UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2004/2005

October 2004

ZCT 211E/2 - Vector Analysis
[Analisis Vektor]

Duration : 2 hours
[Masa : 2 jam]

Please check that this examination paper consists of **FOUR** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **EMPAT** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer all **FIVE** (5) questions. Students are allowed to answer all questions in Bahasa Malaysia or in English.

*[**Arahan:** Jawab kesemua **LIMA** (5) soalan. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]*

1. (a) What vector must be added to the two vectors $\hat{i} - 2\hat{j} + 2\hat{k}$ and $2\hat{i} + \hat{j} - \hat{k}$, so that the resultant may be a unit vector along the x-axis?

[(a) *Apakah vektor yang perlu ditambah kepada dua vektor $\hat{i} - 2\hat{j} + 2\hat{k}$ dan $2\hat{i} + \hat{j} - \hat{k}$ supaya paduannya adalah satu vektor unit sepanjang paksi-x?*]

(20/100)

(b) If $\vec{A} = 4\hat{i} + 6\hat{j} + 2\hat{k}$ and $\vec{B} = \hat{i} + 6\hat{j} + k$, find the magnitudes and direction cosines of $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$.

[(b) *Jika $\vec{A} = 4\hat{i} + 6\hat{j} + 2\hat{k}$ dan $\vec{B} = \hat{i} + 6\hat{j} + k$, cari magnitud dan kosinus arah bagi $(\vec{A} + \vec{B})$ dan $(\vec{A} - \vec{B})$.*]

(40/100)

(c) A car is driven eastward for a distance of 5 kilometers, then northward for 3 kilometers and then in a direction 30° east of north for 10 kilometers. Draw the vector diagram and determine the total displacment of the car from its starting point.

[(c) *Sebuah kereta dipandu ke timur untuk jarak 5 kilometer, kemudian ke utara untuk jarak 3 kilometer, dan seterusnya di dalam arah 30° timur daripada utara untuk jarak 10 kilometer. Lukiskan gambarajah vektor dan tentukan jumlah sesaran kereta tersebut daripada titik permulaannya.*]

(40/100)

2. (a) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, show that

[(a) *Jika $\vec{a}$ dan $\vec{b}$ adalah vektor unit dan θ adalah sudut diantaranya, tunjukkan*]

$$sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$$.

(20/100)

(b) Show that $\vec{a}$ is perpendicular to $\vec{b}$ if $|\vec{a} + \vec{b}| = |\vec{a} - b|$.

[(b) *Tunjukkan bahawa $\vec{a}$ tegak lurus kepada b jika $|\vec{a} + \vec{b}| = |\vec{a} - b|$.*]

(20/100)

(c) If $\vec{A} = 4\hat{i} - 5\hat{j} + 3\hat{k}$, $\vec{B} = 2\hat{i} - 10\hat{j} - 7\hat{k}$ and $\vec{C} = 5\hat{i} + 7\hat{j} - 4\hat{k}$ deduce the values of

[(c) *Jika $\vec{A} = 4\hat{i} - 5\hat{j} + 3\hat{k}$, $\vec{B} = 2\hat{i} - 10\hat{j} - 7\hat{k}$ dan $\vec{C} = 5\hat{i} + 7\hat{j} - 4\hat{k}$ dapatkan nilai bagi*]

(i) $\left(\vec{A} \times \vec{B}\right).\vec{C}$ and *[dan]*

(ii) $\vec{A} \times (\vec{B} \times \vec{C})$.

(60/100)

3. (a) Show that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \vec{b} \times \left(\vec{c} \times \vec{a}\right) + \vec{c} \times \left(\vec{a} \times \vec{b}\right) = 0$.

[(a) Tunjukkan bahawa $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \vec{b} \times \left(\vec{c} \times \vec{a}\right) + \vec{c} \times \left(\vec{a} \times \vec{b}\right) = 0$.]

(15/100)

(b) Find the volume of a paralleopiped whose three coterminus edges are described by the vectors $\hat{i} + 2\hat{j}$, $4\hat{j}$ and $\hat{j} + 3\hat{k}$.

[(b) Cari isipadu bagi paralelepiped di mana tiga sisi koterminus adalah diberi oleh vektor $\hat{i} + 2\hat{j}$, $4\hat{j}$ dan $\hat{j} + 3\hat{k}$.]

(15/100)

(c) Differentiate $\dfrac{\vec{r} \times \vec{a}}{\vec{r}.\vec{a}}$, $\vec{a}$ being constant vector and $\vec{r}$ is the position vector.

[(c) Bezakan $\dfrac{\vec{r} \times \vec{a}}{\vec{r}.\vec{a}}$, $\vec{a}$ adalah vektor malar dan $\vec{r}$ ialah vektor kedudukan.]

(20/100)

(d) Find $\dfrac{\partial s}{\partial y}$, $\dfrac{\partial^2 s}{\partial x^2}$, $\dfrac{\partial^2 s}{\partial y^2}$, $\dfrac{\partial^2 s}{\partial x \partial y}$ for the following functions:

[(d) Cari $\dfrac{\partial s}{\partial y}$, $\dfrac{\partial^2 s}{\partial x^2}$, $\dfrac{\partial^2 s}{\partial y^2}$, $\dfrac{\partial^2 s}{\partial x \partial y}$ bagi fungsi-fungsi berikut]

(i) $\vec{s} = x \cos y\,\hat{i} + x \sin y\,\hat{j} + a e^{my}\,\hat{k}$, where m is an integer *[di sini m adalah integer]*

(ii) $\vec{s} = \dfrac{1}{2}\vec{a}(x+y)\hat{i} + \dfrac{1}{2}\vec{b}(x-y)\hat{j} + \dfrac{1}{2}xy\,\hat{k}$.

(50/100)

4. (a) Prove $\nabla r^n = n r^{n-2}\,\vec{r}$, where n is an integer.

[(a) Buktikan $\nabla r^n = n r^{n-2}\,\vec{r}$, di sini n adalah integer]

(25/100)

(b) Show that $\nabla.\nabla \phi = \nabla^2 \phi$ where $\nabla^2 \equiv \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$.

[(b) Tunjukkan bahawa $\nabla.\nabla \phi = \nabla^2 \phi$ di sini $\nabla^2 \equiv \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$.]

(25/100)

(c)   Find curl of the following functions:
[(c)   Cari keikalan fungsi-fungsi berikut:]

(i) $\bar{f} = \dfrac{x\hat{i} + y\hat{j}}{x + y}$

(ii) $\bar{g} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$

(50/100)

5.   (a)   (i)   Write the Maxwell's equations governing electromagnetic fields in a dielectric medium.
[(a)   (i)   Tuliskan persamaan Maxwell bagi medan elektromagnet dalam medium dielektrik.]

(ii)   What was the Maxwell's contribution toward Ampere's law?
[(ii)   Apakah sumbangan Maxwell terhadap Hukum Ampere?]

(25/100)

(b)   (i)   Write the Stoke's theorem in space.
[(i)   Tuliskan teorem Stokes dalam ruang.]

(ii)   If $\oint \bar{E}.d\bar{r} = -\dfrac{1}{c}\dfrac{\partial}{\partial t}\iint_s \bar{H}.d\bar{s}$,

[(ii)   Jika $\oint \bar{E}.d\bar{r} = -\dfrac{1}{c}\dfrac{\partial}{\partial t}\iint_s \bar{H}.d\bar{s}$,

where s is any surface bounded by curve c,
[di sini sebarang permukaan s disempadani oleh lengkung c,]

show that $\nabla \times \bar{E} = -\dfrac{1}{c}\dfrac{\partial \bar{H}}{\partial t}$

[tunjukkan bahawa $\nabla \times \bar{E} = -\dfrac{1}{c}\dfrac{\partial \bar{H}}{\partial t}$]

(50/100)

(c)   Given that $\bar{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$ when t = 2 and $\bar{r}(t) = 4\hat{i} - 2\hat{j} + 3\hat{k}$ when t = 3.
[(c)   Di beri $\bar{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$ bila t = 2 dan $\bar{r}(t) = 4\hat{i} - 2\hat{j} + 3\hat{k}$ bila t = 3.

Show that $\int_2^3 \bar{r}.\dfrac{d\bar{r}}{dt}dt = 10$.

Tunjukkan $\int_2^3 \bar{r}.\dfrac{d\bar{r}}{dt}dt = 10$.

(25/100)

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