# LOAD FLOW USING NEWTON RAPHSON METHOD 

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Sebagai memenuhi sebahagian daripada syarat keperluan Untuk ijazah dengan kepujian

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#### Abstract

The model of the electric power system that is used in a power flow analysis consist of all the busses in the system, the generating units and load elements connected to these various buses, and the transmission lines that interconnected the buses. The information furnished by a power flow analysis is quite simply the voltage profile of the system. In other words, the important result of the analysis is the determination of the voltage magnitude and its associated phase angle at each bus. Of course, once this information is known, it is a routine matter to determine the currents in each transmission link, and accordingly, the real and reactive power flows. Conclusions regarding overloads on any of the system components are obtained at this point in the study. They are necessary for planning operation, economic scheduling and exchange of power between utilities. In addition, power flow analysis is required for many other analyses such as transient stability and contingency studies. The most common techniques used for the iterative solution of nonlinear algebraic equations are Gauss-Seidel, Newton -Raphson, and Quasi-Newton methods. Because of its quadratic convergence, Newton's method is mathematically superior to the Gauss-Seidel method and is less prone to divergence with ill-conditioned problems. For large power systems, the Newton- Raphson method is found to be more efficient and practical. The number of iterations required to obtain the solution is independent of the system size, but more functional evaluations are required at each iteration. This project presents Newton-Raphson method for conducting load flow. A practical IEEE-5 bus system is considered for illustrator. A software in MATLAB is written and load flow is simulated. The results are compared with the programs written by Hadi Saadat and presented in the Chapter 4.


#### Abstract

ABSTRAK Sebuah model sistem kuasa elektrik di dalam analisis aliran kuasa terdiri daripada bus dalam sistem, unit penjanaan dan elemen beban yang menghubungkan setiap bus dan penghantaran setiap bus. Keputusan yang penting dalam analisa ialah mengenalpasti magnitud voltan dan sudut fasa setiap bus. Maka daripada maklumat yang diketahui ini, kita boleh mengenalpasti aliran arus, kuasa sebenar dan kuasa reaktif. Selain itu sesuatu sistem memerlukan perancangan operasi, ekonomi adan penukaran tenaga kepada pengguna. Teknik biasa yang selalu digunakan adalah kaedah Newton-Raphson, GaussSeidel dan Quasi-Newton. Newton-Raphson adalah kaedah matematik yang lebih berkesan berbanding kaedah Gauss-Seidel. Untuk sistem kuasa yang lebih besar, kaedah Newton-Raphson adalah lebih berkesan dan praktikal. Bilangan iteratif yang diperlukan dalam penyelesaian adalah bergantung kepada saiz sesuatu sistem. Projek ini menunjukkan kaedah Newton-Raphson untuk aliran kuasa. Keputusan IEEE-5 bus sistem digunakan untuk perbandingan. Perisian MATLAB digunakan untuk simulasi. Perbandingan keputusan Hadi Saadat ditunjukkan pada bab 4.


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## CHAPTER 1

## INTRODUCTION

### 1.1 LOAD FLOW:

The electric power system often is composed of many generators, transformers, passive and active load elements, and other accessories, all interconnected by transmission links between buses numbering in the hundreds and even in the thousands. The purpose of the power system is to supply real and reactive electric power to customers all along the network in a reliable and economic fashion on a continuous basis and at a voltage level and frequency that holds within specified limits. Moreover, this must be achieved in a way which ensures that none of the generating units operates in a sustained overloaded condition and that no transmission link is stressed either by excessive line losses or by a serious reduction in its static stability margin. A primary objective of power flow analysis is to guarantee this goal. But there are other reasons too. For example, a power flow analysis( or load flow study as it is alternatively called) is a necessity when the power network is expanded to include additional transmission links and loads to accommodate growth in a region. Such studies help ensure that newly formed system can meet all its needs in a economical, efficient, and secure manner.

The model of the electric power system that is used in a power flow analysis consist of all the busses in the system, the generating units and load elements connected to these various buses, and the transmission lines that interconnected the buses. The information furnished by a power flow analysis is quite simply the voltage profile of the system. In other words, the important result of the analysis is the determination of the voltage magnitude and its associated phase angle at each bus. Of course, once this information is
known, it is a routine matter to determine the currents in each transmission link, and accordingly, the real and reactive power flows. Conclusions regarding overloads on any of the system components are obtained at this point in the study.

The system is assumed to be operating under balanced condition and represented by a single phase network. The network contains hundreds of nodes and branches with impedances specified an per unit on a common MVA base.

Network equations can be formulated systematically in a variety of forms. However, the node-voltage method, which is the most suitable form for many power system analyses, is commonly used. The formulation of the network equations in the nodal admittance form results in complex linear simultaneous algebraic equations in terms of node currents. When node currents are specified, the set of linear equations can be solved for the node voltages. However, in a power system, powers are known as the power flow equation, become nonlinear and must be solved by iterative techniques. Power flow studies, commonly referred to as load flow, are the backbone of power system analysis and design. They are necessary for planning operation, economic scheduling and exchange of power between utilities. In addition, power flow analysis is required for many other analyses such as transient stability and contingency studies.

The load study in a power system constitute a study of paramount importance. The study reveals the electrical performance and power flows(real and reactive) for specified conditions when the system operating under steady state. The load flow study also provides information about the line and transformer loads(as well as losses) throughout the system and voltages at different points in the system for evaluation and regulation of the performance of the power system under conditions known of priori. Further alternative
plans for future expansion to meet new load demands can be analyzed and a complete information is made available through this study.

A bus is a node at which one or many lines, one or many loads and generators are connected. It is not necessary that all of them be connected at every bus. The buses are classified as $\mathrm{P}-\mathrm{Q}$ (load bus), $\mathrm{P}-\mathrm{V}$ (generator or voltage controlled bus) and V-Q(slack bus). The bus is indicated by a vertical line at which several components are connected.

### 1.2 OBJECTIVE

1. Solving load flow 5-bus system problem using Newton-Raphson method.
2. Flow of real and reactive power in the branches of the network.
3. Busbar(node) voltage and angles.
4. Power system augmentation studies to plan expansion to the network to meet future requirements.
5. Difference between Gauss-Seidel and Newton-Raphson method.
6. Optimum system running conditions and load distribution.
7. Generator scheduling and reactive scheduling to minimizes losses.

## SCOPE

In this project generators and loads connected to various nodes of the system, inject and remove power from system. The principal information obtained from a power flow study is the magnitude and phase angle of the voltage at each bus and the real and reactive power flowing in each line. However, much additional information of value is provided by the printout of the solution from computer programs used by electric utility companies. Most of these features are made evident in our discussion of power flow studies. We shall examine
some of the methods upon which solutions to the power flow problem are based. The great value of the power flow computer program in power system design. The load-flow problem can be defined as the calculation of the real and phase angle of the voltage at each bus of a given transmission system for specified generation and load conditions. The information obtained from the load flow studies can be used to test the system's capability to transfer energy from generation to load without overloading lines. Thus ,the solutions are obtained by applying approximations to the Newton-Raphson method. Aim of my project is solving load flow using Newton-Raphson method. The Newton Raphson method is a powerful method that will typically converge rapidly for many function.

## CHAPTER 2

## LOAD FLOW NEWTON-RAPHSON METHOD THEORY

### 2.1 BUS ADMITTANCE MATRIX

In order to obtain the node-voltage equations, consider the simple power system shown in Figure 2.1 where impedance are expressed in per unit on a common MVA base and for simplicity resistances are neglected. Since the nodal solution is based upon Kirchhoff's current law, impedance are converted to admittance,i.e.,

$$
y_{i j}=\frac{1}{z_{i j}}=\frac{1}{r_{i j}+j x_{i j}}
$$



Figure 2.1 The impedance diagram of a simple system


Figure 2.2 The admittance diagram for system

The circuit has been redrawn in figure 2.2 in terms of admittances and transformation to current sources. Node 0 (which is normally ground) is taken as reference. Applying KCL to the independent nodes 1 through 4 results in

Node 1, $\mathrm{I}_{1}=\mathrm{y}_{10} \mathrm{~V}_{1}+\mathrm{y}_{12}\left(\mathrm{~V}_{1}-\mathrm{V}_{2}\right)+\mathrm{y}_{13}\left(\mathrm{~V}_{1}-\mathrm{V}_{3}\right)$

$$
2, \mathrm{I}_{2}=\mathrm{y}_{20} \mathrm{~V}_{2}+\mathrm{y}_{12}\left(\mathrm{~V}_{2}-\mathrm{V}_{1}\right)+\mathrm{y}_{23}\left(\mathrm{~V}_{2}-\mathrm{V}_{3}\right)
$$

$$
3,0=y_{23}\left(\mathrm{~V}_{3}-\mathrm{V}_{2}\right)+\mathrm{y}_{13}\left(\mathrm{~V}_{3}-\mathrm{V}_{1}\right)+\mathrm{y}_{34}\left(\mathrm{~V}_{3}-\mathrm{V}_{4}\right)
$$

$$
4,0=y_{34}\left(\mathrm{~V}_{4}-\mathrm{V}_{3}\right)
$$

Rearranging these equations yields
$\mathrm{I}_{1}=\left(\mathrm{y}_{10}+\mathrm{y}_{12}+\mathrm{y}_{13}\right) \mathrm{V}_{1}-\mathrm{y}_{12} \mathrm{~V}_{2}-\mathrm{y}_{13} \mathrm{~V}_{3}$
$\mathrm{I}_{2}=-\mathrm{y}_{12} \mathrm{~V}_{1}+\left(\mathrm{y}_{20}+\mathrm{y}_{12}+\mathrm{y}_{23}\right) \mathrm{V}_{2}-\mathrm{y}_{23} \mathrm{~V}_{3}$
$0=-y_{13} V_{1}-y_{23} V_{2}+\left(y_{13}+y_{23}+y_{34}\right) V_{3}-y_{34} V_{4}$
$0=-y_{34} V_{3}+y_{34} V_{4}$

We introduce the following admittance
$\mathrm{Y}_{11}=\mathrm{y}_{10}+\mathrm{y}_{12}+\mathrm{y}_{13}$
$\mathrm{Y}_{22}=\mathrm{y}_{20}+\mathrm{y}_{12}+\mathrm{y}_{23}$
$Y_{33}=y_{13}+y_{23}+y_{34}$
$\mathrm{Y}_{44}=\mathrm{y}_{34}$
$Y_{12}=Y_{21}=-y_{12}$
$\mathrm{Y}_{13}=\mathrm{Y}_{31}=-\mathrm{y}_{13}$
$\mathrm{Y}_{23}=\mathrm{Y}_{32}=-\mathrm{y}_{23}$
$\mathrm{Y}_{34}=\mathrm{Y}_{43}=-\mathrm{y}_{34}$
The node equation reduces to
$\mathrm{I}_{1}=\mathrm{Y}_{11} \mathrm{~V}_{1}+\mathrm{Y}_{12} \mathrm{~V}_{2}+\mathrm{Y}_{13} \mathrm{~V}_{3}+\mathrm{Y}_{14} \mathrm{~V}_{4}$
$\mathrm{I}_{2}=\mathrm{Y}_{21} \mathrm{~V}_{1}+\mathrm{Y}_{22} \mathrm{~V}_{2}+\mathrm{Y}_{23} \mathrm{~V}_{3}+\mathrm{Y}_{24} \mathrm{~V}_{4}$
$\mathrm{I}_{3}=\mathrm{Y}_{31} \mathrm{~V}_{1}+\mathrm{Y}_{32} \mathrm{~V}_{2}+\mathrm{Y}_{33} \mathrm{~V}_{3}+\mathrm{Y}_{34} \mathrm{~V}_{4}$
$\mathrm{I}_{4}=\mathrm{Y}_{41} \mathrm{~V}_{1}+\mathrm{Y}_{42} \mathrm{~V}_{2}+\mathrm{Y}_{43} \mathrm{~V}_{3}+\mathrm{Y}_{44} \mathrm{~V}_{4}$
In the above network, since there is no connection between bus 1 and $4, \mathrm{Y}_{14}=\mathrm{Y}_{41}=0$;
similarly $\mathrm{Y}_{24}=\mathrm{Y}_{42}=0$.
Extending the above relation to an $n$ bus system, the node-voltage equations in matrix form is

or $I_{b u s}=Y_{b u s} V_{b u s}$

Where $I_{b u s}$ is the vector of the injected bus currents(i.e., external current sources). The current is positive when flowing towards the bus, and it is negative if flowing away from the bus. $V_{\text {bus }}$ is the vector of bus voltage measured from the reference node (i.e., node voltages). $\mathrm{Y}_{\text {bus }}$ is known as the bus admittance matrix. The diagonal element of each node is the sum of admittance connected to it. It is known as self-admittance or driving point admittance, i.e.,

$$
Y_{i i}=\sum_{j=0}^{n} y_{i j} \quad j \neq i
$$

the off-diagonal element is equal to the negative of the admittance between the nodes. It is known as the mutual admittance or transfer admittance, i.e.,

$$
Y_{i j}=Y_{i j}=-y_{i j}
$$

When the bus currents are known, can be solved for the $n$ bus voltages.

$$
\mathrm{V}_{\mathrm{bus}}=\mathrm{Y}_{\mathrm{bus}}^{-1} \mathrm{I}_{\mathrm{bus}}
$$

The inverse of the bus admittance matrix is known as the bus impedance matrix $Z_{b u s}$. The admittance matrix obtained with one of the buses as reference is nonsingular. Otherwise the nodal matrix is singular.

Inspection of the bus admittance matrix reveals that the matrix is symmetric along the leading diagonal, and we need to store the upper triangular nodal admittance matrix only. In a typical power system network, each bus is connected to only a few nearby buses.

Consequently, many off-diagonal elements are zero. Such a matrix is called sparse, and efficient numerical techniques can be applied to compute its inverse. By means of an appropriately ordered triangular decomposition, the inverse of a sparse matrix can be expressed as a product of sparse matrix factors, thereby giving an advantage in computional
speed, storage and reduction of round off errors. However , $\mathrm{Z}_{\mathrm{b}}$, which is required for short -circuit analysis, can be obtained directly by the method of building algorithm without the need for matrix inversion.

$$
Y_{\text {bus }}=\left[\begin{array}{cccc}
-j 8.50 & j 2.50 & j 5.00 & 0 \\
j 2.50 & -j 8.75 & j 5.00 & 0 \\
j 5.00 & j 5.00 & -j 22.50 & j 12.50 \\
0 & 0 & j 12.50 & -j 12.50
\end{array}\right]
$$

A function called $\mathbf{Y}=\mathbf{y b u s}$ (zdata) is written for the formation of the bus admittance matrix. zdata is the line data input and contains four columns. The first two columns are the line bus numbers and the remaining columns contains the line resistance and reactance in per unit. The function returns the bus admittance matrix. The algorithm for the bus admittance program is very simple and basic to power system programming. Therefore, it is presented here for the reader to study and understand the method of solution. In the program, the line impedances are first converted to admittance. Y is then initialized to to zero. In the first loop, the line data is searched, and the off-diagonal elements are entered. Finally, in a nested loop, line data is search to find the elements connected to a bus and the diagonal elements are thus formed.

### 2.2 SOLUTION OF NONLINEAR ALGEBRAIC EQUATIONS.

The most common techniques used for the iterative solution of nonlinear algebraic equations are Gauss-Seidel, Newton -Raphson, and Quasi-Newton methods. The GaussSeidel and Newton-Raphson methods are discussed for one-dimensionl equation, and are then extended to n -dimensional equations.

### 2.2.1 NEWTON-RAPHSON METHOD.

The most widely used method or solving simultaneous nonlinear algebraic equation is the Newton-Raphson method. Newton's method is a successive approximation procedure based on an initial estimate of the unknown and the use of Taylor's series expansion. Consider the solution of the one-dimensional equation given by

$$
\begin{equation*}
f(x)=c \tag{2.1}
\end{equation*}
$$

if $\mathrm{x}^{(0)}$ is an initial estimate of the solution, and $\Delta \mathrm{x}^{(0)}$ is a small deviation from the correct solution, we must have

$$
\mathrm{f}\left(\mathrm{x}^{(0)}+\Delta \mathrm{x}^{(0)}\right)=\mathrm{c}
$$

Expanding the left- hand side of the above equation in Taylor's series about $\mathrm{x}^{(0)}$ yields

$$
\mathrm{f}\left(\mathrm{x}^{(0)}\right)+(\underline{\mathrm{df}}) \Delta \mathrm{x}^{(0)}+1 / 2!{\left.\underline{\left(\mathrm{d}^{2} \mathrm{f}\right.}\right)^{(0)}}_{\mathrm{dx}}^{\mathrm{dx}^{2}}\left(\Delta \mathrm{x}^{(0)}\right)^{2}+\ldots \ldots . .=\mathrm{c}
$$

assuming the error $\Delta \mathrm{x}^{(0)}$ is very small, the higher-order terms can be neglected, which results in

$$
\Delta \mathrm{c}^{(0)} \approx{(\underline{\mathrm{df}})^{(0)}}_{\mathrm{dx}} \Delta \mathrm{x}^{(0)}
$$

where

$$
\Delta \mathrm{c}^{(0)}=\mathrm{c}-\mathrm{f}\left(\mathrm{x}^{(0)}\right)
$$

adding $\Delta \mathrm{x}^{(0)}$ to the initial estimate will result in the second approximation

$$
\begin{aligned}
& x^{(1)}=x^{(0)} \underline{\Delta c^{(0)}} \\
&\left(\underline{\mathrm{df})^{(0)}}\right. \\
& \mathrm{dx}
\end{aligned}
$$

successive use of this procedure yields the Newton-Raphson algorithm

$$
\begin{align*}
& \Delta c^{(k)}= c-f\left(x^{(k)}\right)  \tag{2.2}\\
& \Delta x^{(k)}= \underline{\Delta c^{(k)}}  \tag{2.3}\\
&\left({\underline{\mathrm{df}})^{(k)}}^{(2)}\right.
\end{align*}
$$

$$
\begin{equation*}
x^{(k+1)}=x^{(k)}+\Delta x^{(k)} \tag{2.4}
\end{equation*}
$$

(2.3) can be rearranged as

$$
\begin{equation*}
\Delta c^{(k)}=j^{(k)} \Delta x^{(k)} \tag{2.5}
\end{equation*}
$$

Where

$$
\mathrm{j}^{(\mathrm{k})}={\frac{(\mathrm{df})}{}{ }^{(\mathrm{k})}}_{\mathrm{dx}}
$$

the relation in (2.5) demonstrates that the nonlinear equation $f(x)-c=0$ is approximated by the tangent line on the curve at $\mathrm{x}^{(\mathrm{k})}$. Therefore, a linear equation is obtained in terms of the small changes in the variable. The intersection of the tangent line with the x -axis results in $\mathrm{x}^{(\mathrm{k}+1)}$.

### 2.3 NEWTON-RAPHSON POWER FLOW SOLUTION.

Because of its quadratic convergence, Newton's method is mathematically superior to the Gauss-Seidel method and is less prone to divergence with ill-conditioned problems. For large power systems, the Newton- Raphson method is found to be more efficient and practical. The number of iterations required to obtain the solution is independent of the system size, but more functional evaluations are required at each iteration. Since in the power flow problem real power and voltage magnitude are specified for the voltagecontrolled buses, the power flow equation is formulated in polar form. For the typical bus of the power system shown in figure 2.3, the current entering bus I is given from equation. This equation can be rewritten in terms of the bus admittance matrix as

$$
\begin{equation*}
I_{i}=\sum_{j=1}^{n} Y_{i j} V_{j} \tag{2.6}
\end{equation*}
$$



Figure 2.3 A typical bus of the power system
In the above equation, $j$ includes bus $i$. Expressing this equation in polar form, we have

$$
\begin{equation*}
I_{i}=\sum_{j=1}^{n}\left|Y_{i j} \| V_{j}\right| \angle \theta_{i j}+\delta_{j} \tag{2.7}
\end{equation*}
$$

The complex power at bus $i$ is

$$
\begin{equation*}
P_{i}-{ }_{j} Q_{i}=V_{i} * I_{i} \tag{2.8}
\end{equation*}
$$

Substituting from (2.7) for $\mathrm{I} i$ in (2.8)

$$
\begin{equation*}
P_{i}-Q_{i}=\left|V_{i}\right| \angle-\delta_{i} \sum_{j=1}^{n}\left|Y_{i j} \| V_{j}\right| \angle \theta_{i j}+\delta_{j} \tag{2.9}
\end{equation*}
$$

Separating the real and imaginary parts,

$$
\begin{align*}
P_{i} & =\sum_{j=1}^{n}\left|V_{i}\left\|V_{j}\right\| Y_{i j}\right| \cos \left(\theta_{i j}-\delta_{i}+\delta_{j}\right)  \tag{2.10}\\
Q_{i} & =-\sum_{j=1}^{n}\left|V_{i}\left\|V_{j}\right\| Y_{i j}\right| \sin \left(\theta_{i j}-\delta_{i}+\delta_{j}\right) \tag{2.11}
\end{align*}
$$

Equations (2.10) and (2.11) constitute a set of nonlinear algebraic equations in terms of the independent variables, voltage magnitude in per unit, and phase angle in radians. We have two equation for each load bus, given by (2.10) and (2.11) and one equation for
each voltage-controlled bus, given by (2.10) . Expanding (2.10) and (2.11) in Taylor's series about the initial estimate and neglecting all higher order terms results in the following set of linear equations.


In the above equation, bus 1 assumed to be the slack bus. The Jacobian matrix gives the linearized relationship between small changes in voltage angle $\Delta \delta i^{(\mathrm{k})}$ and voltage magnitude $\Delta\left|\mathrm{V}_{\mathrm{i}}{ }^{(\mathrm{k})}\right|$ with the small changes in real and reactive power $\Delta \mathrm{P}_{\mathrm{i}}{ }^{(\mathrm{k})}$ and $\Delta \mathrm{Q}_{\mathrm{i}}{ }^{(\mathrm{k})}$.

Elements of the Jacobian matrix are the partial derivatives of (2.5) and (2.6) evaluated at $\Delta \delta i^{(\mathrm{k})}$ and $\Delta\left|\mathrm{V}_{\mathrm{i}}{ }^{(\mathrm{k})}\right|$. In short form, it can be written as

$$
\left[\begin{array}{l}
\Delta P  \tag{2.12}\\
\Delta Q
\end{array}\right]=\left[\begin{array}{ll}
J_{1} & J_{2} \\
J_{3} & J_{4}
\end{array}\right]\left[\begin{array}{l}
\Delta \delta \\
\Delta|V|
\end{array}\right]
$$

For voltage- controlled buses, the voltage magnitudes are known. Therefore, if m buses of the system are voltage-controlled, $m$ equations involving $\Delta \mathrm{Q}$ and $\Delta \mathrm{V}$ and the
corresponding columns of the Jacobian matrix are eliminated. Accordingly, there are $n-1$ real power constraints and $n-1-m$ reactive power constraints, and Jacobian matrix is of order $(2 n-2-m) \times(2 n-2-m) . \mathbf{J}_{\mathbf{1}}$ is of the order $(n-1) \times(n-1), \mathbf{J}_{2}$ is of the order $(n-1) \times(n-1-m), \mathbf{J}_{3}$ is of the order $(n-1-m) \times(n-1), \mathbf{J}_{4}$ is of the order $(n-1-m) \times(n-1-m)$.

The diagonal and the off-diagonal elements of $\mathbf{J}_{\mathbf{1}}$ are

$$
\begin{align*}
& \frac{\partial P_{i}}{\partial \delta_{i}}=\sum_{j \neq i}\left|V_{i}\left\|V_{j}\right\| Y_{i j}\right| \sin \left(\theta_{i j}-\delta_{i}+\delta_{j}\right)  \tag{2.13}\\
& \frac{\partial P_{i}}{\partial \delta_{j}}=-\left|V_{i}\left\|V_{j}\right\| Y_{i j}\right| \sin \left(\theta_{i j}-\delta_{i}+\delta_{j}\right) \mathrm{j} \neq \mathrm{i} \tag{2.14}
\end{align*}
$$

The diagonal and the off-diagonal elements of $\mathbf{J}_{\mathbf{2}}$ are

$$
\begin{align*}
& \frac{\partial P_{i}}{\partial\left|V_{i}\right|}=2\left|V_{i}\left\|Y_{i i}\left|\cos \theta_{i i}+\sum_{j \neq i}\right| V_{j}\right\| Y_{i j}\right| \cos \left(\theta_{i j}-\delta_{i}+\delta_{j}\right)  \tag{2.15}\\
& \frac{\partial P_{i}}{\partial\left|V_{j}\right|}=\left|V_{i} \| Y_{i j}\right| \cos \left(\theta_{i j}-\delta_{i}+\delta_{j}\right) \quad \mathrm{j} \neq \mathrm{i} \tag{2.16}
\end{align*}
$$

The diagonal and the off-diagonal elements of $\mathbf{J}_{3}$ are

$$
\begin{align*}
& \frac{\partial Q_{i}}{\partial \delta_{i}}=\sum_{j \neq i}\left|V_{i}\left\|V_{j}\right\| Y_{i j}\right| \cos \left(\theta_{i j}-\delta_{i}+\delta_{j}\right)  \tag{2.17}\\
& \frac{\partial Q_{i}}{\partial \delta_{j}}=-\left|V_{i}\right|\left|V_{j} \| Y_{i j}\right| \cos \left(\theta_{i j}-\delta_{i}+\delta_{j}\right) \mathrm{j} \neq \mathrm{i} \tag{2.18}
\end{align*}
$$

The diagonal and the off-diagonal elements of $\mathbf{J}_{\mathbf{4}}$ are

$$
\begin{align*}
& \frac{\partial Q_{i}}{\partial\left|V_{i}\right|}=-2\left|V_{i}\left\|Y_{i i}\left|\sin \theta_{i i}-\sum_{j \neq i}\right| V_{j}\right\| Y_{i j}\right| \sin \left(\theta_{i j}-\delta_{i}+\delta_{j}\right)  \tag{2.19}\\
& \frac{\partial Q_{i}}{\partial\left|V_{j}\right|}=-\left|V_{i} \| Y_{i j}\right| \sin \left(\theta_{i j}-\delta_{i}+\delta_{j}\right) j \neq i \tag{2.20}
\end{align*}
$$

The terms $\Delta \mathrm{P}_{i}{ }^{(\mathrm{k})}$ and $\Delta \mathrm{Q}_{i}{ }^{(\mathrm{k})}$ are the difference between the scheduled and calculated values, known as the power residuls, given by

$$
\begin{align*}
& \Delta \mathrm{P}_{\mathrm{i}}{ }^{\mathrm{k})}=\mathrm{P}^{\text {sch }}-\mathrm{P} i^{(\mathrm{k})}  \tag{2.21}\\
& \Delta \mathrm{Q}_{i}{ }^{(\mathrm{k})}=\mathrm{Qi}^{\mathrm{sch}}-\mathrm{Qi}^{(\mathrm{k})} \tag{2.22}
\end{align*}
$$

The new estimates for bus voltages are

$$
\begin{gather*}
\partial_{\mathrm{i}}^{(\mathrm{k}+1)}=\partial_{\mathrm{i}}^{(\mathrm{k})}+\Delta \partial_{\mathrm{i}}^{(\mathrm{k})}  \tag{2.23}\\
\left|\mathrm{V} i^{(\mathrm{k}+1)}\right|=\left|\mathrm{V} i^{(\mathrm{k})}\right|+\Delta\left|\mathrm{V} i^{(\mathrm{k})}\right| \tag{2.24}
\end{gather*}
$$

The procedure for power flow solution by the Newton - Raphson method is as follow:

1. For load buses, where $\mathrm{P}_{\mathrm{i}}{ }^{\text {sch }}$ and $\mathrm{Q} \mathrm{i}^{\text {sch }}$ are specified, voltage magnitudes and phase angles are set equal to the slack bus values, or 1.0 and 0.0 , i.e., $\left|\mathrm{V} i^{(0)}\right|=1.0$ and $\partial_{\mathrm{i}}{ }^{(0)}=0.0$. For voltage-regulated buses, where $|\mathrm{V} i|$ and $\mathrm{P}_{\mathrm{i}}{ }^{\text {sch }}$ are specified, phase angles are set equalto the slack bus angle, or 0 , i.e., $\partial_{\mathrm{i}}{ }^{(0)}=0$.
2. For load buses, $\mathrm{P}_{\mathrm{i}}{ }^{(\mathrm{k})}$ and $\mathrm{Q}_{\mathrm{i}}{ }^{(\mathrm{k})}$ are calculated from (2.10) and (2.11) and $\Delta \mathrm{P}_{\mathrm{i}}{ }^{(\mathrm{k})}$ and $\mathrm{Q}{ }^{(\mathrm{k})}$ are calculated from (2.21) and (2.22).
3. For voltage- controlled buses, $\mathrm{P}_{\mathrm{i}}{ }^{(\mathrm{k})}$ and $\Delta \mathrm{P}_{\mathrm{i}}{ }^{(\mathrm{k})}$ are calculated from (2.21) and (2.10), respectively.
4. The elements of Jacobian matrix $\left(\mathbf{J}_{1}, \mathbf{J}_{2}, \mathbf{J}_{\mathbf{3}}\right.$ and $\left.\mathbf{J}_{\mathbf{4}}\right)$ are calculated from (2.13) (2.20).
5. The linear simultaneous equation (2.12) is solved directly by optimally ordered triangular factorization and Gussian elimination.
6. The new voltage magnitudes and phase angles are computed from (2.23) and (2.24).
7. The process is continued until the residuls $\Delta \mathrm{P}_{\mathrm{i}}{ }^{(\mathrm{k})}$ and $\Delta \mathrm{Q}_{\mathrm{i}}{ }^{(\mathrm{k})}$ are less than the specified accuracy , i.e.,

$$
\begin{aligned}
& \left|\Delta \mathrm{P}_{\mathrm{i}}{ }^{(\mathrm{k})}\right| \leq \varepsilon \\
& \left|\Delta \mathrm{Q}_{\mathrm{i}}{ }^{(\mathrm{k})}\right| \leq \varepsilon
\end{aligned}
$$

Aim of my project is solving load flow using Newton-Raphson method. The Newton Raphson method is a powerful method that will typically converge rapidly for many function.

### 2.4 LINE FLOWS AND LOSSES.

After the iterative solution of bus voltage, the next step is the computation of line flows and line losses. Consider the line connecting the two buses $i$ and $j$ in figure. The line current $\mathrm{I} i j$, measured at bus $i$ and defined positive in the


Figure 2.4 Transmission line model for calcuting line flows
$i \rightarrow j$ is given by

$$
\begin{equation*}
\mathrm{I} i j=\mathrm{I}_{1}+\mathrm{I} i_{0}=\mathrm{y} i j(\mathrm{~V} i-\mathrm{V} \boldsymbol{j})+\mathrm{y} i_{0} \mathrm{~V} i \tag{2.25}
\end{equation*}
$$

Similarly, the line current $\mathrm{I} j i$ measured at bus $j$ and defined positive in the direction $j \rightarrow i$ is given by

$$
\begin{equation*}
\mathrm{I} j i=-\mathrm{I}_{l}+\mathrm{I} j_{0}=\mathrm{y} i j(\mathrm{~V} j-\mathrm{V} i)+\mathrm{y} j_{0} \mathrm{~V} j \tag{2.26}
\end{equation*}
$$

The complex power $\mathrm{S} i j$ from bus $i$ to $j$ and $\mathrm{S} j i$ from bus $j$ to $i$ are

$$
\begin{align*}
& \mathrm{S} i j=\mathrm{V} i I^{*} i j  \tag{2.27}\\
& \mathrm{~S} j i=\mathrm{V} j \mathrm{I}^{*} j i \tag{2.28}
\end{align*}
$$

The power loss in line $i-j$ is the algebraic sum of the power flows determined from, i.e.,

$$
\begin{equation*}
\mathrm{S}_{\mathrm{L}} i j=\mathrm{S} i j+\mathrm{S} j i \tag{2.29}
\end{equation*}
$$



Figure 2.5 Power flow
Equations (2.10) and (2.11) constitute the polar form of the power-flow equations they provide calculated values for the net real power $P_{i}$ and reactive power $Q_{i}$ entering the network at typical bus . Let $\mathrm{P}_{\mathrm{gi}}$ denote the scheduled power being generated at bus and $\mathrm{P}_{\mathrm{di}}$ denote the scheduled power demand of the load at the bus. Then, $\mathrm{P}_{\mathrm{i}, \text { sch }}=\mathrm{P}_{\mathrm{gi}}-\mathrm{P}_{\mathrm{di}}$ is the
net scheduled ower being injected into the network at bus $\bigcirc$, as illustrated in figure. Denoting the calculated value of $P_{i}$ by $P_{i, \text { calc }}$ leads to the definition of mismatch $\Delta P_{i}$ as the scheduled value $\mathrm{P}_{\mathrm{i}, \text { sch }}$ minus the calculated value $\mathrm{P}_{\mathrm{i}, \text { calc, }}$,

$$
\begin{equation*}
\Delta P_{i}=P_{i, s c h}-P_{i, \text { calc }}=\left(P_{g i}-P_{d i}\right)-P_{i, \text { calc }} \tag{2.30}
\end{equation*}
$$

Likewise, for reactive power at bus i we have,

$$
\begin{align*}
& \Delta Q_{i}=Q_{i, s c h}-Q_{i, \text { calc }}=\left(Q_{g i}-Q_{d i}\right)-Q_{i, \text { calc }}  \tag{2.31}\\
& \qquad P_{L}=\sum_{i=1}^{N} P_{i}=\sum_{i=1}^{N} P_{g i}-\sum_{i=1}^{N} P_{d i} \tag{2.32}
\end{align*}
$$

$\mathrm{P}_{\mathrm{L}}=$ real power loss
$\mathrm{P}_{\mathrm{gi}}=$ real power generation
$\mathrm{P}_{\mathrm{di}}=$ real power load

The term $\mathrm{P}_{\mathrm{L}}$ in this equation is evidently the total $\mathrm{I}^{2} \mathrm{R}$ loss in the transmission lines and transformers of the network. The individual currents in the various transmission line of the network cannot be calculated until after the voltage magnitude and angle are known at every bus of the system. Therefore, $\mathrm{P}_{\mathrm{L}}$ is initially unknown and it is not possible to prespecify all the quantities in the summations. In the formation of the power-flow problem we choose one bus,slack bus, at which $\mathrm{Pg}_{\mathrm{g}}$ is not scheduled or otherwise prespecified. After the power-flow problem has been solved, the difference (slack) between the total specified P going into the system at all the other buses and the total output P plus $\mathrm{I}^{2} \mathrm{P}$ losses are assigned to the slack bus. For this reason a generator bus must be selected as the slack bus. The difference between the total megavars supplies by the generators at the buses and the megavars receives by the loads is given by:

$$
\begin{equation*}
\sum_{i=1}^{N} Q_{i}=\sum_{i=1}^{N} Q_{g i}-\sum_{i=1}^{N} Q_{d i} \tag{2.33}
\end{equation*}
$$

### 2.5 MATLAB IEEE POWER FLOW SOLUTION.

Several computer programs have been developed for the power flow solution of practical systems. Each method of solution consists of four programs. The program for the gaussseidel method is lfgauss, which is preceded by lfybus, and is followed by busout and line flow. Programs lfybus,busout, and lineflow are designed to be used with two more power flow programs. These are lfnewton for the Newton-Raphson method and decouple for the fast decoupled method. The following is brief description of the programs used in the Gauss-seidel method.

| Programs | Brief Description of the programs |
| :---: | :--- |
| Ifybus | This program requires the line and transformer parameters and <br> transformer tap settings specified in the input file named <br> linedata. It converts impedances to admittances and obtains <br> the bus admittance matrix. The program is designed to handle <br> parallel lines. |
| lfgauss | This program obtains the power flow solution by the Gauss- <br> Seidel method and requires the files named busdata and <br> linedata. It is designed for the direct use of load and <br> generation in MW and Mvar, bus voltages in per unit, and <br> angle in degrees. Loads and generation are converted to per <br> unit quantities on the base MVA selected. A provision is <br> made to maintain the generator reactive power of the voltage- <br> controlled buses within their specified voltage is either to high <br> or too low. After a few iterations(10 ${ }^{\text {th }}$ iteration in the Gauss <br> method), the var calculated at the generator buses are |


|  | examined. If a limit is reached, the voltage magnitude is adjusted in steps of 0.5 percent up to $\pm 5$ percent to bring the var demand within the specified limits. |
| :---: | :---: |
| busout | This program produces the bus output result in a tabulated form. The bus output result includes the voltage magnitude and angle, real and reactive power of generators and loads, and the shunt capacitor/reactor Mvar. Total generation and total load are also included as outlined in the sample case. |
| lineflow | This program prepares the line output data. It is designed to display the active power flow entering the line terminals and line losses as well as the net power at each bus. Also included are the total real and reactive losses in the system. The output of this portion is also shown in the sample case. |

## Data preparations:

The bus power and voltage specifications are given below:

| Bus | $\mathrm{P}_{\mathrm{L}}$ | $\mathrm{Q}_{\mathrm{L}}$ | $\mathrm{P}_{\mathrm{G}}$ | $\mathrm{Q}_{\mathrm{G}}$ | V | Bus <br> Specification |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 0 | $1.06\llcorner 0$ | Slack bus |
| 2 | 0.2 | 0.1 | 0.4 | 0.3 | 1.045 | PV bus |
| 3 | 0.2 | 1.5 | 0.3 | 0.1 | 1.03 | PV bus |
| 4 | 0.5 | 0.3 | 0 | 0 | Not specified | PQ bus |
| 5 | 0.6 | 0.4 | 0 | 0 | Not specified | PQ bus |

In order to perform a power flow analysis by the Newton-Raphson method in the
MATLAB environment, the following variables must be defined: power system base MVA, power mismatch accuracy,acceleration factor, and maximum number of iterations. The name (in lowercase letters) reserved for these variables are basemva, accuracy, accel, and maxiter, respectively. Typical values are as follows:

Basemva $=100$; accuracy $=0.001$;
Accel $=1.6$; maxiter $=80$;
The initial step in the preparation of input file is the numbering of each bus. Buses are numbered sequentially. Although the numbers are sequentially assigned, the buses need not be entered in sequence. In addition, the following data files are required.

BUS DATA FILE - busdata the format for the bus entry is chosen to facilitate the required data for each bus in a single row. The information required must be included in a matrix called busdata. Column 1 is the bus number. Column 2 contains the bus code.

Column 3 and 4 are voltage magnitude in per unit and phase angle in degrees. Column 5 and 6 are load MW and Mvar. Column 7 through 10 are MW, Mvar, minimum Mva and maximum Mvar of generation, in that order. The last column is the injected Mvar of shunt capacitors.

The bus code entered in column 2 is used for identifying load, voltage-controlled, and slack buses as outlined below:

| \% | Bus | Bus | Voltage | Angle | ---Load---- | --Generator--- |  |  |
| :--- | :---: | :---: | :--- | :---: | :---: | :---: | :---: | :--- |
| $\%$ | No | code | Mag. | Degree | MW | Mvar | MW | Mvar |
| busdata= $[1$ | 1 | 1.06 | 0.0 | 0 | 0 | 0 | 0 |  |
|  | 2 | 2 | 1.045 | 0.0 | 20 | 10 | 40 | 0 |
|  | 3 | 2 | 1.03 | 0.0 | 20 | 15 | 30 | 0 |
|  | 4 | 0 | 1.00 | 0.0 | 50 | 30 | 0 | 0 |
|  | 5 | 0 | 1.00 | 0.0 | 60 | 40 | 0 | 0 |$] ;$


| Code | Description of the code |
| :---: | :--- |
| 1 | This code is used for the slack bus. The only necessary information <br> for this bus is the voltage magnitude and its phase angle. |
| 0 | This code is used for load buses. The loads are entered positive in <br> megawatts and megavars. For this bus,initial voltage estimate must be <br> specified. This is usually 1 and 0 for voltage magnitude and phase <br> angle for this type of bus specified, they will be taken as the initial <br> starting voltage for the bus instead of a flat start of 1 and 0. |
| 2 | This code is used for the voltage-controlled buses. For this bus, <br> voltage magnitude,real power generation in megawatts, and the <br> minimum and maximum limits of the megavar demand must be <br> specified. |

LINE DATA FILE - linedata lines are identified by the node-pair method. The information required must be included in a matrix called linedata. Column 1 and 2 are the line bus numbers. Column 3 and 5 contain the line resistance, reactance, and one-half of the total line charging susceptance in per unit on the specified MVA base. The last column is for the transformer tap setting; for lines, 1 must be entered in this column. The lines may be entered in any sequence or order with the only restriction being that if the entry is a transformer, the left bus number is assumed to be the tap side of the transformer.

| \% | Line code |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \% | Bus | bus | R | X | 1/2 B | $=1$ for lines |
| \% | nl | nr | p.u. | p.u. | p.u. | > 1 or $<1$ tr. tap at bus nl |
| linedata $=[$ | 1 | 2 | 0.02 | 0.06 | 0 | 1 |
|  | 1 | 3 | 0.08 | 0.24 | 0 | 1 |
|  | 2 | 3 | 0.06 | 0.18 | 0 | 1 |
|  | 2 | 4 | 0.06 | 0.18 | 0 | 1 |
|  | 2 | 5 | 0.04 | 0.12 | 0 | 1 |
|  | 3 | 4 | 0.01 | 0.03 | 0 | 1 |
|  | 4 | 5 | 0.08 | 0.24 | 0 | 1]; |

A program named Ifnewton is developed for power flow solution by the Newton-Raphson method for practical power systems. This program must be preceded by the lfybus program. busout and lineflow programs can be used to print the load flow solution and the line flow results. The format is the same as the gauss-seidel. The following is a brief description of the Ifnewton program.

Lfnewton this program obtains the power flow solution by the Newton-Raphson method and requires the busdata and the linedata. It is designed for the direct use of load and generation in MW and Mvar, bus voltages in per unit quantities on the base MVA selected. A provision is made to maintain the generator reactive power of the voltage-controlled buses within their specified limits. The violation of reactive power limit may occur if the specified voltage is either too high or too low. In the second iteration, the var calculated at the generator buses are examined. If a limit is reached, the voltage magnitude is adjusted in steps of 0.5 percent up to $\pm 5$ percent to bring the var demand within the specified limits.

## CHAPTER 3

## POWER FLOW 5 -BUS SYSTEM SOLUTION USING NEWTON RAPHSON METHOD



Figure 3.1 5-bus power system.
Figure shows a five bus power system. The line shunt admittances may be neglected. The bus power and voltage specifications are given below:

| Bus | $\mathrm{P}_{\mathrm{L}}$ | $\mathrm{Q}_{\mathrm{L}}$ | $\mathrm{P}_{\mathrm{G}}$ | $\mathrm{Q}_{\mathrm{G}}$ | V | Bus <br> Specification |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 0 | $1.06\llcorner 0$ | Slack bus |
| 2 | 0.2 | 0.1 | 0.4 | 0.3 | 1.045 | PV bus |
| 3 | 0.2 | 1.5 | 0.3 | 0.1 | 1.03 | PV bus |
| 4 | 0.5 | 0.3 | 0 | 0 | Not specified | PQ bus |
| 5 | 0.6 | 0.4 | 0 | 0 | Not specified | PQ bus |

