
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2007/2008

April 2008

MAA 102 – Calculus For Science Students II
[Kalkulus Untuk Pelajar Sains II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

Arahan: Jawab **semua sepuluh** [10] soalan.]

1. Determine whether the sequence $\left\{ \frac{4^{n+1} + 3^n}{4^n} \right\}$ is monotonic and bounded.

[8 marks]

2. Test the convergence of the series:

(a)
$$\sum_{n=1}^{\infty} \frac{1}{3^{n-1} + 1}$$

(b)
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2\sqrt{n+1}}{\sqrt{n+1}}$$

[10 marks]

3. The series,

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \dots$$

converges for all x . Find the first six terms of a series for $\cos x$. Hence, find the first six terms of a series for $2 \sin x \cos x$.

[5 marks]

4. Determine whether the integral $\int_{\pi}^{\infty} \frac{2 + \cos x}{x} dx$ is convergent or divergent.

[5 marks]

5. (a) Find the domain of the function $f(x, y, z) = \frac{1}{\sqrt{1 - \ln(x^2 + y^2 + z^2)}}$.

- (b) If $z = f(x, y)$ where f is differentiable, $x = s + t$ and $y = s - t$, show

that
$$\left(\frac{\partial z}{\partial x} \right)^2 - \left(\frac{\partial z}{\partial y} \right)^2 = \frac{\partial z}{\partial s} \frac{\partial z}{\partial t}.$$

[10 marks]

6. The gas law for a fixed mass m of an ideal gas at absolute temperature T , pressure P and volume V is $PV = mRT$, where R is the gas constant. Show

that
$$T \frac{\partial P}{\partial T} \cdot \frac{\partial V}{\partial T} = mR.$$

[6 marks]

1. Tentukan sama ada jujukan $\left\{ \frac{4^{n+1} + 3^n}{4^n} \right\}$ monotonik dan terbatas.
[8 markah]

2. Uji penumpuan siri:

(a) $\sum_{n=1}^{\infty} \frac{1}{3^{n-1} + 1}$

(b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2\sqrt{n+1}}{\sqrt{n+1}}$

[10 markah]

3. Siri,

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \dots$$

menumpu bagi semua x . Dapatkan enam sebutan pertama siri $\cos x$.
Seterusnya, dapatkan enam sebutan pertama siri $2 \sin x \cos x$.

[5 markah]

4. Tentukan sama ada kamiran $\int_{\pi}^{\infty} \frac{2 + \cos x}{x} dx$ menumpu atau mencapah.
[5 markah]

5. (a) Cari domain bagi fungsi $f(x, y, z) = \frac{1}{\sqrt{1 - \ln(x^2 + y^2 + z^2)}}$.

- (b) Jika $z = f(x, y)$, f terbezakan, $x = s + t$ dan $y = s - t$, tunjukkan bahawa $\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \frac{\partial z}{\partial s} \frac{\partial z}{\partial t}$.

[10 markah]

6. Hukum gas ideal yang mempunyai berat tetap m pada suhu mutlak T , tekanan P dan isipadu V adalah $PV = mRT$, R adalah pemalar gas. Tunjukkan bahawa $T \frac{\partial P}{\partial T} \cdot \frac{\partial V}{\partial T} = mR$.

[6 markah]

7. Over a certain region of space, the electrical potential V is given by

$$V(x, y, z) = 5x^2 - 3xy + xyz.$$

- (a) Find the rate of change of the electrical potential at $P(3, 4, 5)$ in the direction of the vector $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.
- (b) In which direction does V decrease most rapidly at P and what is the rate of change in this direction?

[10 marks]

8. Find the points on the sphere $x^2 + y^2 + z^2 = 25$ where $f(x, y, z) = x + 2y - 3z$ has its maximum and minimum values.

[14 marks]

9. Evaluate the integrals.

(a)
$$\int_0^4 \int_0^{\sqrt{4y-y^2}} \frac{1}{\sqrt{x^2+y^2}} dx dy.$$

- (b)
$$\iint_D \frac{2y}{x} dA,$$
 where D is the region above the line $x + y = 1$ and inside the circle $x^2 + y^2 = 1$.

[12 marks]

10. (a) Determine whether the differential equation, $x \frac{dy}{dx} + y = 2xy^2$ is linear or Bernoulli equation. Hence, find its general solution.
- (b) Find the orthogonal trajectories of the family of curves $y = (x+k)^{-1}$ where k is an arbitrary constant.

[20 marks]

7. Pada suatu rantau, keupayaan elektrik V diberikan oleh

$$V(x, y, z) = 5x^2 - 3xy + xyz.$$

- (a) Dapatkan kadar perubahan keupayaan elektrik pada $P(3, 4, 5)$ dalam arah vektor $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.
- (b) Arah manakah V menyusut dengan cepatnya pada P dan apakah kadar perubahan di arah ini?

[10 markah]

8. Dapatkan titik-titik di sfera $x^2 + y^2 + z^2 = 25$, yang mana $f(x, y, z) = x + 2y - 3z$ mempunyai nilai maksimum dan minimum.

[14 markah]

9. Nilaikan kamiran.

(a)
$$\int_0^4 \int_0^{\sqrt{4y-y^2}} \frac{1}{\sqrt{x^2 + y^2}} dx dy.$$

(b)
$$\iint_D \frac{2y}{x} dA, D \text{ merupakan rantau di atas garis } x + y = 1 \text{ dan didalam bulatan } x^2 + y^2 = 1.$$

[12 markah]

10. (a) Tentukan sama ada persamaan pembezaan, $x \frac{dy}{dx} + y = 2xy^2$ adalah persamaan linear atau Bernoulli. Seterusnya, dapatkan penyelesaiannya.

(b) Dapatkan famili trajektori ortogonal bagi lengkung $y = (x + k)^{-1}$, k pemalar sebarang.

[20 markah]