P}^{x} = 0$

ΣF on 3 up + $F_{23}^{y} + F_{43}^{y} + F_{P}^{y} = 0$

ΣM on 3 about P CCW + $- F_{23}^{x}\ r_{B/P}^{y} + F_{23}^{y}\ r_{B/P}^{x} - F_{43}^{x}\ r_{C/P}^{y} + F_{43}^{y}\ r_{C/P}^{x} = 0$

ΣF on 4 right + $F_{34}^{x} + F_{14}^{x} + = 0$

ΣF on 4 up + $F_{34}^{y} + F_{14}^{y} + = 0$

ΣM on 4 about Q CCW + $- F_{34}^{x} + F_{34}^{y} - F_{14}^{x} + F_{14}^{y} = 0$

By using matrix solution, we will obtain the forces acting on each bar.

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -r_{B/P}^{y} & r_{B/P}^{x} & r_{C/P}^{y} & -r_{C/P}^{x} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} F_{12}^{x} \\ F_{12}^{y} \\ F_{23}^{x} \\ F_{23}^{y} \\ F_{34}^{x} \\ F_{34}^{x} \\ F_{14}^{x} \\ F_{14}^{y} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -F_{P}^{x} \\ -F_{P}^{y} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Total force acting on each bar as shown below:

$$\begin{Bmatrix} F_{12}^{x} \\ F_{12}^{y} \\ F_{23}^{x} \\ F_{23}^{y} \\ F_{34}^{x} \\ F_{34}^{x} \\ F_{14}^{x} \\ F_{14}^{y} \\ \end{Bmatrix} = \begin{Bmatrix} 4847.93\text{ N} \\ -2990.90\text{ N} \\ -1991.30\text{ N} \\ 4304.59\text{ N} \\ 1991.30\text{ N} \\ -4304.59\text{ N} \\ 4847.93\text{ N} \\ -2990.90\text{ N} \\ \end{Bmatrix}$$

Stiffness matrix for static bar linkage will be obtained by using equation 3.24 to 3.34.Detail explanation will be given in dynamic analysis of four bar mechanism. Stiffness matrix four both, static analysis and dynamic analysis are same. Displacement of each bar linkage will be calculated easily by using equation 3.24.

3.2 Dynamic analysis of four bar mechanism

3.2.1 Analytical method for angle.

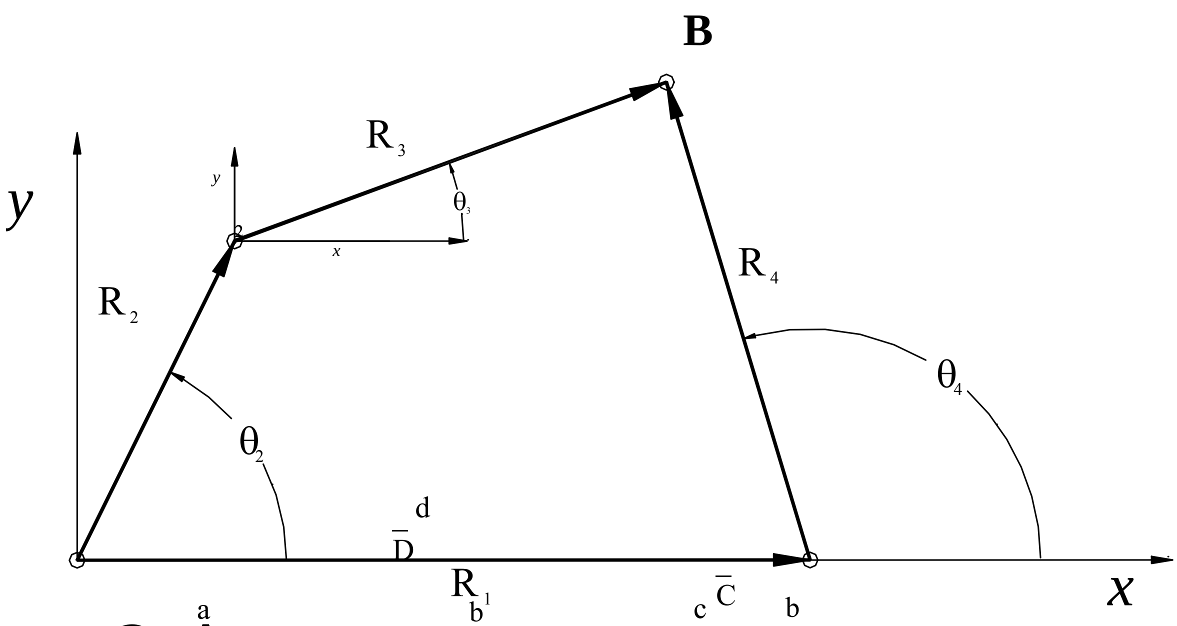

Figure 3.3: Position vector loop

The four bar linkage is based on dynamic analysis without external load. There are several approaches to solve the position analysis, all of which require insight and manipulation to obtain the desired output as a function of the input angle. The procedure used, in this research, is loop closure. Loop closure uses vector equations based on closed loops obtained from the mechanism. The vectors that describe the links are defined based upon the angle. Figure 3.3, shown the four-bar linkage operates in a vertical plane. Assume that the masses of the bearings and the effects of friction are negligible. Do not neglect the effects of gravity. Each link is modeled by one planar beam finite element, which allows the bending deformation.

The angle θ_2 is measured at the fixed pivot, thus $\mathbf{R}_2$ has its root at that point. A similar logic is used to define the vectors $\mathbf{R}_3$, $\mathbf{R}_4$, and $\mathbf{R}_1$. For convenience the direction of $\mathbf{R}_1$ is taken along the x-axis. From Figure 3.3, one can see the vectors that define a closed loop of the mechanism. Using the directions of the vectors in Figure3.4 the vector loop equation is given by

$$\mathbf{R}_1 + \mathbf{R}_4 - \mathbf{R}_3 - \mathbf{R}_2 = 0 \qquad (3.1)$$

where $\mathbf{R_1}$ is the vector describing the ground link, $\mathbf{R_2}$ is the vector that represents the second link often called the crank, $\mathbf{R_3}$ is the vector that describes the third link in the mechanism often called the coupler. $\mathbf{R_4}$ is the final vector that represents the fourth link often referred to as the output.

Using complex number notation, the position analysis can be completed for the four-bar linkage.

Although, one loop can describe the mechanism, using two simpler loops makes the position analysis easier. As mentioned earlier, position analysis requires insights and manipulations to obtain the output angles. In this instance, using two smaller loop equations is easier than a large complex loop.

$$\mathbf{A} + \mathbf{L_2} + \mathbf{L_3} = \mathbf{P}, \qquad (3.2)$$

where **A** is the vector from the origin to the first fixed pivot, $\mathbf{L_2}$ is the vector from the fixed pivot to the moving pivot that describes the link with the same name, $\mathbf{L_3}$ is the vector from the fixed pivot to the user input **P**.

Using complex notation, equation 3.2 is given by

$$Ae^{j\theta_A} + l_2 e^{j\theta_2} + l_3 e^{j\theta_3} = Pe^{j\theta_P} \tag{3.3}$$

where A is the distance from the origin to the

Equation 3.2 represents one vector equation with two unknowns. A vector equation can be broken down into two scalar equations. Therefore, the result of loop is two equations with two unknowns, a simple system of equations. The angles, θ_2 and θ_3 are embedded in transcendental functions, i.e. trigonometric functions of sine and cosine. Multiplying equation 3.3 by its complex conjugate allows one of the unknown angles to be isolated and solved for. The complex conjugate of equation 3.3 is,

$$Ae^{-j\theta_A} + l_2 e^{-j\theta_2} + l_3 e^{-j\theta_3} = Pe^{-j\theta_P} \tag{3.4}$$

Multiplying equation 3.2 and 3.3 together yields the following vector equation 3.4:

$$l_3^{\ 2} = l_2^{\ 2} + P^2 + A^2 - AP\left(e^{j(\theta_P-\theta_A)} + e^{j(\theta_A-\theta_P)}\right) - Pl_2\left(e^{j(\theta_P-\theta_2)} + e^{j(\theta_2-\theta_P)}\right) + Al_2\left(e^{j(\theta_A-\theta_2)} + e^{j(\theta_2-\theta_A)}\right)$$

Using basic trigonometric identities to simplify,

$$e^{j(\beta)} = \left(\cos\beta - \vec{i}\sin\beta\right) \tag{3.5}$$

$$l_3^{\ 2} = l_2^{\ 2} + P^2 + A^2 - 2AP(\cos\theta_P\cos\theta_A + \sin\theta_P\sin\theta_A) - 2Pl_2(\cos\theta_2\cos\theta_P + \sin\theta_P\sin\theta_2) + 2Al_2(\cos\theta_2\cos\theta_A + \sin\theta_2\sin\theta_A)$$

In the above equation, the unknowns are the two angles, since the lengths of the links are known. Using other trigonometric identities to simplify plug 3.6 and 3.7 into the above equation and group like terms yields a quadratic equation.

$$\cos(\beta) = \frac{1-t^2}{1+t^2} \tag{3.6}$$

$$\sin(\beta) = \frac{2t}{1+t^2} \tag{3.7}$$

An equation in a form of the quadratic equation is obtained.

$$At^2 + Bt + C = 0 \tag{3.8}$$

Using the standard quadratic equation to solve for t,

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{3.9}$$

Once t is obtained, trigonometric tangent half-angle identity is used to find θ_2,

$$t = \frac{\tan(\theta_2)}{2} \Rightarrow 2t = \tan(\theta_2) \therefore \theta_2 = \tan^{-1}(2t) \tag{3.10}$$

Once θ_2 has been solved for, θ_3 can be solved for

$$\theta_3 = \cos^{-1}\left(\frac{P\cos(\theta_P) - A\cos(\theta_A) - l_2\cos(\theta_2)}{l_3}\right) \quad (3.11)$$

3.2.2 Analytical method for force analysis

Force acting on each bar linkage is calculated by using matrix below with Matlab software. Force acting on each bar linkages are shown in figure 3. 4.

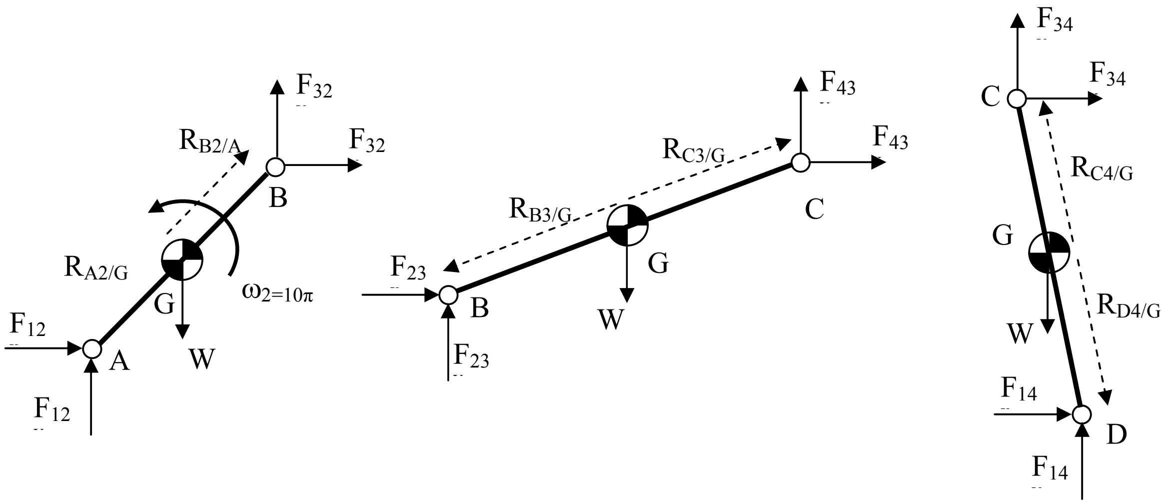

Figure 3.4: Free body diagram for four bar mechanism

ΣF on 2 right + $F_{12}^x + F_{32}^x = m_2 A_{G2}^x$

ΣF on 2 up + $F_{12}^y + F_{32}^y + W_2 = m_2 A_{G2}^y$

ΣM on 2 about G_2 CCW + $-F_{12}^x r_{A2/G2}^y + F_{12}^y r_{A2/G2}^x - F_{32}^x r_{B2/G2}^y + F_{32}^y r_{B2/G2}^x = J_{G2}' \alpha_2$

ΣF on 3 right + $F_{23}^x + F_{43}^x = m_3 A_{G3}^x$

ΣF on 3 up + $F_{23}^y + F_{43}^y + W_3 = m_2 A_{G3}^y$

ΣM on 3 about G_3 CCW + $-F_{23}^x r_{B3/G3}^y + F_{23}^y r_{B3/G3}^x - F_{43}^x r_{C3/G3}^y + F_{43}^y r_{C3/G3}^x = J_{G3}' \alpha_3$

ΣF on 4 right + $F_{34}^{x} + F_{14}^{x} = m_4 A_{G4}^{x}$

ΣF on 4 up + $F_{34}^{y} + F_{14}^{y} + W_4 = m_4 A_{G4}^{y}$

ΣM on 4 about G_4 CCW + $-F_{34}^{x} r_{C4/G4}^{y} + F_{34}^{y} r_{C4/G4}^{x} - F_{14}^{x} r_{D4/G4}^{y} + F_{14}^{y} r_{D4/G4}^{x} = J_{G4}' \alpha_4$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ -r_{A2/G2}^{y} & r_{A2/G2}^{x} & r_{B2/G2}^{y} & -r_{B2/G2}^{x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -r_{B3/G3}^{y} & r_{B3/G3}^{x} & r_{C3/G3}^{y} & -r_{C3/G3}^{x} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} F_{12}^{x} \\ F_{12}^{y} \\ F_{23}^{x} \\ F_{23}^{y} \\ F_{34}^{x} \\ F_{34}^{x} \\ F_{14}^{x} \\ F_{14}^{y} \end{Bmatrix} = \begin{Bmatrix} m_2 A_{G2}^{x} \\ m_2 A_{G2}^{y} - W_2 \\ J_{G2}^{'} \alpha_2 \\ m_3 A_{G3}^{x} \\ m_3 A_{G3}^{y} - W_3 \\ J_{G3}^{'} \alpha_3 \\ m_4 A_{G4}^{x} \\ m_4 A_{G4}^{y} - W_4 \end{Bmatrix}$$

3.2.3 Velocity analysis of four bar mechanism

Figure 3, shown the four-bar linkage operates in a vertical plane. Assume that the masses of the bearings and the effects of friction are negligible. Do not neglect the effects of gravity. Each link is modeled by one planar beam finite element, which allows the bending deformation.

Member MA is moving with angular velocity $\omega_2 = 10\pi$ rad/s in a counterclockwise direction. The objective is to develop explicit mathematical relationships giving V_A, V_B, α_3, α_4, A_A and A_B of the coupler and the output links. The relative positions of the four links QM, MA, AB and QB are described using vectors A,B,C and D where

$$\mathbf{D} = de^{i\theta_1}$$
$$\mathbf{A} = ae^{i\theta_2}$$
$$\mathbf{B} = be^{i\theta_3} \qquad (3.12)$$

$$\mathbf{C} = ce^{j\theta_4}$$

For vector polygon QMAB, the vector equation can be written as:

D + A + B = C (3.13)

-

That is ,

$$de^{j\theta_1} + ae^{j\theta_2} + be^{j\theta_3} = ce^{j\theta_4} \tag{3.14}$$

Angles θ_1, θ_2, θ_3 and θ_4 are measured in counterclockwise direction. In order to obtain velocity of each of the moving links, we must take the time derivative of Eq (3.14). Since a, b, c, d and θ1 ($\theta_1 = 180$ °) do not vary with time, the time derivative of Eq (3.14) yields

$$ja\frac{d\theta_2}{dt}e^{j\theta_2} + jb\frac{d\theta_3}{dt}e^{j\theta_3} = jc\frac{d\theta_4}{dt}e^{j\theta_4} \tag{3.15}$$

By definition

$$\omega_2 = \frac{d\theta_2}{dt} \quad \omega_3 = \frac{d\theta_3}{dt} \quad \text{and} \quad \omega_4 = \frac{d\theta_4}{dt}$$

Therefore, Eq (3.15) can be written as

$$ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} = jc\omega_4 e^{j\theta_4} \tag{3.16}$$

Separating the real and the complex parts, Eq (3.16) yields,

$$c\omega_4 \sin\theta_4 = a\omega_2 \sin\theta_2 + b\omega_3 \sin\theta_3 \tag{3.17}$$

$$c\omega_4 \cos\theta_4 = a\omega_2 \cos\theta_2 + b\omega_3 \cos\theta_3 \tag{3.18}$$

Simultaneous solution of Eqs (3.17) and (3.18) yields

$$\omega_3 = \frac{a\omega_2}{b} \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \tag{3.19}$$

$$\omega_4 = \frac{a\omega_2}{c} \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \tag{3.20}$$

3.2.4Acceleration analysis of a four bar mechanism

The explicit mathematical relationships for the calculation of acceleration of coupler and output links of a four bar mechanism are obtain by taking the time derivative of Eq (3.16); this yields

$$ja\alpha_2 e^{j\theta_2} + j^2 a\omega_2^2 e^{j\theta_2} + j^2 b\omega_3^2 e^{j\theta_3} + jb\alpha_3 e^{j\theta_3} = j^2 c\omega_4^2 e^{j\theta_4} + jc\alpha_4 e^{j\theta_4} \tag{3.21}$$

Where

$$\alpha_2 = \frac{d\omega_2}{dt}$$

$$\alpha_3 = \frac{d\omega_3}{dt}$$

$$\alpha_4 = \frac{d\omega_4}{dt}$$

Simultaneous solution of the above two equation in two unknowns α_3 and α_4 yields

$$\alpha_3 = \frac{CD - AF}{AE - BD} \tag{3.22}$$

$$\alpha_4 = \frac{CE - BF}{AE - BD} \quad (3.23)$$

Where,

$$A = c \sin\theta_4$$
$$B = b \sin\theta_3$$
$$C = a\alpha_2 \sin\theta_2 + a\omega_2^2 \cos\theta_2 + b\omega_3^2 \cos\theta_3 - c\omega_4^2 \cos\theta_4$$
$$D = c \cos\theta_4$$
$$E = b \cos\theta_3$$
$$F = a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 - b\omega_3^2 \sin\theta_3 + c\omega_4^2 \sin\theta_4$$

Equation 3.22 and 3.23 permit us to calculate the angular acceleration α_3 and α_4.

3.3 FEA analysis of 2D Bar Element

The stiffness matrix will be used in optimization of static loading condition as well as dynamic condition of four bar linkages. The stiffness matrix for an arbitrarily oriented beam element can be derive in a manner similar to that used for the bar element. The local axes $\hat{x}$ and $\hat{y}$ are located along the beam element and transverse to the beam element respectively and the global axes *x* and *y* are located to be convenient for the local structure. Local displacements can be relate to global displacements by using equation (3.24)

$$\begin{Bmatrix} \hat{d}_x \\ \hat{d}_y \end{Bmatrix} = \begin{bmatrix} C & S \\ -S & C \end{bmatrix} \begin{Bmatrix} d_x \\ d_y \end{Bmatrix} \quad (3.24)$$

Applying the equation for the beam element, local nodal degrees of freedom relate to global degrees of freedom.

$$\begin{Bmatrix} \hat{d}_{1y} \\ \hat{\phi}_{1y} \\ \hat{d}_{2y} \\ \hat{\phi}_{2} \end{Bmatrix} = \begin{bmatrix} -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{1y} \\ \phi_1 \\ d_{2x} \\ d_{2y} \\ \phi_2 \end{Bmatrix} \tag{3.25}$$

where, for a beam element, defines

$$\underline{T} = \begin{bmatrix} -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.26}$$

as the transformation matrix.

The axial effects are not yet included. Equation (3.25) indicates that rotation is invariant with respect to either coordinate system. For example, $\hat{\phi}_1 = \phi_1$, and moment $\hat{m}_1 = m_1$ can be considered to be a vector pointing normal to the $\hat{x} - \hat{y}$ plane or to the *x*-*y* plane by the usual right hand rule. From either viewpoint, the moment is in the $\hat{z} = z$ direction. Therefore, moment is unaffected as the element changes orientation in the *x*-*y* plane.

Substituting equation (3.26) for $\underline{T}$ and equation

$$k = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{3.27}$$

into equation $k = \underline{T}^T \underline{k}\underline{T}$ (3.27), global stiffness matrix was obtained as

$$\underline{k} = \frac{EI}{L^3}\begin{bmatrix} 12S^2 & -12SC & -6LS & -12S^2 & 12SC & -6LS \\ & 12C^2 & 6LC & 12SC & -12C^2 & 6LC \\ & & 4L^2 & 6LS & -6LC & 2L^2 \\ & & & 12S^2 & -12SC & 6LS \\ & & & & 12C^2 & -6LC \\ Symmetry & & & & & 4L^2 \end{bmatrix}$$

(3.28)

where, C = cos θ and S = sin θ. It is not necessary here to expand $\underline{T}$ given by equation (3.26) to make it a square matrix to be able to use equation (3.27). Because the equation is a generally applicable equation, the matrices used must merely be of the correct order for matrix multiplication.

The stiffness matrix equation (3.28) is the global element stiffness matrix for a beam element that includes shear and bending resistance. Local axial effects are not yet included.

The transformation from local to global stiffness by multiplying matrices $\underline{T}^T \underline{k}\underline{T}$, as done in equation (3.28), is usually done on the computer.

Axial effects will now be included in the element. The element now has three degrees of freedom per node $(\hat{d}_{ix}, \hat{d}_{iy}, \hat{\phi}_i)$. For axial effects, the equation (3.29) comes into the picture.

$$\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{2x} \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \hat{d}_{1x} \\ \hat{d}_{1y} \end{Bmatrix} \qquad (3.29)$$

Combining the axial effects of the above equation with the shear and principal bending moment effects, in local coordinates.