
UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2007/2008

Jun 2008

MAA 102 – Calculus for Science Students II
[Kalkulus untuk Pelajar Sains II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

Arahan: Jawab **semua 10** [10] soalan.]

1. Determine whether the sequence $\left\{ \frac{2n}{n+1} \right\}$ is monotonic. [5 marks]

2. Test the convergence of the series:

(a) $\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$ (b) $\sum_{n=1}^{\infty} (-1)^n \frac{2^{2n}}{3^n}$ [10 marks]

3. (a) Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} n!(2x-1)^n.$$

- (b) Find the power series representation for $\frac{1}{1-x^2}$. Hence, find the first three terms of the power series representation of $f(x) = \frac{1+x^2}{1-x^2}$ which holds for $|x| < 1$.

[15 marks]

4. Determine whether the integral $\int_2^{\infty} \frac{2+\cos x}{x} dx$ is convergent or divergent. [5 marks]

5. (a) Find the domain of the function $f(x, y) = \ln(9 - 9y^2 - x^2)$.

- (b) Show that $w = \ln(x^2 + y^2 + z^2)$ satisfies the partial differential equation

$$x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} + z \frac{\partial w}{\partial z} = 2.$$

[7 marks]

6. Let $z = f(x, y)$, where f is differentiable, $x = r + \cos r$ and $y = e^r$.

If $f_x(1, 1) = 5$ and $f_y(1, 1) = 0$, find $\frac{dz}{dr}$ when $r = 0$.

[8 marks]

1. Tentukan sama ada jujukan $\left\{ \frac{2n}{n+1} \right\}$ monotonik.

[5 markah]

2. Uji penumpuan siri:

(a)
$$\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$$

(b)
$$\sum_{n=1}^{\infty} (-1)^n \frac{2^{2n}}{3^n}$$

[10 markah]

3. (a) Dapatkan jejari dan selang penumpuan bagi siri kuasa

$$\sum_{n=1}^{\infty} n!(2x-1)^n.$$

- (b) Dapatkan perwakilan siri kuasa bagi $\frac{1}{1-x^2}$. Seterusnya, dapatkan tiga sebutan pertama perwakilan siri kuasa $f(x) = \frac{1+x^2}{1-x^2}$ yang sah untuk $|x| < 1$.

[15 markah]

4. Tentukan sama ada kamiran $\int_2^{\infty} \frac{2 + \cos x}{x} dx$ menumpu atau mencapah.

[5 markah]

5. (a) Dapatkan domain bagi fungsi $f(x, y) = \ln(9 - 9y^2 - x^2)$.

- (b) Tunjukkan bahawa $w = \ln(x^2 + y^2 + z^2)$ memenuhi persamaan

pembezaan separa $x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} + z \frac{\partial w}{\partial z} = 2$.

[7 markah]

6. Biar $z = f(x, y)$, f fungsi yang terbezakan, $x = r + \cos r$ dan $y = e^r$.

Jika $f_x(1,1) = 5$ dan $f_y(1,1) = 0$, dapatkan $\frac{dz}{dr}$ bila $r = 0$.

[8 markah]

7. Find the rate of change of $f(x, y, z) = x^2y + y^2z + z^2x$ at $P(2, 1, 3)$ in the direction of the vector $\mathbf{v} = -2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$.

[10 marks]

8. Find the shortest distance from the point $(2, 1, -1)$ to the plane $x + y - z = 1$.

[12 marks]

9. Evaluate the integrals.

(a)
$$\int_0^4 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{x^2 + y^2}} dx dy.$$

(b)
$$\iint_D \frac{y}{x^2 + 1} dA, \text{ where } D \text{ is the region bounded by } y = \sqrt{x}, y = 0 \text{ and } x = 1.$$

[12 marks]

10. (a) Find the integrating factor for the differential equation $x \frac{dy}{dx} - 3y = x^3$.
Hence, find the particular solution when $y(1) = 1$.

- (b) Find the orthogonal trajectories of the family of curves $y = (x + k)^{-1}$, where k is an arbitrary constant.

[16 marks]

7. Dapatkan kadar perubahan $f(x, y, z) = x^2y + y^2z + z^2x$ pada $P(2, 1, 3)$ dalam arah vektor $\mathbf{v} = -2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$.
[10 markah]

8. Dapatkan jarak terdekat dari titik $(2, 1, -1)$ ke satah $x + y - z = 1$.
[12 markah]

9. Nilaikan kamiran.

(a)
$$\int_0^4 \int_0^{\sqrt{4y-y^2}} \frac{1}{\sqrt{x^2 + y^2}} dx dy.$$

(b)
$$\iint_D \frac{y}{x^2 + 1} dA, D \text{ merupakan rantau yang dibatasi oleh } y = \sqrt{x}, y = 0$$

dan $x = 1$.

[12 markah]

10. (a) Dapatkan faktor pengamir bagi persamaan pembezaan $x \frac{dy}{dx} - 3y = x^3$.
Seterusnya, dapatkan penyelesaian khusus bila $y(1) = 1$.

- (b) Dapatkan famili trajektori ortogonal bagi lengkung $y = (x + k)^{-1}$,
 k pemalar sebarang.

[16 markah]