STRUCTURAL ASSESSMENT FOR A THREE STOREY BUILDING UNDER DIFFERENT USAGE USING S-FRAME

By

Chin Wai Yet

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CONTENT

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CHIN WAI YET
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ABSTRACT

Structural analysis is one of the important constituents of structural engineering. This report deal with the effect of increased the area load onto the structural. The purpose of this project is to analyze the structure behavior under different load cases which content 2.0kN/m^2, 2.5kN/m^2 , 3.0kN/m^2 , 4.0kN/m^2 and 5.0kN/m^2. The building is originally for office usage. Tested loading(imposed load)are start from 2.0kN/m^2 to 5kN/m^2. From the analysis the critical elements of the structure are obtained. The performance of the building is then analyzed through the relationship between maximum moment and area load. This to prevent any structure failure occurs during the change of structure function. S-Frame version 6.1 is used to model and analyze the structure. A thorough analysis of the building frame, using stiffness, flexibility and finite element methods. To remain the structure safety and reliability, the regular inspections and condition assessment of engineering structures are necessary to ensure that early detection of any defect can be made. From the final result, this structure are fail during the 5.0kN/m^2, which failed element from second floor of the building.

ABSTRAK

Struktur analisis merupakan salah sebuah bidang utama dalam kejuruteraan Awam. Sebelum sebuah bangunan itu dibina atau diubah fingsi kegunaannya, proses analisa hendaklah dijalankan untuk mengelakkan kegagalan struktur berlaku. Kajian ini menitikberat tentang kesan terhadap penambahan beban khidmat kepada sesuatu struktur. Tujuan kajian ini adalah untuk menganalisah kesan-kesan yang mungkin berlaku terhadap beban khidmat yang ditanggung. Kegunaan asal bangunan ini adalah sebagai office. Beban khidmat yang akan dibentang dalam kajian ini adalah 2.0kN/m^2, 2.5kN/m^2 , 3.0kN/m^2 , 4.0kN/m^2 and 5.0kN/m^2 . Daripada kajian ini, dapat ketahui perhubungan antara momen dengan beban khidmat. Tujuan kajian ini dapat mengelakkan daripada kegagalan berlaku pada struktur bangunan. Perisian komputer yang digunakan dalam analisis ini adalah S-Frame version 6.1. daripada kajian ini, didapati kegagalan mula berlaku pada rasuk di tingkat 2 apabila beban khidmat bertambah sehingga 5.0kN/m^2.

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General

C_s	Seismic response coefficient
D	dead load
E	earthquake load
L	live load
L_r	roof live load
Mu	the applied ultimate bending moment on the section
Mr	is the moment resistance of the section
q	dynamic pressure
V	wind speed
V	total lateral forces or base shear
Vu	the applied ultimate shear force on the section
Vr	the shear resistance at the section
W	wind load
W	dead load of the building

Parameter

ρ	mass density

Vectors

$\{ F \}_e$	the element internal load vector
$\{ R \}_e$	the element applied equivalent load vector
$\{ \acute{R} \}_s$	a shifted system applied load vector

$\{X\}_s$ the system displacement vector

$\{\ddot{X}\}_s$ the system acceleration vector

$\{\dot{X}\}_s$ the system velocity vector

Matrices

$[C]_s$ the system damping matrix

$[\check{D}]$ a diagonal matrix

$[K]_s$ the system stiffness matrix

$[\acute{K}]_s$ a shifted system stiffness matrix

[L] a lower matrix

$[M]_s$ the system mass matrix

Other Variations

maxpvt maximum pivot encountered

numelm the number of element

tstval a test value.

CHAPTER 1 : INTRODUCTION

1.1 INTRODUCTION OF STRUCTURAL ANALYSIS

Structural analysis, structural design and appraisal are three important constituents of structural engineering. For a specific country and a typical site, it is important to study the building code of the country prior to the analysis and design of buildings. It is important to keep in mind certain criteria related to the analysis and design. A thorough analysis of the building frame are carried out, using stiffness, flexibility and finite element methods. In spite of recorded evidence of major structural failures world-wide, no serious attempts have been made towards investigation on the structural damage assessment topic, in par with the areas of analysis and design. Only recently the important and complexity of damage assessment is being discussed openly and professionally. The structure are subjected to outstanding loading, such as change in usage, environment and accident (impact and fired). Evaluation or prediction of structural damage in such cases is very important for the structural engineer.

The criteria are intended to ensure that building will be fit for purpose, safe and durable from a structural point of view. The criteria will apply to all types of building system for which agreement certificates may be issued. Structural strength (resistance to all likely loadings), structural stability (under gravity and wind loading) and in-service performance (resistance to door slamming and accidental loading) are aspects of building performance

National Building Regulation,1990,Clause B1 states :

"Any building and any structural element or component thereof shall be designed to provide strength, stability, serviceability and durability in accordance with accepted principles of structural design, and so that it will not impair the integrity of any other building or property."

BS 8110 : Part 1 :1997 states:

"It is assumed that the quality of the concrete, steel and other materials and of the workmanship, as verified by inspection, is adequate for safety, serviceability and durability

A rational approach of the damage assessment should be based on a balanced combination of heuristic art and science of investigation applicable to the domain.

To design a structure, engineer will learn to carry out a structural analysis that establishes the internal forces and deflections at all points produced by the design loads. In recent years, structural analysis already become a famous topic of research for engineering job, such as civil infrastructures, building, mechanical system, aircraft and aerospace structures. In particular, it would be very useful if same scientific and systematic method can be developed to detect and even quantify damage in structures. For civil engineering structures, damage may cause of material or structural defect formed during the construction stage or during its service life span resulting from natural disasters or man-made actions.

Services loads, environmental and accidental actions may cause damage to construction. In concrete structure, for example, may experience tensile cracking, compression crushing and other forms of damage due to various types of loading. If not detected and rectified early, such damage would increase maintenance cost, render the structures unserviceable and, in the extreme event, cause them to collapse catastrophically involving fatalities and injuries.

To remain safety and reliability of the structure, the regular inspections and condition assessment of engineering structures are necessary to ensure that early detection of any defect can be made. The detection cannot be done visually if the structural damage is small or it is in the interior of the system. It relies on the fact that occurrence of damage or loss of integrity in a structural system leads to changes in the dynamic properties of the structure.

1.2 PROBLEM STATEMENT

Nowadays, some of the building changed their usage function, such as from residential to commercial, from apartment or condominium to hotel service, from commercial to shopping complex. Some of this change does not carry out the structural assessment properly. Without knowing the condition and behavior of the structure, it will cause the structure damage and fail. To prevent the damage occur on the structure, the analysis should be done before any changes of the structure function take place.

1.2 OBJECTIVES

The main purpose of this research is to analyze the behavior of the structural element during the change of design loads due to usage changes. The objectives of the project are :

1. To analyze the behaviors of the structural element
2. To studies the relationship between area load and maximum y-moment.
3. To find out the relationship between area load and axial forces, y-shear and z-shear.
4. To define the performance of the building under different loading circumstances.
5. To proposed allowable usage changes for the building.

CHAPTER 2 : LITERATURE REVIEW AND PROJECT BACKGROUND

2.1 STRUCTURAL DAMAGE ASSESSMENT

Assessment of structural damage is a complex subject imbued with uncertainty and vagueness. This complexity arises from the use of subjective opinion and imprecise numerical data. An analysis of the structural integrity can be accomplished using combined nonnumeric and numeric information. Expert opinions on structural damage are used to develop the nonnumeric portion of the code. The structural damage can occur in the following sections.

2.1.1 Blasting and Vibration Damage

1. Assessment and evaluation of buildings, slabs, pads, and pavements subject to ground vibrations from sources which include, among others, surface mining, quarry and construction blasting, and demolition. Damage assessment and analysis are based on seismograph data, threshold damage, structure response, and vibration characteristics. It is necessary to separate the vibration effects of blasting from damaging influence of environmental force on structures. Vibration effect of blasting, environmental forces decreases gradually the strength of materials. There is rarely a clear distinction between damage from blasting and damage from other courses. The blasting operation is to shutter the material before it can be remove. Beside the shattered and displaced effect of the blasting, plastic deformation and cracking is the remaining energy to propagate as an elastic wave in the ground. If the charge is near the surface there may also be propagation through the air. At short range, wave radiates spherically and the amplitude

decrease as the distance from the blast. At long ranges, there are 2 factors affects the propagation process. First, the wave splits into 3 type of wave, which travels at different speeds. Second, variations in the medium, such as layering or fissuring. The distance relations reported in relatively homogeneous elastic media at sufficiently close range that little dispersion has taken place.*(I.Engel* , Structural Principles, , School of Architecture, Washington University. TA645.E54, 1984)

2.1.2 Impact Damage

Assessment of the extend of damage, evaluation of structural integrity and repair requirement to building impacted by vehicles, trees, construction equipment and falling object .Temperature also can cause to the impact damage of the building. As floor framing and supported slabs above and in a fired area are heated, they expand. As a structure expands, it can developed additional, potentially large, secondary stresses in some elements. If the resulting stress state exceeds the capacity of some member or their connections, this can initiate a serious of failures, potentially including buckling in columns of failure of floors.

2.1.3 Wind Damage

Determination of structural damage to building and other structures caused by the force of wind. Extreme wind can cause several kind of damage to a building, wind speeds, even in these extreme wind events, rapidly increase and decrease. An obstruction, such as a house in the path of the wind causes the wind to change direction. This change in wind direction increases pressure on parts of the house. The combination of increased pressures and fluctuating wind speeds creates stress on the house that frequently causes connections between building components to fail. Building that fail under the effects of extreme winds often appear to have exploded giving rise to the misconception that the damage is caused by unequal wind pressure inside and outside the building.

2.2 LOADING ON STRUCTURES

A basic functional requirement of structures is to carry load. The objective of structural design that will be able to support all the loads to which it is subjected while serving its mean purpose throughout its intended life span. Many other requirements may also be important to the mission of the structure, and once these structural requirements have been determined, the following step is the determination of loading. In designing a structure, an engineer must consider all the loads that can realistically be expected to act on the structure during its planned life span. Loading effect on structural system that is the beginning of the understanding of structural behavior.

During the conceptual design phase, this feeling for loading and structural response allows the designer to match structural systems to specific types of loadings. For example, designers of tall building, knowing the cost premium paid for carrying lateral loads by frame action at multistory heights, select shear walls and tubular systems instead. To estimate the magnitudes of the design loads, an engineer must consider the possibility that some of these loads might act simultaneously on the structure. The structure is designed to be able withstand the combination of loads that is likely to occur in its lifetime. (KENNETH M.LEET and CHIA-MING UANG,2002, Fundamentals of structural Analysis.)

The minimum design loads are often specified by building codes, but the designer must determine the specific loads to be used, which may be more than the code minimums. The minimum design loads and the load combinations for which the structures must be designed are usually specified in building codes. It is finally left to the judgment of the experienced engineer to determine the specific loads to be used for design of the structure.

2.2.1 Dead Loads

Dead loads are the weights of every element or gravity loads of the building that act permanently on the structure. The dead loads for a building structure include the weight of frames, framing and bracing systems, floors, roofs ceilings, walls, stairways, air-conditioning systems and equipment. The typical unit weight of construction materials are given in Table 2.1. (James M. Gere, 2001, Mechanics of Materials, fifth edition, A division of Thomson Learning)

Table 2.1 Unit Weight of Construction Material

Material	Unit weight
Aluminum	25.9 kN/m^3
Brick	18.8 kN/m^3
Concrete, reinforced	23.6 kN/m^3
Structural steel	77.0 kN/m^3
Wood	6.3 kN/m^3
Roofing felt	0.30 kN/m^3
Roof sheathing	0.4 kN/m^2
Roof tiles	0.75 kN/m^2

Dead load are static loads, they remain fixed and do not vary in intensity or location. Only when activated by an earthquake do dead take on a dynamic nature in the form of inertial forces.

2.2.2 Live Load

Live load is a nonpermanent load on the structure, other than wind, snow, or special types of equipment, which are usually treated separately. Nonpermanent loads include people, furniture, storage, automobiles, minor equipment and other items of a similar nature. These also can define as the weight superimposed on the structure by the use and occupancy of the building. Live loads are extremely variable by nature. They normally change, sometimes significantly, during a structure's lifetime as occupancy changes.

2.2.3 Combine Load

When the design loads for a structure have been estimated, an engineer must consider all loads that might act simultaneously on the structure at a given time. It realizes that the structure must be designed to have adequate strength to resist the most unfavorable of all the load combination. The forces produced by various combinations of load need to be combined in proper manner and increased by a factor of safety to produce the desired level of safety. The combined load also called as required factored strength, which represents the minimum strength for the member need to be designed. The BS 8110 : Part 1 : 1997 standard requires that the following load combinations need to be considered: (Table 2.2). For load combination 1 and 2 in table 2.2, the 'adverse' partial factor is applied to any loads that tend to produce a more critical design condition while the 'beneficial factor is applied to any loads that tend to produce a less critical design condition at the section considered. In addition to the aforementioned strength or safety requirements, a structure must also satisfy any serviceability requirements related to its intended use. (ROBERT A. COLEMAN, Structural System Design, *P.E* Enwright Associates, Inc. TA658.C56, 1983)

Table 2.2 Load combinations and values of γ_f for the ultimate limit state.(British Standard, BS6399 : Part 1 : 1996)

Load combination	Load type					
	Dead		Imposed		Earth and water pressure	Wind
	Adverse	Beneficial	Adverse	Beneficial		
1.Dead and imposed(and earth and water pressure	1.4	1.0	1.6	0	1.4	-
2.Dead and wind(and earth and water pressure)	1.4	1.0	-	-	1.4	1.4
3.Dead, wind and imposed(and earth and water pressure)	1.2	1.2	1.2	1.2	1.2	1.2

2.2.4 Impact

Normally the values of live loads specified by building codes are treated as static loads because the majority of loads are stationary. When live loads are applied rapidly to a structure, it will cause larger stresses than those that would be produced if the same loads would have been applied gradually. The dynamic effect of the load that cause this increase in the structure is referred to as impact. As an alternative to a dynamic analysis, moving loads are often treated as static forces and increased empirically by an impact factor.

2.2.5 Wind Loads

Wind loads are produced by the flow of wind around the structure. The magnitude of wind pressures on a structure depends on the wind velocity, the shape and stiffness of the structure, the roughness and profile of the surrounding ground, and the influence of adjacent structures. Although the procedures described in the various codes for the estimation of wind loads usually vary in detail, most of them are based on the same basic relationship between the wind speed V and the dynamic pressure q induced on a flat surface normal to the wind flow, which can be obtained by applying Bernoulli's principle and expressed as

$$q = \frac{1}{2} \rho V^2 \qquad (3.1)$$

in which ρ is the mass density of the air. Thus the pressure of the wind varies with the density of the air (a function of temperature) and with the square of the wind velocity. (KENNETH M.LEET and CHIA-MING UANG,2002, Fundamentals of structural Analysis)

2.2.6 Earthquake loads

The ground motions created by major earthquake forces cause buildings to sway back and forth. During an earthquake, as the foundation of the structure moves with the ground, the above-ground portion of the structure, because of the inertia of its mass, resists the motion, thereby causing the structure to vibrate in the horizontal direction. These vibrations produce horizontal shear forces in the structure. (KENNETH M.LEET and CHIA-MING UANG,2002, Fundamentals of structural Analysis)

2.3 THEORY OF STRUCTURAL ANALYSIS

The objective of this chapter is to present the analysis of internal forces and moments that may develop in beams and the member of plane frames under the action of coplanar system of external forces and couples. The determination of these of these internal forces and moments is necessary for the design of such structure. Beginning with defining three types of stress resultants – axial forces, shear forces and bending moments – that may act on the cross section of beams and the member of plane frames.

The internal axial force *Q* at any section of a beam is equal in magnitude but opposite in direction to the algebraic sum (resultant) of the components in the direction parallel to the axis of the beam of all the external loads and support reactions acting on either side of the section under consideration.

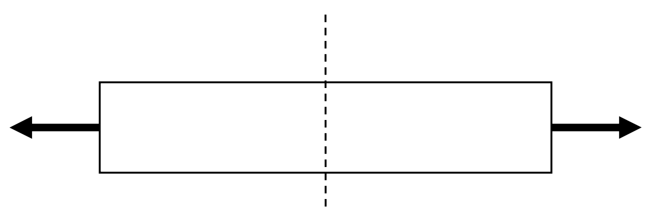

Figure 2.1 External Forces Causing Positive Axial Force

As indicate in Figure 2.1, the internal axial force *Q* is consider to be positive when the external forces acting on the member produce tension or have the tendency to pull the member apart at the section. (M.Y.H. Bangash, Prototype Building Structures: Analysis and design, 1999. p48-p63)

The shear *S* at any section of a beam is equal in magnitude but opposite in direction to the algebraic sum (resultant) of the components in the direction perpendicular to the axis of the beam of all the external loads and support reactions acting on either side of the section under consideration.

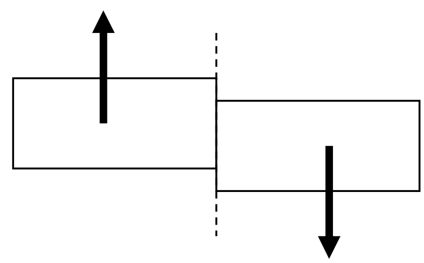

Figure 2.2 External Forces Causing Positive Shear

The shear *S* is consider to be positive when the external forces tend to push the portion of the member on the left of the section upward with respect to the portion on the right of the section.

The bending moment *M* at any section of a beam is equal in magnitude but opposite in direction to the algebraic sum of the moments about the centroid of the cross section of the beam at the section under consideration of all the external loads and support reactions acting on either side of the section.

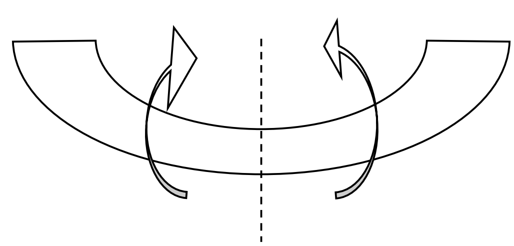

Figure 2.3 External Forces Causing Positive Bending Moment

The bending moment M is consider being positive when the external forces and couples tend to bend the beam concave upward, causing compression in upper fiber and tension in the lower fiber of the beam at the section.

A qualitative deflected shape of a structure is simply a rough sketch of a neutral surface of the structure, in the deformed position, under the action of a given loading condition. Such sketches, which can be constructed without any knowledge of the numerical values of deflections, provide valuable insights into the behavior of structures and are often useful in computing the numerical values of deflections. It is important to realize that a qualitative deflected shape is approximate, because it is based solely on the signs of curvatures and the numerical values of deflections along the axis of the beam are not known. (ROBERT A. COLEMAN, Structural System Design, *P.E* Enwright Associates, Inc. TA658.C56, 1983)

To ensure that structures are serviceable, that is their function is not impaired because of excessive flexibility that permits large deflections or vibrations under service loads. So the designers must be able to compute deflections at all critical points in a structure and compare them to allowable values specified by building codes. Deflections in well-designed beams and frames are usually small compared to the dimensions of the structure. An accurate sketch must satisfy the following rules:

1. The curvature must be consistent with the moment curve.
2. The deflected shape must satisfy the constraints of the boundaries.
3. The original angle (usually 90º) at a rigid joint must be preserved.
4. The length of the deformed member is the same as the original length of the unloaded member.

5. The horizontal projection of a beam or the vertical projection of a column is equal to the original length of the member.
6. Axial deformations, trivial compared to flexural deformations, are neglected.

2.4 Project Background

The selected building for this project is Komplex Pusat Matrikulasi at Lot 1243, 1248 & 3173, Majlis Tanah, Daerah Alor Gajah, Melaka. The original purpose of the building is for office use. Based on BS6399, Part 1, 1996, the minimum imposed load for this building is 2.0kN/m ^{2}. The are 3 levels with the dimension of 48m by 45m in Figure 2.4, Figure 2.5 and Figure 2.6.

GROUND FLOOR PLAN

Figure 2.4 Ground Floor Plan View

FIRST FLOOR PLAN

Figure 2.5 First Floor Plan View

SECOND FLOOR PLAN

Figure 2.6 Second Floor Plan View

CHAPTER 3 : COMPUTER MODELING

3.1 INTRODUCTION

S- Frame version 6.1 is used for this structural analysis. From the entry level of 2D elastic analysis to the sophisticated 3D non-linear finite element analysis, the entire analysis products share exactly the same simple method of working. S-Frame are virtually unique in that they come in several editions. Each edition builds on the features functionality and analysis capabilities of the previous one while maintaining the same basic interface. S-Frame version 6.1 model type are show in Table 3.1.

Table 3.1 Categories of S-Frame

<table>
<tr><th>Edition</th><th>Model Type</th><th>Detail</th></tr>
<tr><td>S-Frame Standard</td><td>3D space frame</td><td>Allow linear elastic (static) analysis (including plate and shell element.</td></tr>
<tr><td>S-Frame Professional</td><td>3D space frame</td><td>As S-Frame standard but with access to :
P-Delta analysis –applicable wherever interest in such second order effects, for example in un-braced and sway frames.
Moving load analysis, and
Dynamic analysis- includes Response Spectrum Analysis and many features focused on design for earthquake conditions.</td></tr>
<tr><td>S-Frame Enterprise</td><td>3D space frame</td><td>As S-Frame Professional but with access to
Non-linear analysis (a Full Newton Raphson Iterative Solver with increment. Non-linear analysis allow definition of a range of non-linear elements, including:
Tension/compression only member and support.
Non-Linear axial and torsion spring element.
Non-Linear supports (including gab/hook characteristics)
Cable/Tension- only membrane elements
Moving load analysis for non-linear structures.
Dynamic analysis for non-linear structures.</td></tr>
</table>

3.2 ANALYSIS PROCEDURES

S-Frame uses the displacement analysis method of the finite-element method. The structure to be analyzed is approximated by an assembly of structural regions connected at a finite number of nodes to ensure that the displacements are continuous. The equilibrium equations for each element are known,

$$\{ F \}_e = \{ R \}_e \qquad (3.1)$$

where $\{ F \}_e$ is the element internal load vector and $\{ R\}_e$ is the element applied equivalent load vector. Their contribution to the behavior of the overall structure can be accounted for by an assembly of the element equation (3.1) using standard matrix procedures,

$$\sum_{e=1}^{numelm}(\{F\}_e = \{R\}_e \qquad (3.2)$$

where *numelm* is the number of element. To yield the system equations,

$$\{F\}_s = \{R\}_s \qquad (3.3)$$

where $\{F\}_s$ is the system applied load vector and $\{R\}_s$ is the system applied load vector. When the internal load vector, for linear static analysis, is defined as,

$$\{F\}_s = [K]_s\{X\}_s \qquad (3.4)$$

where $[K]_s$ is the system stiffness matrix and $\{X\}_s$ is the system displacement vector. For linear dynamic analysis, is defined as,

$$\{F\}_s = [M]_s\{ \ddot{X} \}_s + [C]_s\{ \dot{X} \}_s + [K]_s\{ X \}_s \qquad (3.5)$$

where $[M]_s$ is the system mass matrix, $\{ \ddot{X} \}_s$ is the system acceleration vector, $[C]_s$ is the system damping matrix and $\{ \dot{X} \}_s$ is the system velocity vector.

And for non-linear static analysis,

$$\{F\}_s = \{R\}_s = \{ 0 \}$$

Below are the type of analysis inside S-Frame version 6.1:

1. Linear Static Analysis

2. Free-Vibration Analysis

3. Stressed Free-Vibration Analysis

4. Linear Dynamic Analysis

5. P-Delta Analysis

6. Linear Buckling Analysis

7. Response Spectrum Analysis

8. Non-Linear Static Analysis

9. Non-linear stressed Free-Vibration Analysis

10. Stressed Response Spectrum Analysis

11. Non-Linear Stressed response Spectrum Analysis

3.2.1 Linear Static Analysis

The purpose of the static analysis is to determine the displacements and stress due to time-independent loading conditions under the following assumptions:

1. Stiffness effects and applied loads do not depend on time.
2. Inertial and damping effects are ignored.
3. Static acceleration, such as gravity, maybe included.
4. Time independent load, displacement, pressures and temperature effect may be applied.

3.2.1.1 *Analysis Description for Linear Static Analysis*

The system equilibrium equations for the linear static analysis is

$$[K]_s\{X\}_s = \{R\}_s \quad (3.7)$$

Where the system load vector $\{R\}_s$ includes the contribution of :

1. Applied nodal loads.
2. Loading due to static acceleration fields (such as gravity)
3. Element thermal / pressure loads.

The system stiffness matrix will have a degeneracy equal in number to the rigid-body modes of the structure. On application of adequate boundary conditions, rigid-body modes and the degeneracy of the stiffness matrix are removed and hence, the solution of the system of equations becomes possible. The system of equilibrium equations after application of the boundary conditions becomes

$$[\acute{K}]_s\{X\}_s = \{\acute{R}\}_s \quad (3.8)$$

and a unique solution $\{X\}_s$ may be obtained. Where $[\acute{K}]_s$ is a shifted system stiffness matrix, $\{X\}_s$ is system displacement vector and $\{\acute{R}\}_s$ is a shifted system applied load vector. Once the nodal displacement are obtained, the element stresses and nodal forces may be computed. For the element at the boundaries these nodal forces will be in equilibrium with the reaction forces.