UNIVERSITI SAINS MALAYSIA
MASTER OF BUSINESS ADMINISTRATION

Second Semester Examination
Academic Session 1999/2000

February 2000

AGW615 ADVANCED BUSINESS STATISTICS

Time: [3 hours]

INSTRUCTION:
Please ensure that this examination paper consists of **SEVENTEEN (17)** printed pages before you begin.
Answer ALL questions in **SECTION A** and any ONE question from **SECTION B**

SECTION A

1. Media Management Consultants Sdn. Bhd. is a management consultant company and uses an aptitude test to aid in the selection of new consultant to join the company. Prior experience suggests that 60 percent of all applicants would be successful if they were hired. Furthermore, past experience with the aptitude test indicates that 85 percent of applicants who turn out to be successful consultants pass the test and 90 percent of applicants who turn out not to be successful consultant fail the test.

a. If an applicant passes the aptitude test, what is the probability that the applicant will become a successful consultant?
b. Calculate the accuracy of the attitude test. Do you think that the aptitude test is a valuable way to screen applicants to be consultants with the company?

(10 marks)

2. US Professional Golf Association (USPGA) requires golf ball of weight 1.62 ± 0.05 oz. You are the manufacturer of the golf balls used by USPGA in the US circuit. Your machine capability indicates that the golf balls produced are normally distributed with mean 1.62 oz. and a standard deviation of 0.04 oz.

a. What percentage of golf balls you produce will meet the USPGA specifications?
b. If in one of the tournament, the USPGA selected 25 golf balls that you supplied, what is the likelihood of finding at least 20 balls that meet the specification? *Use the closest value of p given in the tables.*
c. If each golf ball that does not meet the specifications has to be destroyed at a cost of RM0.50, and if in each production run you produce 1 million golf balls, what is probability of you losing not more than RM105,500 in each production run? *Use Normal approximation for Binomial distribution.*

(15 marks)

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3. A wholesaler has recently developed a computerized sales invoicing system. Prior to implementation of this system, a manual system as used. The distribution of the number of errors per invoice for the manual system is as follows:

Errors per invoice	0	1	2	3	More than 3
Number of invoices	87%	8%	3%	1%	1%

After the implementation of the computerized system, a random sample of 500 invoices gives the following error distribution:

Errors per invoice	0	1	2	3	More than 3
Number of invoices	479	10	8	2	1

At 5% level of significance can we conclude that the error percentages of the computerized system differ from those for the manual system? What can you conclude? Please indicate your hypothesis.

(10 marks)

4. A telemarketing firm has performed an experiment to study the effects of two factors on the response to the television advertisements. The first factor studied is the time of day at which the ad is run, while the second factor is the position of the ad within the hour. The data obtained using a completely randomized experimental design gives the number of calls placed to a 600-number following a sample broadcast of the advertisement.

Time of Day	Position of advertisement				
	On the hour(H)	On the half-hour (HH)	Early in the program (E)	Late in the program (L)	Mean
10:00 a.m.	42 37 41	36 41 38	62 68 64	51 47 48	47.08
4:00 p.m.	62 60 58	57 60 55	88 85 81	67 60 66	66.58
9:00 p.m.	100 96 103	97 96 101	127 120 126	105 101 107	106.58
Mens	66.56	64.55	91.22	72.44	73.41

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The SPSS output gives the following ANOVA table

Source of variation	Degrees of Freedom	Sum of Squares	Mean Squares	F value	Sig. F
Time		21560.81			0.0001
Position		3989.42			0.0001
Time*Position					0.8212
Error		214.00			
Total	35	25789.64			

a. Identify the hypotheses that can be tested using the above ANOVA analysis.
b. Fill in the blanks in the above ANOVA table.
c. What conclusions can you draw about the impact of the two factors on the number of calls following the screening of the advertisement.

(30 marks)

SECTION B

Answer any ONE (1) of the following questions.

5. A multiple regression analysis was carried out to explain (and predict) the movement of the Kuala Lumpur Composite impact, using some explanatory variables as listed below. The description of the variables are as follows:

Variables	Description	Classification
Dependent: KLCI	KLSE Composite index	Metric
Independent:		
Ringgit	Ringgit exchange rate against US dollars	Metric
KLIBOR	3-month KLIBOR	Metric
CPOFUT	1-month CPO (Crude Palm Oil) Futures prices	Metric
KLCIFUT	KLCI Futures spot index	Metric
DJIA	Dow Jones Industrial Average	Metric
DAY	Day of the week: Monday(1), Tuesday(2), Wednesday(3), Thursday(4) and Friday.(5)	Nonmetric
PERIOD	Period of the month First half of the month (1); Second half of the month (0)	Nonmetric

The regression results are as given in Fig. 1 below.

a. How good is the model?
b. What are the significant explanatory variables? Rank them in terms of their impact on the dependent variable.
c. Are the assumptions of multiple regression met? Use the results given in Fig. 1 to support your answer, and if not available, describe how you would go about testing the assumptions.

(35 marks)

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Fig. 1 Regression

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	Period of the month, DDAY3, KLCI Futures spot index, Dow Jones Industrial Average Index, DDAY1, DDAY, 1-Month CPO Futures price, DDAY2, 3-Month KLIBOR	.	Enter

a All requested variables entered.
b Dependent Variable: KLCI Composite Index
Note: DDAY = 1 for Monday and 0 otherwise; DDAY1 = 1 for Tuesday and 0 otherwise; DDAY2 = 1 for Wednesday and 0 otherwise; DDAY3 = 1 for Thursday and 0 otherwise;
Period of month = 1 for first half of the month & 0 for later half of the month

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	.997	.994	.994	8.8062	.973

a Predictors: (Constant), Period of the month, DDAY3, KLCI Futures spot index, Dow Jones Industrial Average Index, DDAY1, DDAY, 1-Month CPO Futures price, DDAY2, 3-Month KLIBOR
b Dependent Variable: KLCI Composite Index

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1629959.828		181106.648	2335.354	.00
	Residual	9926.395	12	77.550		
	Total	1639886.223	13			

a Predictors: (Constant), Period of the month, DDAY3, KLCI Futures spot index, Dow Jones Industrial Average Index, DDAY1, DDAY, 1-Month CPO Futures price, DDAY2, 3-Month KLIBOR
b Dependent Variable: KLCI Composite Index

Coefficients

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.	Collinearity Statistics Tolerance	VIF
1	(Constant)	-60.436	31.536		-1.916	.058		
	1-Month CPO Futures price	2.527E-02	.009	.024	2.853	.005	.645	1.552
	DDAY	3.344	2.400	.012	1.393	.166	.620	1.614
	DDAY1	1.147	2.421	.004	.474	.637	.627	1.594
	DDAY2	.328	2.367	.001	.139	.890	.604	1.654
	DDAY3	1.328	2.362	.005	.562	.575	.607	1.647
	Dow Jones Industrial Average Index	5.036E-03	.002	.021	2.082	.039	.475	2.105
	Dow Jones Industrial Average Index	5.036E-03	.002	.021	2.082	.039	.475	2.105
	KLCI Futures spot index	1.033	.010	1.019	108.654	.000	.538	1.860
	KLCI Futures spot index	1.033	.010	1.019	108.654	.000	.538	1.860
	3-Month KLIBOR	-4.972	2.567	-.020	-1.937	.055	.450	2.224
	3-Month KLIBOR	-4.972	2.567	-.020	-1.937	.055	.450	2.224
	Period of the month	-4.986	1.575	-.023	-3.167	.002	.907	1.102
	Period of the month	-4.986	1.575	-.023	-3.167	.002	.907	1.102

a Dependent Variable: KLCI Composite Index

Collinearity Diagnostics

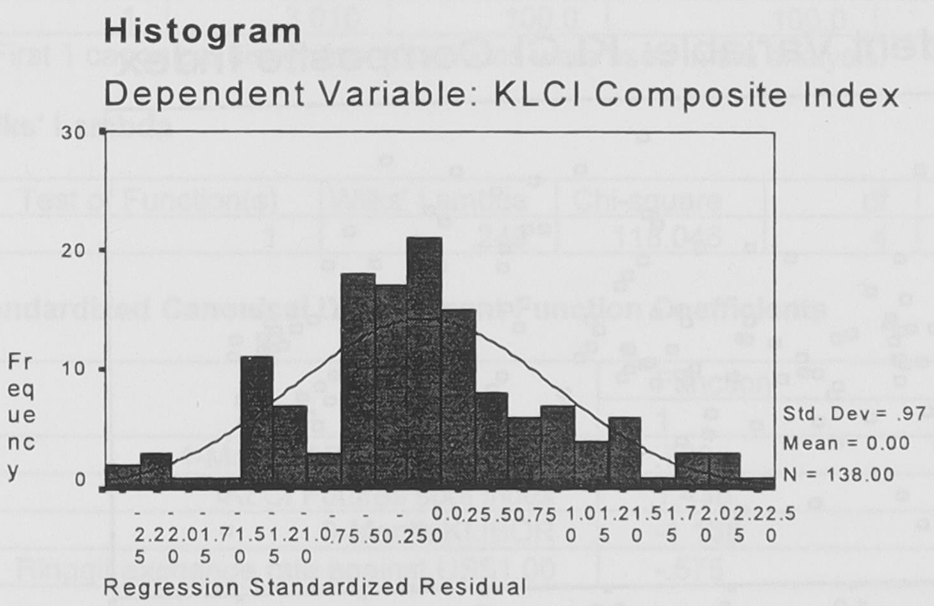
Dimension	Eigenvalue	Condition Index	Variance Proportions (Constant)	1-Month CPO Futures price	DDAY	DDAY1	DDAY2	DDAY3	Dow Jones Industrial Average Index	KLCI Futures spot index	3-Month KLIBOR	Period of the month
1	6.355	1.000	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01
2	1.004	2.515	.00	.00	.12	.05	.31	.00	.00	.00	.00	.00
3	1.000	2.521	.00	.00	.22	.27	.01	.00	.00	.00	.00	.00
4	1.000	2.521	.00	.00	.03	.08	.03	.35	.00	.00	.00	.00
5	.441	3.794	.00	.00	.02	.02	.00	.01	.00	.00	.00	.89
6	.166	6.189	.00	.00	.60	.59	.64	.63	.00	.00	.00	.02
7	3.019E-02	14.508	.00	.00	.00	.00	.00	.00	.00	.46	.00	.00
8	2.332E-03	52.204	.01	.24	.00	.00	.00	.00	.21	.03	.04	.00
9	5.815E-04	104.539	.13	.23	.00	.00	.00	.00	.71	.45	.51	.02
10	3.823E-04	128.936	.86	.52	.00	.00	.00	.00	.08	.05	.45	.06

a Dependent Variable: KLCI Composite Index

Residuals Statistics

	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	379.4268	751.2330	581.6297	109.0757	138
Residual	-20.2460	22.2710	4.893E-13	8.5121	138
Std. Predicted Value	-1.854	1.555	.000	1.000	138
Std. Residual	-2.299	2.529	.000	.967	138

a Dependent Variable: KLCI Composite Index



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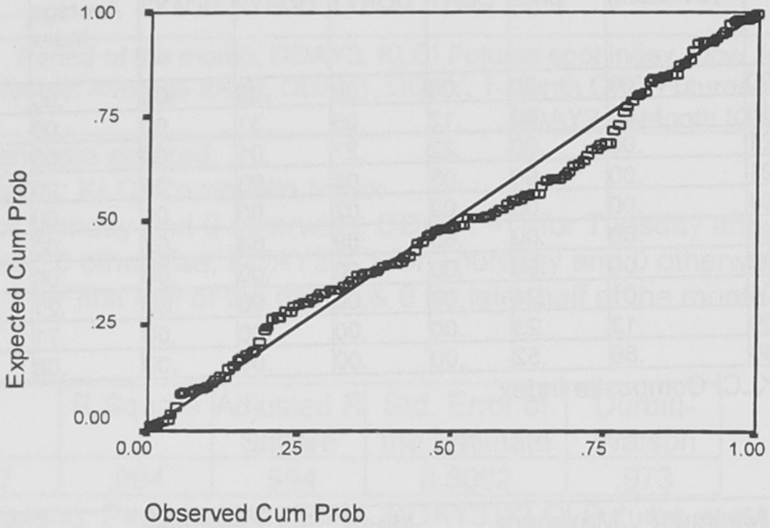
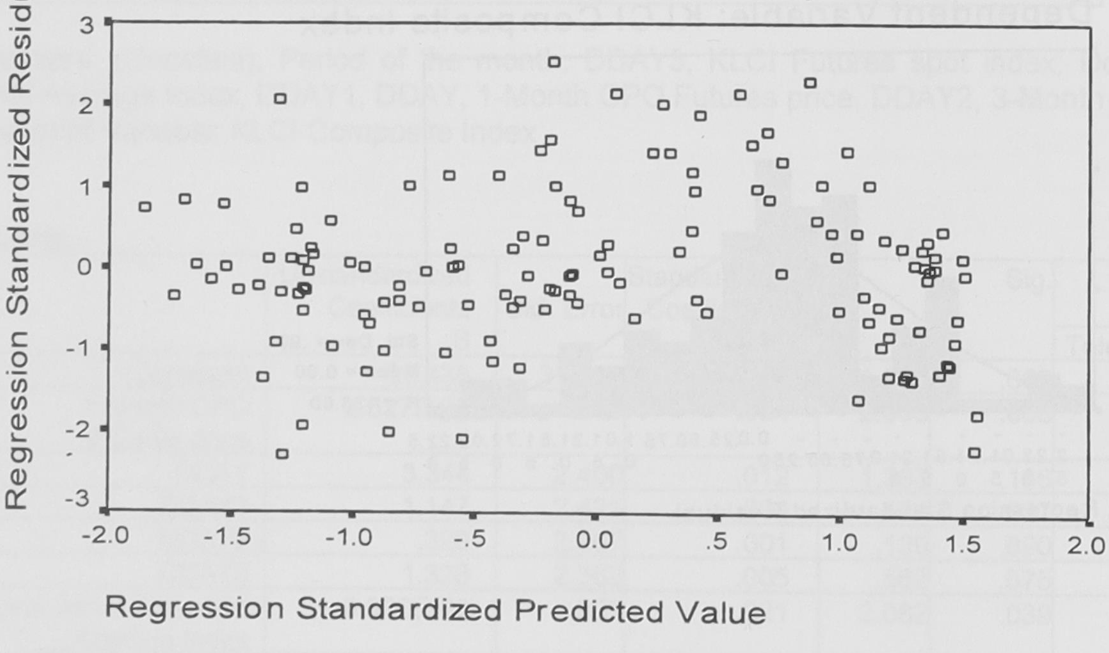
Normal P-P Plot of Regression Standardized Residual

Dependent Variable: KLCI Composite Index



Scatterplot

Dependent Variable: KLCI Composite Index



6. Using the same variables as in Question 6 (please refer to the above question), the dependent variable was classified as low (1) or high (2) and the discriminant analysis was conducted using all the variables with the categorized KLCI as the dependent variable. Fig. 2 gives the SPSS output of the analysis. In answering the following questions, please quote the appropriate statistics from Fig. 2.

a. Is there a significant discriminant model?
b. State the discriminant function.
c. Supposing we have 1-month CPO future price = 556; KLCI futures spot price = ; Ringgit exchange rate against US$1.00 = ; 1-Month CPO Futures price = ; Dow Jones Industrial Average Index = ; and the day is a Tuesday the 12th of January. Will the KLCI be high or low?
d. What does the structure matrix indicate?
e. How accurate are the classification results? Compare with highest chance criterion, proportional chance criterion and the Press Q statistics.

(35 marks)

Fig. 2 Summary of Canonical Discriminant Functions

Eigenvalues

Function	Eigenvalue	% of Variance	Cumulative %	Canonical Correlation
1	3.010	100.0	100.0	.866

a First 1 canonical discriminant functions were used in the analysis.

Wilks' Lambda

Test of Function(s)	Wilks' Lambda	Chi-square	df	Sig.
1	.249	118.046	4	.000

Standardized Canonical Discriminant Function Coefficients

	Function
	1
1-Month CPO Futures price	.799
KLCI Futures spot index	1.438
3-Month KLIBOR	-1.168
Ringgit exchange rate against US$1.00	-.575

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Structure Matrix

	Function
	1
KLCI Futures spot index	.640
Ringgit exchange rate against US$1.00	-.233
1-Month CPO Futures price	-.189
Dow Jones Industrial Average Index	-.133
DDAY2	.118
DDAY1	-.087
3-Month KLIBOR	-.082
DDAY	.077
Period of the month	-.054
DDAY3	-.015

Pooled within-groups correlations between discriminating variables and standardized canonical discriminant functions Variables ordered by absolute size of correlation within function.
a This variable not used in the analysis.

Canonical Discriminant Function Coefficients

	Function
	1
1-Month CPO Futures price	.008
KLCI Futures spot index	.020
3-Month KLIBOR	-2.310
Ringgit exchange rate against US$1.00	-2.289
(Constant)	4.268

Unstandardized coefficients

Functions at Group Centroids

	Function
klci	1
low	-3.087
high	.953
high	.953

Unstandardized canonical discriminant functions evaluated at group means

Classification Processing Summary

Processed		148
Excluded	Missing or out-of-range group codes	0
Excluded	Missing or out-of-range group codes	0
	At least one missing discriminating variable	10
	At least one missing discriminating variable	10
Used in Output		138
Used in Output		138

Classification Function Coefficients

	klci	
	low	high
1-Month CPO Futures price	.187	.218
KLCI Futures spot index	.358	.440
3-Month KLIBOR	115.167	105.835
Ringgit exchange rate against US$1.00	260.136	250.888
(Constant)	-1476.280	-1454.728

Fisher's linear discriminant functions

Classification Results

				Predicted Group Membership		Total
			klci	low	high	
Cases Selected	Original	Count	low	20	1	21
			high	2	66	68
		%	low	95.2	4.8	100.0
			high	2.9	97.1	100.0
Cases Not Selected	Original	Count	low	18	1	19
			high	4	26	30
		%	low	94.7	5.3	100.0
			high	13.3	86.7	100.0

a 96.6% of selected original grouped cases correctly classified.
b 89.8% of unselected original grouped cases correctly classified.

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AGW 615 ADVANCED BUSINESS STATISTICS FORMULAE

1. Mean

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \cdots or \cdots \bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \cdots$$

2. Variance

$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1} = \frac{\sum x^2 - n\bar{x}^2}{n-1}$$

3. Sampling Distribution

$$\mu_{\bar{x}} = \mu \cdots\cdots and \cdots\cdots \mu_{\hat{p}} = p$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \cdots\cdots and \cdots\cdots \sigma_{\hat{p}} = \sqrt{\frac{p(1-P)}{n}}$$

4. Confidence Interval

$$\bar{x} \pm z_{\alpha/2} \bullet \frac{\sigma}{\sqrt{n}} \cdots\cdots or \cdots\cdots \bar{x} \pm t_{\alpha/2,n-1} \bullet \frac{\sigma}{\sqrt{n}} \cdots\cdots or \cdots\cdots \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

5. Sample Sizes

$$n = \left(\frac{z_{\alpha/2}\sigma}{B}\right)^2 \cdots\cdots or \cdots\cdots n = \left(\frac{t_{\alpha/2,n-1}s}{B}\right)^2 \cdots\cdots or \cdots\cdots n = \left(\frac{z_{\alpha/2}\sqrt{p(1-p)}}{B}\right)$$

6. Sampling distribution of Differences

$$\mu_{x_1-x_2} = \mu_1 - \mu_2 \cdots\cdots or \mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2$$

$$\sigma_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \cdots\cdots or \cdots\cdots s_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \cdots$$

$$df = n_1 + n_2 - 2 \cdots\cdots or \cdots\cdots df = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\left(\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}\right)}$$

$$s_{\bar{x}_1-\bar{x}_2} = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \cdots\cdots and \cdots\cdots s_p = \sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}}$$

$$\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

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WILCOXON RANK SUM TEST

$$E(T)=\frac{n_1(n_1+n_2+1)}{2}\cdots\cdots and\cdots\cdots\sigma_T=\sqrt{\frac{n_1n_2(n_1+n_2+1)}{12}}$$

WILCOXON SIGNED RANK SUM TEST

$$E(T)=\frac{n\,(\,n\,+1)}{2}\cdots\cdots and\cdots\cdots\sigma_T=\sqrt{\frac{n\,(n\,+1)(2n_1+1)}{24}}$$

KRUSKAL-WALLIS

$$H=\left[\frac{12}{n(n+1)}\sum\frac{T_j^2}{n_j}\right]-3(n+1)$$

FRIEDMAN

$$F_r=\left[\frac{12}{bk(k+1)}\sum T_j^2\right]-3b(k+1)$$

CHI-SQUARED

$$\chi^2=\sum\frac{(o_i-e_i)^2}{e_i}\cdots\cdots where\cdots e_i=np_i$$

$$\chi^2=\sum\sum\frac{(o_{ij}-e_{ij})^2}{e_{Ij}}\cdots\cdots where\cdots e_{ij}=\frac{T_iT_j}{N}$$

$$and\cdots\cdots T_i=row\cdots i\cdots total,\cdots and\cdots T_j=column\cdots j\cdots total$$

ANALYSIS OF VARIANCE

$$\bar{x}_j=\frac{\sum_{i=1}^{n_j}x_{ij}}{n_j}\cdots\cdots\bar{\bar{x}}_j=\frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j}x_{ij}}{n}$$

$$SST=\sum_{j=1}^{k}n_j(\bar{x}_j-\bar{\bar{x}})^2\cdots\cdots SSE=\sum_{j=1}^{k}\sum_{i=1}^{n_j}n_j(x_{ij}-\bar{x}_j)^2$$

$$SS(Total)=SSE+SST\cdots\cdots MST=\frac{SST}{k-1}\cdots\cdots MSE=\frac{SSE}{n-k}\cdots\cdots F=\frac{MST}{MSE}$$

$$SS(Total)=\sum_{j=1}^{k}\sum_{i=1}^{b}(x_{ij}-\bar{\bar{x}})^2\cdots\cdots SST=b\sum_{j=1}^{k}\left(\bar{x}_{T_j}-\bar{\bar{x}}\right)^2$$

$$SSB=k\sum_{i=1}^{b}\left(\bar{x}_{B_i}-\bar{\bar{x}}\right)^2\cdots\cdots SSE=\sum_{j=1}^{k}\sum_{i=1}^{b}(x_{ij}-\bar{x}_{T_j}-\bar{x}_{B_i}+\bar{\bar{x}})^2$$

$$MST=\frac{SST}{k-1}\cdots\cdots MSB=\frac{SSB}{b-1}\cdots\cdots MSE=\frac{SSE}{n-k-b+1}$$

$$SS(Total) = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}(x_{ijk} - \bar{\bar{x}})^2 \cdots\cdots SS(A) = rb\sum_{i=1}^{a}\left(\bar{x}[A]_i - \bar{\bar{x}}\right)^2$$

$$SS(B) = ra\sum_{i=1}^{b}\left(\bar{x}[B]_j - \bar{\bar{x}}\right)^2 \cdots\cdots SS(AB) = r\sum_{j=1}^{k}\sum_{i=1}^{b}(\bar{x}[AB]_{ij} - \bar{x}[A]_{ij} - \bar{x}[B]_j + \bar{\bar{x}})^2$$

$$SSE = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}\left(x_{ijk} - \bar{x}[AB]_{ij}\right)^2$$

$$MS(A) = \frac{SS(A)}{a-1} \cdots\cdots MS(B) = \frac{SS(B)}{b-1}$$

$$MS(AB) = \frac{SS(AB)}{(a-1)(b-1)} \cdots\cdots MSE = \frac{SSE}{n-ab}$$

REGRESSION

$$y = \beta_0 + \beta_1 x \cdots\cdots \hat{\beta}_1 = \frac{SS_{xy}}{SS_x} \cdots\cdots \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{x}$$

$$SS_{xy} = \sum(x_i - \bar{x})(y_i - \bar{y}) \cdots\cdots SS_x = \sum(x_i - \bar{x})^2 = \sum x_i^2 - n\bar{x}^2$$

$$SS_y = \sum(y_i - \bar{y})^2 = \sum y_i^2 - n\bar{y}^2 \cdots\cdots\cdots SSE = SS_y - \frac{SS^2{}_{xy}}{SS_x}$$

Standard Error of Estimate & Coefficient of Determination,

$$S_\varepsilon = \sqrt{\frac{SS}{n-2}} \qquad\qquad R^2 = 1 - \frac{SSE}{SS_y}$$

Prediction and Estimation Interval at x_g

$$\hat{y} \pm t_{\alpha/2,n-2}\sqrt{1 + \frac{1}{n} + \frac{(x_g - \bar{x})^2}{SS_x}} \cdots\cdots \hat{y} \pm t_{\alpha/2,n-2}\sqrt{\frac{1}{n} + \frac{(x_g - \bar{x})^2}{SS_x}}$$

Coefficient of Correlation

$$r = \frac{SS_{xy}}{\sqrt{SS_x \cdot SS_y}} \cdots\cdots t = \sqrt{\frac{n-}{1-r^2}}$$

Spearman Rank Correlation

$$r_s = \frac{SS_{ab}}{\sqrt{SS_a \cdot SS_b}}$$

Standard Deviation of i-th residual

$$s_{r_i} = s_\varepsilon \bullet \sqrt{1 - h_i} \cdots\cdots h_i = \frac{1}{n} + \frac{(x_g - \bar{x})^2}{SS_x}$$

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TABLE 1 *(concluded)* **Cumulative Binomial Distribution** P(X ≤ x)

n	x	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
							p							
15	0	.860	.463	.206	.035	.005	.000	.000	.000	.000	.000	.000	.000	.000
	1	.990	.829	.549	.167	.035	.005	.000	.000	.000	.000	.000	.000	.000
	2	1.000	.964	.816	.398	.127	.027	.004	.000	.000	.000	.000	.000	.000
	3	1.000	.995	.944	.648	.297	.091	.018	.002	.000	.000	.000	.000	.000
	4	1.000	.999	.987	.836	.515	.217	.059	.009	.001	.000	.000	.000	.000
	5	1.000	1.000	.998	.939	.722	.403	.151	.034	.004	.000	.000	.000	.000
	6	1.000	1.000	1.000	.982	.869	.610	.304	.095	.015	.001	.000	.000	.000
	7	1.000	1.000	1.000	.996	.950	.787	.500	.213	.050	.004	.000	.000	.000
	8	1.000	1.000	1.000	.999	.985	.905	.696	.390	.131	.018	.000	.000	.000
	9	1.000	1.000	1.000	1.000	.996	.966	.849	.597	.278	.061	.002	.000	.000
	10	1.000	1.000	1.000	1.000	.999	.991	.941	.783	.485	.164	.013	.001	.000
	11	1.000	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.352	.056	.005	.000
	12	1.000	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.602	.184	.036	.000
	13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.833	.451	.171	.010
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.794	.537	.140
20	0	.818	.358	.122	.012	.001	.000	.000	.000	.000	.000	.000	.000	.000
	1	.983	.736	.392	.069	.008	.001	.000	.000	.000	.000	.000	.000	.000
	2	.999	.925	.677	.206	.035	.004	.000	.000	.000	.000	.000	.000	.000
	3	1.000	.984	.867	.411	.107	.016	.001	.000	.000	.000	.000	.000	.000
	4	1.000	.997	.957	.630	.238	.051	.006	.000	.000	.000	.000	.000	.000
	5	1.000	1.000	.989	.804	.416	.126	.021	.002	.000	.000	.000	.000	.000
	6	1.000	1.000	.998	.913	.608	.250	.058	.006	.000	.000	.000	.000	.000
	7	1.000	1.000	1.000	.968	.772	.416	.132	.021	.001	.000	.000	.000	.000
	8	1.000	1.000	1.000	.990	.887	.596	.252	.057	.005	.000	.000	.000	.000
	9	1.000	1.000	1.000	.997	.952	.755	.412	.128	.017	.001	.000	.000	.000
	10	1.000	1.000	1.000	.999	.983	.872	.588	.245	.048	.003	.000	.000	.000
	11	1.000	1.000	1.000	1.000	.995	.943	.748	.404	.113	.010	.000	.000	.000
	12	1.000	1.000	1.000	1.000	.999	.979	.868	.584	.228	.032	.000	.000	.000
	13	1.000	1.000	1.000	1.000	1.000	.994	.942	.750	.392	.087	.002	.000	.000
	14	1.000	1.000	1.000	1.000	1.000	.998	.979	.874	.584	.196	.011	.000	.000
	15	1.000	1.000	1.000	1.000	1.000	1.000	.994	.949	.762	.370	.043	.003	.000
	16	1.000	1.000	1.000	1.000	1.000	1.000	.999	.984	.893	.589	.133	.016	.000
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.965	.794	.323	.075	.001
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.992	.931	.608	.264	.017
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.988	.878	.642	.182
25	0	.778	.277	.072	.004	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.974	.642	.271	.027	.002	.000	.000	.000	.000	.000	.000	.000	.000
	2	.998	.873	.537	.098	.009	.000	.000	.000	.000	.000	.000	.000	.000
	3	1.000	.966	.764	.234	.033	.002	.000	.000	.000	.000	.000	.000	.000
	4	1.000	.993	.902	.421	.090	.009	.000	.000	.000	.000	.000	.000	.000
	5	1.000	.999	.967	.617	.193	.029	.002	.000	.000	.000	.000	.000	.000
	6	1.000	1.000	.991	.780	.341	.074	.007	.000	.000	.000	.000	.000	.000
	7	1.000	1.000	.998	.891	.512	.154	.022	.001	.000	.000	.000	.000	.000
	8	1.000	1.000	1.000	.953	.677	.274	.054	.004	.000	.000	.000	.000	.000
	9	1.000	1.000	1.000	.983	.811	.425	.115	.013	.000	.000	.000	.000	.000
	10	1.000	1.000	1.000	.994	.902	.586	.212	.034	.002	.000	.000	.000	.000
	11	1.000	1.000	1.000	.998	.956	.732	.345	.078	.006	.000	.000	.000	.000
	12	1.000	1.000	1.000	1.000	.983	.846	.500	.154	.017	.000	.000	.000	.000
	13	1.000	1.000	1.000	1.000	.994	.922	.655	.268	.044	.002	.000	.000	.000
	14	1.000	1.000	1.000	1.000	.998	.966	.788	.414	.098	.006	.000	.000	.000
	15	1.000	1.000	1.000	1.000	1.000	.987	.885	.575	.189	.017	.000	.000	.000
	16	1.000	1.000	1.000	1.000	1.000	.996	.946	.726	.323	.047	.000	.000	.000
	17	1.000	1.000	1.000	1.000	1.000	.999	.978	.846	.488	.109	.002	.000	.000
	18	1.000	1.000	1.000	1.000	1.000	1.000	.993	.926	.659	.220	.009	.000	.000
	19	1.000	1.000	1.000	1.000	1.000	1.000	.998	.971	.807	.383	.033	.001	.000
	20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.910	.579	.098	.007	.000
	21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.967	.766	.236	.034	.000
	22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.902	.463	.127	.002
	23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.973	.729	.358	.026
	24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.928	.723	.222

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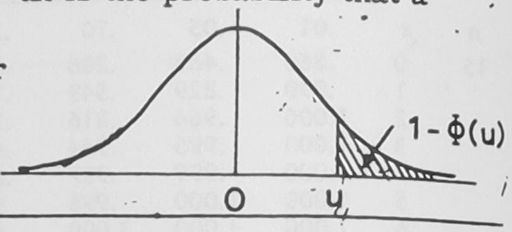
Table 3

AREAS IN TAIL OF THE NORMAL DISTRIBUTION

The function tabulated is $1 - \Phi(u)$ where $\Phi(u)$ is the cumulative distribution function of a standardised Normal variable u. Thus $1 - \Phi(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-x^2/2}\, dx$ is the probability that a standardised Normal variable selected at random will be greater than a value of u $\left(= \frac{x-\mu}{\sigma}\right)$.



$\frac{(x-\mu)}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135									
3.1	.00097									
3.2	.00069									
3.3	.00048									
3.4	.00034									
3.5	.00023									
3.6	.00016									
3.7	.00011									
3,8	.00007									
3.9	.00005									
4.0	.00003									

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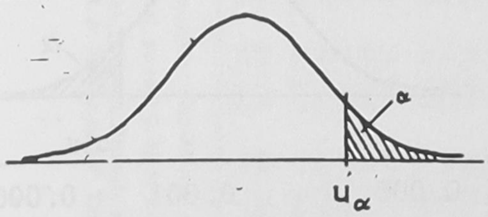
Table 4

PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

The table gives the 100α percentage points, u_α, of a standardised Normal distribution where $\alpha = \frac{1}{\sqrt{2\pi}} \int_{u_\alpha}^{\infty} e^{-x^2/2}\, dx$. Thus u_α is the value of a standardised Normal variate which has probability α of being exceeded.



α	u_α	α	u_α	α	u_α	α	u_α	α	u_α	α	u_α
.50	0.0000	.050	1.6449	.030	1.8808	.020	2.0537	.010	2.3263	.050	1.6449
.45	0.1257	.048	1.6646	.029	1.8957	.019	2.0749	.009	2.3656	.010	2.3263
.40	0.2533	.046	1.6849	.028	1.9110	.018	2.0969	.008	2.4089	.001	3.0902
.35	0.3853	.044	1.7060	.027	1.9268	.017	2.1201	.007	2.4573	.0001	3.7190
.30	0.5244	.042	1.7279	.026	1.9431	.016	2.1444	.006	2.5121	.00001	4.2649
.25	0.6745	.040	1.7507	.025	1.9600	.015	2.1701	.005	2.5758	.025	1.9600
.20	0.8416	.038	1.7744	.024	1.9774	.014	2.1973	.004	2.6521	.005	2.5758
.15	1.0364	.036	1.7991	.023	1.9954	.013	2.2262	.003	2.7478	.0005	3.2905
.10	1.2816	.034	1.8250	.022	2.0141	.012	2.2571	.002	2.8782	.00005	3.8906
.05	1.6449	.032	1.8522	.021	2.0335	.011	2.2904	.001	3.0902	.000005	4.4172

Table 5

ORDINATES OF THE NORMAL DISTRIBUTION

The table gives ϕ (u) for values of the standardised Normal variate, u, in the interval 0.0(0.1)4.0 where $\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}$

u	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.0	.3989	.3970	.3910	.3814	.3683	.3521	.3332	.3123	.2897	.2661
1.0	.2420	.2179	.1942	.1714	.1497	.1295	.1109	.0940	.0790	.0656
2.0	.0540	.0440	.0355	.0283	.0224	.0175	.0136	.0104	.0079	.0060
3.0	.0044	.0033	.0024	.0017	.0012	.0009	.0006	.0004	.0003	.0002
4.0	.0001									

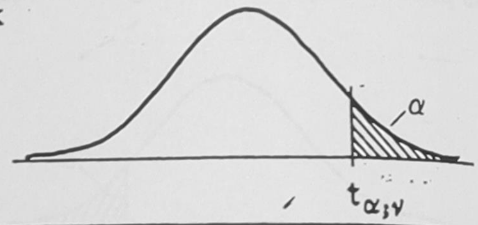
Table 7

PERCENTAGE POINTS OF THE t DISTRIBUTION

The table gives the value of $t_{\alpha;\nu}$ — the 100α percentage point of the t distribution for ν degrees of freedom.

The values of t are obtained by solution of the equation:-

$$\alpha = \Gamma\{\tfrac{1}{2}(\nu+1)\}\,\{\Gamma(\tfrac{1}{2}\nu)\}^{-1}\,(\nu\pi)^{-1/2}\int_t^{\infty}(1 + x^2/\nu)^{-(\nu+1)/2}dx$$



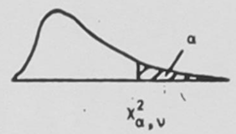
Note. The tabulation is for one tail only i.e. for positive values of t. For |t| the column headings for α must be doubled.

$\alpha =$	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
ν = 1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Table 8

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

Table of $\chi^2_{\alpha;\nu}$ — the 100 α percentage point of the χ^2 distribution for ν degrees of freedom



α =	.995	.99	.98	.975	.95	.90	.80	.75	.70	.50	.30	.25	.20	.10	.05	.025	.02	.01	.005	.001	= α
ν = 1	$.0^4393$	$.0^3157$	$.0^3628$	$.0^3982$	.00393	.0158	.0642	.102	.148	.455	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827	ν = 1
2	.0100	.0201	.0404	.0506	.103	.211	.446	.575	.713	1.386	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815	2
3	.0717	.115	.185	.216	.352	.584	1.005	1.213	1.424	2.366	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268	3
4	.207	.297	.429	.484	.711	1.064	1.649	1.923	2.195	3.357	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465	4
5	.412	.554	.752	.831	1.145	1.610	2.343	2.675	3.000	4.351	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517	5
6	.676	.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457	6
7	.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322	7
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125	8
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877	9
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588	10
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264	11
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909	12
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528	13
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123	14
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.036	11.721	14.339	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697	15
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252	16
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790	17
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312	18
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820	19
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315	20
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797	21
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268	22
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	18.137	19.021	22.337	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728	23
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179	24
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620	25
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336	29.246	30.434	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052	26
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336	30.319	31.528	32.912	36.741	40.113	43.194	44.140	46.963	49.645	55.476	27
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.893	28
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302	29
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703	30
40	20.706	22.164	23.838	24.433	26.509	29.051	32.345	33.660	34.872	39.335	44.165	45.616	47.269	51.805	55.759	59.342	60.436	63.691	66.766	73.402	40
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	42.942	44.313	49.335	54.723	56.334	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.661	50
60	35.535	37.485	39.699	40.482	43.188	46.459	50.641	52.294	53.809	59.335	65.227	66.981	68.972	74.397	79.082	83.298	84.580	88.379	91.952	99.607	60
70	43.275	45.442	47.893	48.758	51.739	55.329	59.898	61.698	63.346	69.334	75.689	77.577	79.715	85.527	90.531	95.023	96.388	100.425	104.215	112.317	70
80	51.171	53.539	56.213	57.153	60.391	64.278	69.207	71.145	72.915	79.334	86.120	88.130	90.405	96.578	101.880	106.629	108.069	112.329	116.321	124.839	80
90	59.196	61.754	64.634	65.646	69.126	73.291	78.558	80.625	82.511	89.334	96.524	98.650	101.054	107.565	113.145	118.136	119.648	124.116	128.299	137.208	90
100	67.327	70.065	73.142	74.222	77.929	82.358	87.945	90.133	92.129	99.334	106.906	109.141	111.667	118.498	124.342	129.561	131.142	135.807	140.170	149.449	100

For values of $\nu > 30$, approximate values for χ^2 may be obtained from the expression $\nu\left[1 - \frac{2}{9\nu} \pm \frac{x}{\sigma}\sqrt{\frac{2}{9\nu}}\right]^3$, where $\frac{x}{\sigma}$ is the normal deviate cutting off the corresponding tails of a normal distribution. If $\frac{x}{\sigma}$ is taken at the 0.02 level, so that 0.01 of the normal distribution is in each tail, the expression yields χ^2 at the 0.99 and 0.01 points. For very large values of ν, it is sufficiently accurate to compute $\sqrt{2\chi^2}$, the distribution of which is approximately normal around a mean of $\sqrt{2\nu - 1}$ and with a standard deviation of 1. This table is taken by consent from Statistical Tables for Biological, Agricultural, and Medical Research, by R. A. Fisher and F. Yates, published by Oliver and Boyd, Edinburgh, and from Table 8 of Biometrika Tables for Statisticians, Vol. 1, by permission of the Biometrika Trustees.