UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2007/2008

Jun 2008

MAA 111 – Algebra for Science Students
[Aljabar untuk Pelajar Sains]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all fifteen** [15] questions.

*[**Arahan** : Jawab **semua lima belas** [15] soalan.]*

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1. Consider the augmented matrix $C = (A|\mathbf{b})$ of a given linear nonhomogeneous system $A\mathbf{x} = \mathbf{b}$. Suppose $G = (M|\mathbf{d})$ is the reduced row echelon form of C.
   (a) What will you see in G if the system is inconsistent?
   (b) What will you see in G if the system has exactly one solution?
   (c) What will you see in G if the system has infinitely many solutions?

[6 marks]

2. If a matrix A is in row-echelon form, then write which vectors form a basis for the row space of A, and which vectors form a basis for the column space of A.

[4 marks]

3. Suppose $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ are orthonormal vectors in $\mathbb{R}^4$. Compute the length of the vector $\mathbf{v} = 2\mathbf{x}_1 - 3\mathbf{x}_2 - 2\mathbf{x}_3$.

[2 marks]

4. Describe the method to convert an arbitrary basis $V = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ of an inner product space $\mathbb{R}^n$ into an orthonormal basis.

[6 marks]

5. Write what it means for a vector $\mathbf{u}$ to be called a least squares solution of a linear system $A\mathbf{x} = \mathbf{b}$ of m equations in n variables. Then describe how to solve a least squares problem. When would you get a unique least square solution?

[7 marks]

6. What is the null space of a 5×4 matrix with linearly independent columns?
   What is the null space of a 4×5 matrix with linearly independent columns?

[4 marks]

7. Find *three* different ways of writing $(3,-4,7) \in \mathbb{R}^n$ as a linear combination of $\mathbf{x}_1 = (-1,0,1)$, $\mathbf{x}_2 = (1,-1,0)$, $\mathbf{x}_3 = (0,1,-1)$, and $\mathbf{x}_4 = (1,1,1)$.

[8 marks]

8. If $\det\begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix} = 3$, compute $\det(2C^{-1})$ where $C = \begin{pmatrix} 2p & -a+u & 3u \\ 2q & -b+v & 3v \\ 2r & -c+w & 3w \end{pmatrix}$.

[4 marks]

1. *Pertimbangkan matriks imbuhan* $C = (A|\mathbf{b})$ *bagi suatu sistem tak homogen linear* $A\mathbf{x} = \mathbf{b}$. *Andaikan* $G = (M|\mathbf{d})$ *adalah bentuk eselon baris terturun bagi* C.
   (a) *Apa yang dapat anda perhatikan dalam* G *jika sistem yang diberi adalah tak konsistent?*
   (b) *Apa yang dapat anda perhatikan dalam* G *jika sistem yang diberi mempunyai penyelesaian unik?*
   (c) *Apa yang dapat anda perhatikan dalam* G *jika sistem yang diberi mempunyai penyelesaian tak terhingga banyak?*

[6 markah]

2. *Andaikan suatu matriks* A *sudah dalam bentuk eselon baris. Berikan vector-vector yang membentukkan suatu asas bagi ruang baris* A, *dan juga vector-vector yang membentukkan suatu asas bagi ruang lajur* A.

[4 markah]

3. *Andaikan* $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ *adalah vektor-vektor ortonormal dalam* $\mathbb{R}^4$. *Carikan panjang bagi vektor* $\mathbf{v} = 2\mathbf{x}_1 - 3\mathbf{x}_2 - 2\mathbf{x}_3$.

[2 markah]

4. *Huraikan cara untuk menukarkan suatu asas sembarangan* $V = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ *bagi ruang hasil darab terkedalaman* $\mathbb{R}^n$ *menjadi suatu asas ortonormal.*

[6 markah]

5. *Berikan apa maksudnya kita katakan suatu vektor* $\mathbf{u}$ *ialah penyelesaian kuasa dua terkecil bagi sistem linear* $A\mathbf{x} = \mathbf{b}$ *yang mempunyai* m *persamaan dan* n *pembolehubah. Huraikan juga bagaimana menyelesaikan suatu masalah kuasa dua terkecil. Akhirnya sekali nyatakan bila kita akan dapat suatu penyelesaian kuasa dua terkecil yang unik?*

[7 markah]

6. *Apa itu ruang nol bagi matriks* 5×4 *yang mempunyai lajur tak bersandar linear? Apa itu ruang nol bagi matriks* 4×5 *yang mempunyai lajur tak bersandar linear?*

[4 markah]

7. *Berikan tiga cara berlainan untuk menuliskan* $(3, -4, 7) \in \mathbb{R}^n$ *sebagai suatu gabungan linear bagi* $\mathbf{x}_1 = (-1,0,1)$, $\mathbf{x}_2 = (1,-1,0)$, $\mathbf{x}_3 = (0,1,-1)$, *dan* $\mathbf{x}_4 = (1,1,1)$.

[8 markah]

8. *Diberi det* $\begin{pmatrix} a & b & c \\ p & q & r \\ u & v & w \end{pmatrix} = 3$ *dan* $C = \begin{pmatrix} 2p & -a+u & 3u \\ 2q & -b+v & 3v \\ 2r & -c+w & 3w \end{pmatrix}$. *Kirakan* $det(2C^{-1})$.

[4 markah]

9. If A is an $n \times n$ matrix and the homogeneous system $A^T\mathbf{x} = \mathbf{0}$ has a unique solution, what can you say about the dimension of the row space and the column space of A?

[2 marks]

10. Consider the augmented matrix of a linear system $\left(\begin{array}{cc|c} 1 & h & 2 \\ 2 & 4 & k \end{array}\right)$.

(a) Find h and k such that the system has one solution
(b) Find h and k such that the system is inconsistent
(c) Find h and k such that the system has infinitely many solutions

[9 marks]

11. If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $T\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$ and $T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$. Find a formula for $T\begin{pmatrix}x\\y\end{pmatrix}$.

[8 marks]

12. Let U be a subspace of $\mathbb{R}^3$ spanned by $\left\{\begin{pmatrix}1\\2\\0\end{pmatrix}, \begin{pmatrix}2\\4\\0\end{pmatrix}, \begin{pmatrix}1\\3\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$. Find the dimension of U.

[5 marks]

13. Let U be the subspace of $\mathbb{R}^4$ spanned by $\mathbf{u}_1 = (2,0,1,-1)$, $\mathbf{u}_2 = (3,2,0,1)$ and $\mathbf{u}_3 = (1,0,4,0)$. Find the projection of $\mathbf{w} = (-1,5,3,1)$ onto U.

[15 marks]

14. Find the quadratic polynomial that best fits the given points/data in the sense of least squares.

x	-3	-1	0	1	3
y	3	1	1	2	4

[10 marks]

15. Suppose h_k and r_k denote the number of hares and rabbits respectively in the year k. Assume that the number of those hares and rabbits changes over the years according to the following rule :

$$r_{k+1} = 10r_k - 3h_k$$
$$h_{k+1} = 4r_k - 2h_k$$

If there are 2 hares and 100 rabbits initially, what is the number of hares and rabbits in 2 years? in 20 years?

[10 marks]

9. *Jika A ialah suatu matriks* $n \times n$ *dan sistem homogen* $A^T\boldsymbol{x} = \boldsymbol{0}$ *mempunyai penyelesaian unik, apa yang boleh anda simpul tentang dimensi ruang baris dan ruang lajur bagi A?*

[2 markah]

10. *Pertimbangkan matriks imbuhan bagi sistem linear* $\left(\begin{array}{cc|c} 1 & h & 2 \\ 2 & 4 & k \end{array}\right)$.

(a) *Carikan h dan k sedemikian hingga sistem ini mempunyai penyelesaian unik.*

(b) *Carikan h dan k sedemikian hingga sistem ini tak konsisten*

(c) *Carikan h dan k sedemikian hingga sistem ini mempunyai penyelesaian tak terhingga banyak*

[9 markah]

11. *Jika* $T: \mathbb{R}^2 \to \mathbb{R}^2$ *ialah suatu transformasi linear sedemikian hingga* $T\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$ *dan* $T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$. *Cari suatu rumus bagi* $T\begin{pmatrix}x\\y\end{pmatrix}$.

[8 markah]

12. *Biar U suatu subruang bagi* $\mathbb{R}^3$ *yang direntang oleh* $\left\{\begin{pmatrix}1\\2\\0\end{pmatrix}, \begin{pmatrix}2\\4\\0\end{pmatrix}, \begin{pmatrix}1\\3\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$.

Carikan dimensi bagi U.

[5 markah]

13. *Biar U suatu subruang bagi* $\mathbb{R}^4$ *yang direntang oleh* $\boldsymbol{u}_1 = (2,0,1,-1)$, $\boldsymbol{u}_2 = (3,2,0,1)$ *and* $\boldsymbol{u}_3 = (1,0,4,0)$. *Cari unjuran bagi* $\boldsymbol{w} = (-1,5,3,1)$ *ke dalam U.*

[15 markah]

14. *Cari suatu polinomial kuadratik yang paling bersepadanan dengan data/titik yang diberi dari sudut kuasa dua terkecil.*

x	*-3*	*-1*	*0*	*1*	*3*
y	*3*	*1*	*1*	*2*	*4*

[10 markah]

15. *Andaikan* h_k *dan* r_k *menandakan bilangan kelinci dan arnab masing-masing pada tahun ke-k. Katakan bilangan kelinci dan arnab akan berubah dari setahun ke setahun mengikut peraturan seperti berikut:*

$$r_{k+1} = 10r_k - 3h_k$$
$$h_{k+1} = 4r_k - 2h_k$$

Jika pada mulanya ada 2 kelinci dan 100 arnab, berapa bilangan kelinci dan arnab akan diperolehi pada tahun ke-2? Pada tahun ke-20?

[10 markah]

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