ANALISIS HABA KE ATAS DINDING RENCAM
YANG DIKENAKAN
PERUBAHAN SUHU BERSANDAR MASA

THERMAL ANALYSIS OF THE COMPOSITE WALL SUBJECTED TO TIME DEPENDENT TEMPERATURE VARIATION

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NOMENCLATURE

k	Thermal conductivity (W/m K)
c_p	Specific heat (J/kg K)
T	Temperature (^{o}C)
h	Heat transfer coefficient (W/m^2 K)
L	Thickness (m)
x	Space coordinate (m)
t	Time (s)

Greek Letter

α	Thermal diffusivity (m^2/s)
θ	Temperature difference (^{o}C)
ρ	Density (kg/m^3)
Δt	Time step (s)
Δx	control volume (m)
δx	distance between two nodes (m)

Subscripts

a	Air inside building
o	Initial
sa	Sol-air
i	Inner surface
e	Outer surface, east side
E	East node
w	West side
W	West node
F	Fluid
P	Center node

ABSTRACT

The analysis of thermal behaviour of composite wall subjected to unsteady-state heat conduction is becoming increasingly important in building designs today. Many researches have been carried out in this field in order to develop new methods that will lead to progressions and improvements.

The main objective of this project is to develop a program that can be used to analyze thermal behaviour of composite walls. In order to simplify the computations, the problem is assumed to be one-dimensional (only in the direction normal to the surface of walls) unsteady-state heat conduction through the multilayer composite walls. The wall inner surface is subjected to convective boundary condition and the wall outer surface is subjected to periodic surface temperature boundary condition. The problem was solved by using finite volume method with Tri-Diagonal Matrix Algorithm (TDMA) and fully implicit scheme was chosen.

For code validation purposes, a program code for solving transient heat conduction through a one-layer wall was developed and it was used to solve two different types of boundary conditions, the step change in surface temperature and the surface convection. The results obtained were compared with the analytical solutions. Later, a program code for solving unsteady-state heat conduction through a multilayer composite wall, which was incorporated with convective boundary condition on the wall inner surface and periodic surface temperature boundary condition on the wall outer surface, was developed.

By using the program, the wall inner surface temperature at any time of a day was determined. Time lags and decrement factors of the composite walls were also calculated. The effects of the thermophysical properties and thickness of the walls were investigated. From the results obtained, it was found that the thermal conductivity, specific heat and the thickness have a very deep effect on the time lag and decrement factor. The investigations were repeated for different wall materials and thickness and the results were discussed.

ABSTRAK

Analisis kelakuan haba ke atas dinding rencam yang dikenakan konduksi haba tak-mantap menjadi semakin penting di dalam rekabentuk bangunan hari ini. Banyak kajian telah dijalankan dalam bidang ini bagi tujuan membangunkan kaedah-kaedah baru yang akan mendatangkan kemajuan dan pembaikan.

Tujuan utama projek ini adalah untuk membangunkan satu program yang dapat digunakan untuk menganalisa kelakuan haba pada dinding rencam. Untuk tujuan memudahkan pengiraan, masalah ini dianggap sebagai konduksi haba tak-mantap satu-dimensi (hanya pada arah normal kepada permukaan dinding), menerusi dinding rencam berbilang lapis. Permukaan dalam dinding dikenakan keadaan sempadan berolak manakala permukaan luar dinding dikenakan keadaan sempadan suhu permukaan berkala. Masalah ini diselesaikan dengan menggunakan kaedah isipadu terhingga dengan algoritma matriks tiga-perpenjuru (TDMA) dan skema tersirat penuh dipilih.

Bagi tujuan pengesahan kod, satu kod program bagi menyelesaikan masalah koduksi haba fana melalui dinding satu lapis telah dibangunkan. Program ini digunakan untuk menyelesai dua jenis keadaan sempadan iaitu perubahan langkah dalam suhu permukaan dan permukaan olakan. Keputusan yang didapati dibandingkan dengan penyelesaian analisis. Kemudian, satu kod program dibangunkan bagi menyelesaikan masalah konduksi haba tak-mantap menerusi dinding rencam berbilang lapis, yang mana telah digabungkan dengan keadaan sempadan berolak pada permukaan dalam dinding dan keadaan sempadan suhu permukaan berkala pada permukaan luar dinding.

Dengan menggunakan program tersebut, suhu permukaan dalam dinding pada bila-bila masa untuk satu hari dapat ditentukan. Ekoran masa dan faktor susutan untuk dinding rencam juga dikira. Kesan ciri-ciri haba fizik dan ketebalan dinding disiasat. Dari keputusan, didapati bahawa keberkondukan haba, haba tentu dan ketebalan mempunyai kesan yang amat dalam terhadap ekoran masa dan faktor susutan. Penyiasatan ini diulang bagi bahan dinding dan ketebalan yang berlainan, dan keputusan dibincangkan.

CHAPTER 1: INTRODUCTION

1.1 Introduction of Heat Conduction

Heat transfer by conduction is defined as the transfer of energy caused by physical interaction among molecular, atomic, and subatomic particles of a substance at different temperatures. Very energetic molecules, which located in the high temperature region, will lose energy in the transfer process, and the lower energy molecules, which located in the low temperature region, will receive energy. Motion, as we understand the term in fluid dynamic, is not necessary.

Heat conduction can occur in gases, liquids as well as solids. Conduction in gases involves the collision and exchange of energy and momentum among molecules in continuous random motion. This same molecular transport mechanism also occurs in liquid mediums, but is complicated by the effects of molecular force fields, and can be augmented by the transport of free electrons in liquids that are good electrical conductors. Conduction in solids occurs as a result of the movement of free electrons and vibration in the atomic lattice structure of the material.

The French scientist, J. B. J. Fourier, proposed the basic relation for heat transfer by conduction in 1822. The relation states that the rate of heat flow by conduction, q in a material, is equal to the product of the following three quantities:

1. k, the thermal conductivity of the material.
2. A, the area of the section through which heat flows by conduction, perpendicular to the direction of heat flow.
3. $\frac{dT}{dx}$, the temperature gradient at the section.

The relation above can be written in mathematical form:

$$q = -k.A.\frac{dT}{dx} \tag{1.1}$$

The negative sign in the equation denotes that heat flows from point of higher temperature to point of lower temperature, according to the second law of thermodynamics. The increasing distance x is to be the direction of positive heat flow. Heat flow will be positive when the temperature gradient is negative. Equation (1.1) is the elementary equation for one-dimensional conduction in steady state.

1.2 Introduction of Unsteady-state Heat Conduction

Unsteady-state conduction is very important in many applications of heat transfer. In technological areas, designers are often faced with start-up, operating, and instability transients. These must be well understood in order to guide material selection, for example, in solid-fuel rocket nozzles, in reentry heat shields, in reactor components, and in combustion devices.

Unsteady-state conduction mechanisms are also important in the many earth sciences due to the ever-changing effects of solar radiation and atmospheric conditions. For example, both daily and seasonal temperature changes cause complicated time-dependent temperature variations in the soil. Geophysics problems are also analyzed on the basis of conduction mechanisms in the steady state and unsteady state. The growth characteristics of ice in soil and on the surfaces of bodies of water are also considered in terms of unsteady-state conduction and diffusion mechanisms.

There are actually two different general kinds of unsteady-state process. One is a transient, wherein the temperature field varies with time, from an initial condition, toward an eventual steady state. For example, an object at an initial temperature of T_i is immersed in a surrounding at a temperature T_1. The temperature difference decays with time. Another unsteady process is a periodic, in which the temperature at each location in the region continues to vary periodically with time. This arises approximately on the surface layer downward into soil, due to both annual and daily variations of atmospheric conditions. The annual periodic component has a time scale of 365 days, whereas the daily period is 24 hours. Another common examples are the daily variation of air temperature or of solar loading at a bounding surface.

1.3 Prediction of Heat Transfer Process

Heat transfer is a part of science studies, which seeks to predict not only how heat energy may be transferred but also the rate of heat transfer between material bodies due to the temperature difference. The prediction of heat transfer processes can be done through experimental investigations and theoretical analysis.

1.3.1 Experimental Investigations

An experiment investigation of heat transfer process requires high accuracy and precision full-scale equipments in order to obtain accurate results. Such equipments are very expensive and also difficult to manufacture. The alternative way is to perform the experiments on small-scale models. The resulting information, however, must be extrapolated to full scale but general rules for doing this are often unavailable. Furthermore, the small-scale models do not always simulate all the features of the full-scale equipment. This reduces the reliability of the test results. In conclusion, it is hard to obtain experimental results with high accuracy due to the unavailability of full-scale equipments.

1.3.2 Theoretical Analysis

A theoretical analysis uses mathematical models rather than actual physical models to predict the heat transfer processes. The mathematical model mainly consists of a set of differential equations, which can be solved by means of analytical solutions as well as numerical methods. Analytical solutions are only used to solve simple heat transfer problems, which have simple boundary conditions. For more complicated problems, the numerical methods are more practical solutions.

The development of numerical methods and availability of high-speed, digital computers hold the promise that the implications of a mathematical model can be solved for almost any practical problems. The examples of the types of numerical method are Finite Element Methods, Finite Difference Methods and Finite Volume Methods.

The advantages of theoretical analysis compared with the experimental investigations are low cost, high speed, more detail and complete results, and ability to simulate both practical and ideal conditions. In the other hand, the disadvantages of theoretical analysis may happen if the prediction has a very limited objective and the computation may be more expensive than an experiment test due to expensive software.

1.4 Introduction of Thermal Characteristics of Composite Walls

Walls are the basic structural elements of any buildings. Generally, the walls are composed of several layers with different thermophysical and mechanical properties depend on the types of building. Most ordinary houses are made of brick walls, which generally consists of three layers (brick and plastering on both sides). For taller buildings (5-storage and above), concrete walls are used for load retaining purposes, such as wind load.

The function of walls of passive solar buildings [9,11] is to provide a comfortable indoor environment for living. In this context, walls are used as heat storage elements. During day periods, the ambient temperature is much higher than the normal human body temperature due to the solar radiation. The walls isolate the solar heat from directly heating the inside building space and at the same time absorb some portion of the heat energy. As a result, the temperature inside the building is maintained at a bed comfort level. During nights, when the ambient temperature turns lower, the heat energy stored by the walls is dissipated to the environment to keep the inside building space comfortably warm.

There are different temperature profiles during any instant of one-day period at the cross-section of the outer wall of a building. These temperature profiles are functions of inside air temperature, outside air temperature and thermophysical properties of the wall. The outside air temperature changes periodically during one-day period. Thus, there will be new temperature profiles at any instant of time of the day. Since the outside air temperature changes periodically with time, it is an unsteady-state process.

A heat wave in terms of temperature and time flows from the outside to inside through the thickness of the wall. The amplitude of the heat waves is represented by the temperature magnitudes, and the wavelength of the heat wave is represented by the time. The amplitude of the heat wave on the outer surface of the wall is depending on solar radiation and convection between the outer wall surface and the outside air. When the heat wave propagates through the thickness of the wall, its amplitude will decrease accordingly to the thermophysical properties of the wall materials. When the wave reaches the inner wall surface, the amplitude of the wave is significantly smaller than that at the outer wall surface. This effect is caused by the decrement factor characteristic of the wall materials. The maximum temperature at the inner surface may occur later or earlier than that of the outer surface. This effect is caused by the time lag characteristic of the wall material.

This project is mainly focused on developing a program code using finite volume method and the fully implicit scheme is applied. The code was used to solve the one-dimensional unsteady-state heat conduction, without internal heat generations, through a multi-layer composite wall. The corresponding boundary conditions are convective type on the wall inner surface and periodic temperature type on the wall outer surface. Two important characteristics, time lag and decrement factor were also calculated in the computations. The effects of thermophysical properties and thickness of the wall on time lag and decrement factor were investigated and the results were discussed. The results obtained from the investigations are important for effective passive solar building designs.

The inputs of the code are: number of layers, the thickness of each layer, initial condition and boundary conditions, thermal conductivity, specific heat and density of each layer. The outputs of the code are: time lag, decrement factor, wall inner surface temperatures and the temperature of any location at any time of the day.

1.5 Literature Survey

H. Asan's published papers [1, 2, 3] provide the basic theory of the thermal analysis on composite walls. The researches were mainly focus on developing program code by using numerical method to investigate the effect of thermophysical properties, wall thickness, and insulation position on the time lag and decrement factor.

C. Carter and J. DeWilliers's book [9] and J.D. Balcomb's [11] paper provide the researches for heating of passive solar building via direct heat gain and thermal storage methods.

R. J. Duffin's research paper [12] emphasizes the design of special walls, which have very high time lags and very low decrement factors to prevent the propagation of big fluctuations of outside temperature to inside and almost constant inside temperature can be obtained which results to a good comfort level.

P. T. Tsilingiris's research paper [4] focus on the study of thermal behaviour of wall subjected to transient heat conduction. The thermal time constants were evaluated for number of typical walls.

Frank Kreith's book [5] provides the basic theory of heat conduction. The analytical and numerical solutions are introduced in details. The book also provides the visual FORTRAN codes for solving heat transfer problems.

Lindon C. Thomas's book [6], J. P. Holman's book [7] and Benjamin Gebhart's book provide the analytical solutions for one-dimensional transient heat conduction through infinite and semi-infinite slabs with step change and surface convection boundary conditions.

S. V. Patankar's book [8] introduces the finite volume method for solving the unsteady-state heat conduction problems.

E. R. G. Eckert and Robert M. Drake, Jr.'s book [13] and Lindon C. Thomas's book [6] provide the analysis of surface convection and periodic surface temperature on solids.

N. Ozisik's book [15] develops a range of formulae for infinite, semi-infinite and finite boundary problems of transient heat conduction subjected to general boundary conditions. The book also introduces the solutions for multi-layer slab.

CHAPTER 2: THEORY

2.1 Introduction

There are several techniques available to solve the heat conduction problem in the form of differential formulation. These solution techniques include analytical, analogical, and graphical methods. The differential formulation can also be solved by applying the numerical methods. In this chapter, only the analytical solutions of the one-dimensional unsteady-state heat conduction problems are introduced.

The analytical solutions can be applied to either one-dimensional or multidimensional heat transfer problems. Two popular methods, which are often used to develop analytical solutions, are the *separation-of-variables* technique and the *approximate integral* technique [6, 7]. There are frequently used to develop multidimensional steady and unsteady heat conduction analytical solutions due to their simplicity. The separation-of-variables solution will be discussed in detail in this chapter.

Sometimes, the analytical solutions are difficult to obtain due to the complexity of the heat transfer problems. In this case, numerical methods are introduced. The numerical methods provide the basis for analyzing the more complex problems such as nonlinear boundary conditions, temperature-dependent properties, and complex geometries. The numerical approach will be discussed in detail in the Chapter 3.

2.1.1 Time Lag and Decrement Factor

Two very important characteristics, which are very useful in determining the heat storage capabilities of any wall material, are introduced in this chapter. There are "time lag" and "decrement factor"[12].

Time lag is defined as the time taken for a heat wave to propagate from outer wall surface to the inner wall surface. Decrement factor is defined as the decreasing ratio of the heat wave amplitude when the heat wave propagates from outer wall surface to the inner wall surface. From studies done by H. Asan [1, 2, 3], it was shown that the thermophysical properties and the thickness of wall have an important effect on time lag and decrement factor.

The schematic diagram shown in Figure 2.1 illustrates the concept of time lag and decrement factor. In mathematical form, the time lag, which is denoted as ϕ is defined as follows [1, 2, 3]:

$$\phi = \begin{cases} t_{T_i^{max}} > t_{T_e^{max}} \Rightarrow t_{T_i^{max}} - t_{T_e^{max}} \\ t_{T_i^{max}} < t_{T_e^{max}} \Rightarrow t_{T_i^{max}} - t_{T_e^{max}} + P \\ t_{T_i^{max}} = t_{T_e^{max}} \Rightarrow P \end{cases} \tag{2.1}$$

where $t_{T_i^{max}}$ and $t_{T_e^{max}}$ represent the time in hours when the inside and outside wall surfaces are at their maximum value, respectively. P is the period of the wave. In this study, it is taken as 24 h (one-day period).

Similarly, the decrement factor, which is denoted as f is defined as [1, 2, 3]:

$$f = \frac{A_i}{A_e} = \frac{T_i^{max} - T_i^{min}}{T_e^{max} - T_e^{min}} \tag{2.2}$$

Where A_i and A_e are the amplitudes of the wave in the inner and outer wall surface, respectively.

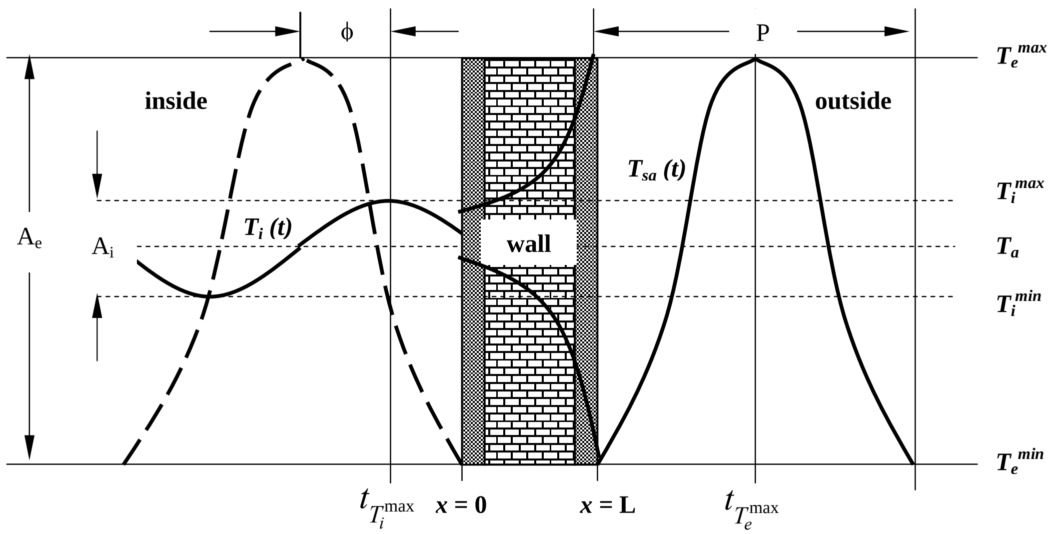

Figure 2.1 The schematic illustration of time lag, ϕ and decrement factor, *f*.

A detailed computational study was made to determine the effects of the thickness and thermophysical properties of a wall on time lag and decrement factor. The computations were carried out on three selected cases in Chapter 4. Comparisons were made between the results in each case and these results are useful for designing more effective passive solar buildings [9,11] and other related areas.

2.1.2 Sol-air Temperature

The sol-air temperature, denoted as $T_{sa,}$ is the outside air temperature affected by the heat flux of solar radiation. This temperature changes periodically with time [6, 13] and it is assumed to oscillate in the sinusoidal way during one-day period (24 h). A very general equation [1, 2, 3] for sol-air temperature is taken as follows:

$$T_{sa}(t) = T_{min} + \frac{|T_{max} - T_{min}|}{2} + \frac{|T_{max} - T_{min}|}{2} \cdot sin\left(\frac{2\pi t}{P} - \frac{\pi}{2}\right) \qquad (2.3)$$

Here, T_{min} and T_{max} are the minimum and maximum ambient air temperature over the one-day period, respectively. P is taken as a value of 24 hours as the problem is investigated based on one-day period.. Refer to Equation (2.3), the sol-air temperature changes between T_{max} and T_{min} during the 24-hour period.

For example, T_{max} and T_{min} are 40°C and 22°C, respectively over a 24-hour period. The sol-air temperature profile is shown in Figure 2.2.

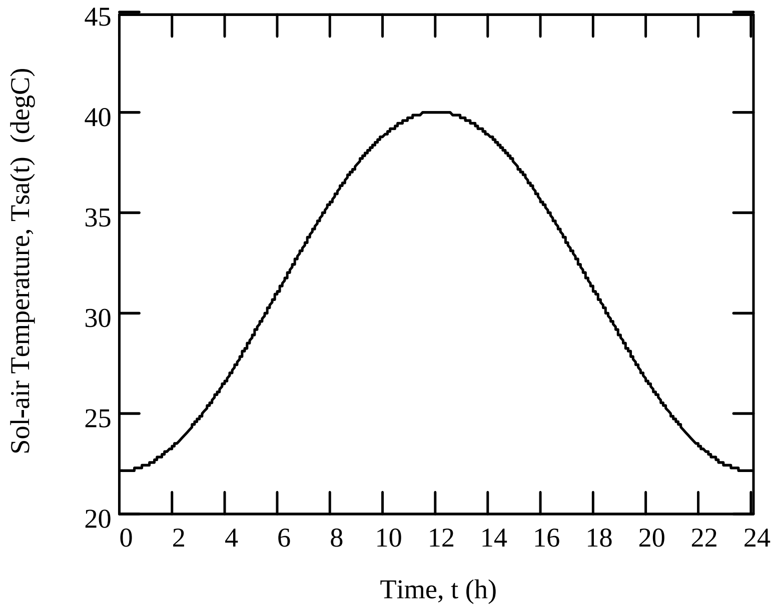

Figure 2.2 The Sol-air temperature profile

According to Equation (2.3), the minimum ambient air temperature occurs exactly at 12 a.m. midnight and the maximum ambient air temperature occurs at 12 p.m. noon. The comparison between the actual sol-air temperature profile, which is taken from a real climatological data by Threlkeld [10] and the sol-air temperature in present study is shown in Figure 2.3 [2]. Equation (2.3) is a reasonable choice for sol-air temperature as shown in Figure 2.3.

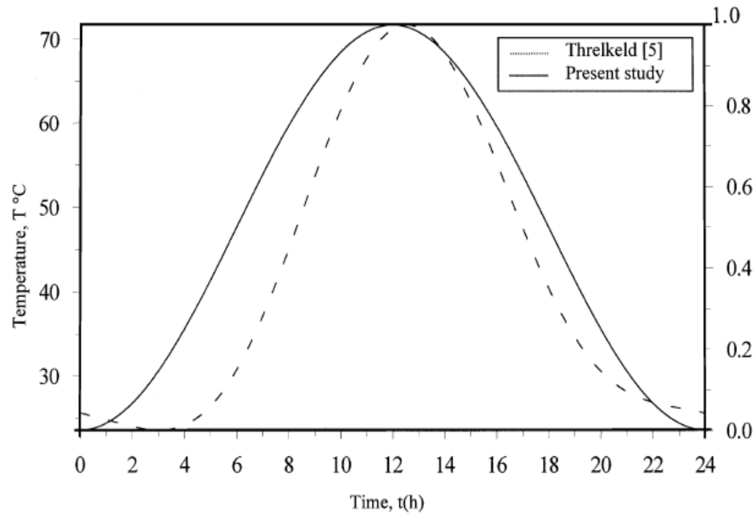

Figure 2.3 Comparison of sol-air temperature

2.2 Methods

During the thermal analysis on the multi-layer composite wall [15], the problem is assumed to be one-dimensional (only in *x*-direction) and time dependent. The problem geometry is illustrated in Figure 2.4.

The one-dimensional, Fourier energy equation for transient heat conduction is used to solve this problem geometry. The energy equation is as follows:

$$k\frac{\partial^2 T}{\partial x^2} = \rho c_p \frac{\partial T}{\partial t} \tag{2.4}$$

where k is the thermal conductivity, ρ is the density and c_p is the heat capacity of the wall material. In order to solve Equation (2.4) base on the problem geometry in Figure 2.1, two boundary conditions and one initial condition are required. In this case, the internal heat generation term is neglected.

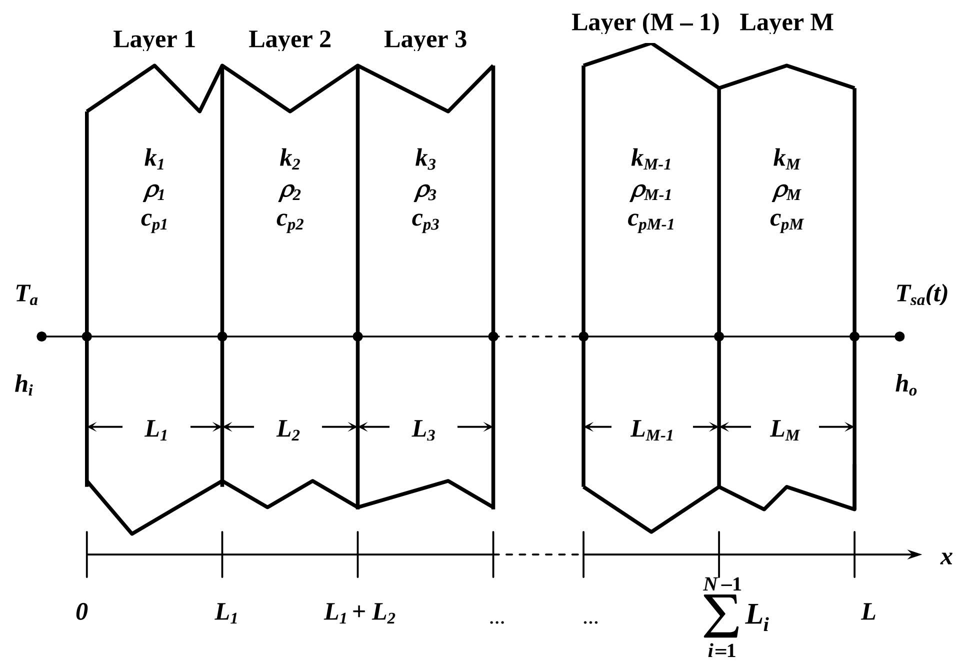

Figure 2.4 The Schematic of the problem geometry

The convective boundary conditions on both outer and inner surface of the walls are indicated as follows:

At the inner surface, the boundary condition is written as:

$$k\left(\frac{\partial T}{\partial x}\right)_{x=0} = h_i\left[T_{x=0}(t) - T_a\right] \tag{2.5}$$

At the outer surface, the boundary condition is written as:

$$k\left(\frac{\partial T}{\partial x}\right)_{x=L} = h_o\left[T_{sa}(t) - T_{x=L}(t)\right] \tag{2.6}$$

Refer to the Equation (2.5) and (2.6), h_o and h_i are heat transfer coefficient of the wall outer surface and wall inner surface, respectively. $T_{sa}(t)$ is the sol-air temperature, which was defined in Equation (2.3). $T_{x=0}$(t) is the wall outer surface temperature whereas $T_{x=L}(t)$ is the wall inner surface temperature. T_a is the inside air temperature.

As for initial condition, it is assumed that the wall is isothermal at a temperature T_o, at t = 0. During the investigations, the inside air temperature, T_a is taken to be a constant at any time during the one-day period. In order to simplify the computations, the problem is assumed to be one-dimensional transient heat conduction, which has a periodic boundary condition on the outer surface of the wall, the sol-air temperature boundary condition. This simply means that the temperature profile at the wall outer surface is given in the relation as follows:

$$T_{x=L}(t) = T_{sa}(t) \tag{2.7}$$

The boundary condition on the wall inner surface is remained as Equation (2.5). The thermophysical properties across each layer are assumed to be homogeneous.

2.2 Analytical Solutions

The separation-of-variables method of analytical solution for one-dimensional transient heat transfer differential formulation is given as follows [5, 6, 7]:

Consider an infinite plane slab of thickness L as shown in Figure 2.5. At the initial state, which is $t < 0$, the slab is at a uniform temperature of T_o, and at $t = 0$, the surfaces of the slab are suddenly brought to a temperature T_I. The governing differential equation is:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \cdot \frac{\partial T}{\partial t} \tag{2.8}$$

The thermal diffusivity, α is a property of the slab material and is given by:

$$\alpha = \frac{k}{\rho c_p} \tag{2.9}$$

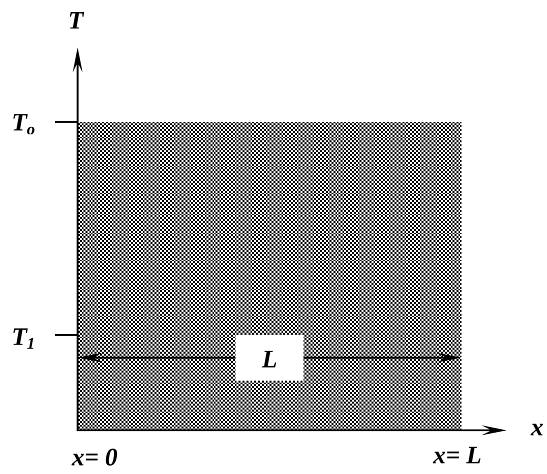

Figure 2.5 Infinite plane slab subjected to sudden cooling of surfaces

Equation (2.8) is arranged in a more convenient form by introducing the variable θ, where

$$\theta = T - T_1 \tag{2.10}$$

Then, the Equation (2.8) becomes

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{\alpha}\cdot\frac{\partial\theta}{\partial t} \tag{2.11}$$

The initial and boundary conditions are

$$\theta = \theta_o = T_o - T_1 \qquad \text{at } t = 0,\ 0 \le x \le L \qquad \text{(a)}$$

$$\theta = 0 \qquad \text{at } x = 0,\ t > 0 \qquad \text{(b)}$$

$$\theta = 0 \qquad \text{at } x = L,\ t > 0 \qquad \text{(c)}$$

Assuming that the solution consists of the product of a function of x and a function of t, or

$$\theta(x,t) = X(x)\cdot\Psi(t) \tag{2.12}$$

Double differentiating the left term with respect to *x*, Equation (2.12) becomes

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 X}{\partial x^2} \cdot \Psi \tag{2.13}$$

Differentiating the left term with respect to *t*, Equation (2.12) becomes

$$\frac{\partial \theta}{\partial t} = \frac{\partial \Psi}{\partial t} \cdot X \tag{2.14}$$

Introducing Equation (2.13) and (2.14) into Equation (2.11) gives

$$\frac{\partial^2 X}{\partial x^2} \cdot \Psi = \frac{\partial \Psi}{\partial t} \cdot \frac{X}{\alpha} \tag{2.15}$$

Separating the variables, Equation (2.15) becomes

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \frac{1}{\alpha \Psi} \cdot \frac{\partial \Psi}{\partial t} = -\lambda^2 \tag{2.16}$$

The term $(-\lambda^2)$ is the separation constant. Two ordinary differential equations can be obtained. They are

$$\frac{d^2 X}{d x^2} + \lambda^2 X = 0 \tag{2.17}$$

$$\frac{d^2 \Psi}{d t^2} + \alpha \lambda^2 \Psi = 0 \tag{2.18}$$

The solution for Equation (2.17) is given by

$$X(x) = C_1 \cos \lambda x + C_2 \sin \lambda x \tag{2.19}$$

whereas the solution for Equation (2.18) is given by

$$\Psi(t) = C_3 e^{-\lambda^2 \alpha t} \tag{2.20}$$

C_1, C_2 and C_3 are constants.

Substituting Equation (2.19) and (2.20) into Equation (2.12), the solution becomes

$$\theta(x,t) = \left(A\,cos\,\lambda\,x + B\,sin\,\lambda\,x\right)\cdot e^{-\lambda^2\,\alpha\,t} \tag{2.21}$$

where $A = C_1\,C_3$ and $B = C_2\,C_3$. The evaluations of the constants A and B depend on the on the physical boundary conditions. In order to satisfy the boundary conditions, it is necessary that $\lambda^2 > 0$.

From boundary condition (b), $A = 0$ for $t > 0$. From boundary condition (c), B cannot be zero, therefore $sin\ \lambda\ L = 0$, or

$$\lambda_n = \frac{n\,\pi}{L} \qquad n = 0, 1, 2, 3,\ldots$$

The final series form of the Equation (2.21) is

$$\theta(x,t) = \sum_{n=1}^{\infty} B_n\, e^{-\left[\frac{n\pi}{L}\right]^2 \alpha\, t} \cdot sin\,\frac{n\,\pi\,x}{L} \tag{2.22}$$

Equation (2.22) is known as a Fourier sine expansion with the constants B_n. B_n is determined from the initial condition (a) and is written as follows:

$$B_n = \frac{2}{L}\int_0^L \theta_o\ \ sin\,\frac{n\pi x}{L}\ dx = \frac{2}{\pi}\cdot\frac{1-(-1)^n}{n} \qquad n = 1, 2, 3, \ldots \tag{2.23}$$

Substituting Equation (2.23) into Equation (2.22) gives

$$\frac{\theta(x,t)}{\theta_o} = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n}\, e^{-\left[\frac{n\pi}{L}\right]^2 \alpha\, t} \cdot sin\,\frac{n\,\pi\,x}{L} \tag{2.24}$$

Equation (2.24) is the analytical solution for one-dimensional transient heat conduction over an infinite plane slab subjected to step change in surfaces temperature with a thickness of L. The *T-x* (temperature-distance) plots of Equation (2.24) at different t values are shown in Figure 2.6 whereas the *T-t* (temperature-time) plots at different x values are shown in Figure 2.7.

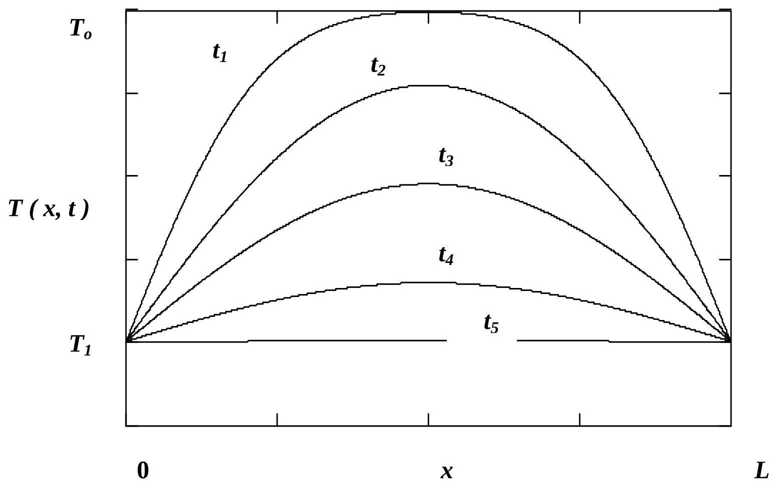

Figure 2.6 Temperature distribution across the thickness *L* of infinite plane slab at different time *t*.

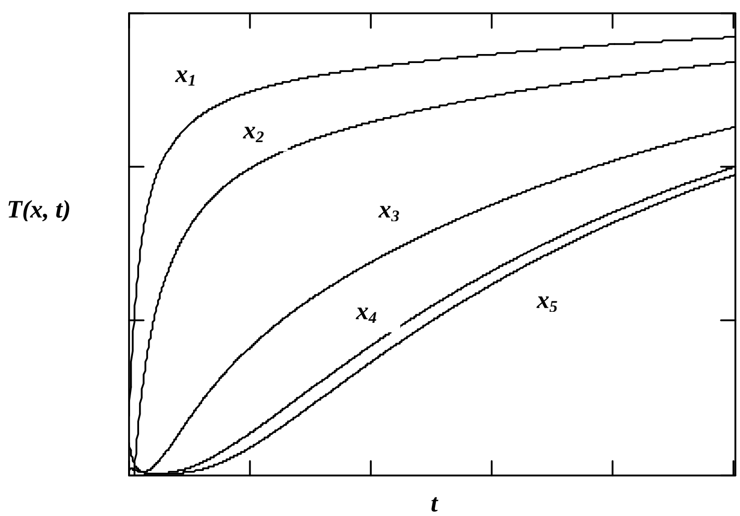

Figure 2.7 The temperature profiles of different distances, *x* from the surface of the infinite slab at any time, *t*.

For solving the one-dimensional transient heat conduction in a semi-infinite slab with convective boundary condition, The following solution [6, 7] is used. The schematic diagram of the semi-infinite slab is shown in Figure 2.8.

Initial condition: T(x,0) = T_o

Boundary conditions: $-k \cdot \left.\frac{\partial T}{\partial x}\right|_{x=0} = h\left[T_F - T(0,t)\right]$

$$T(\infty,t) = T_o$$

Solution:

$$\frac{T(x,t)-T_o}{T_F - T_o} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha \cdot t}}\right) - \left[\exp\left(\frac{h\cdot x}{k} + \frac{h^2\cdot\alpha\cdot t}{k}\right)\right].\left[\text{erfc}\left(\frac{x}{2\sqrt{\alpha\cdot t}} + \frac{h\sqrt{\alpha\cdot t}}{k}\right)\right] \quad (2.25)$$

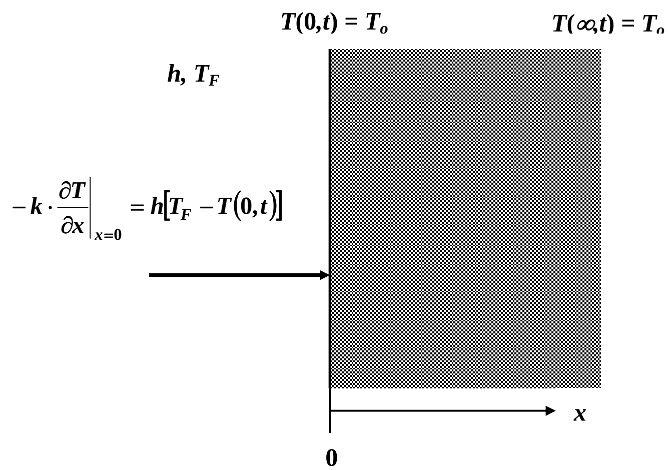

Figure 2.8 Semi-infinite slab subjected to convection at the surface.

The T-x plots of Equation (2.22) at different t values are shown in Figure 2.9 whereas the T-t plots at different x values are shown in Figure 2.10.

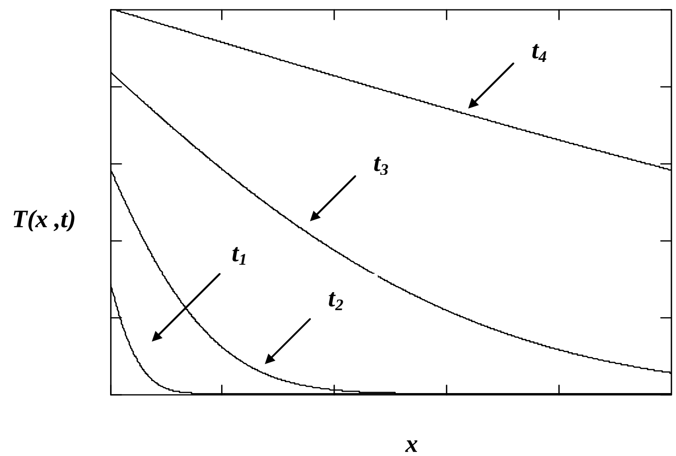

Figure 2.9 The temperature distributions across the thickness of the semi-infinite slab at different time *t*

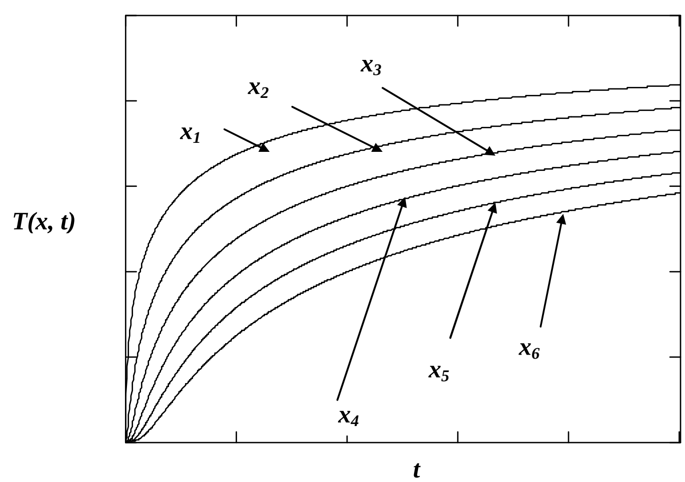

Figure 2.10 The temperature profiles of different distances, *x* from the surface of the semi-infinite slab at any time, *t*.

CHAPTER 3 NUMERICAL APPROACH

3.1 Introduction of Numerical Methods

In many practical situations, the geometry of the system and the boundary conditions are too complex to yield either analytical or analogical solutions. In this case, numerical methods can be very useful tools in solving such problems. These methods are based on finite difference techniques, which are ideally suited for solution by means of computer simulations.

Before numerical methods can be applied to a heat-transfer problem, or any other physical problem described by a differential equation, some preliminary steps are necessary. The purpose of these preliminary steps is to approximate the differential equation and the boundary conditions by a set of algebraic equations. This is accomplished by replacing the continuous domain by a pattern of discrete points within the domain and introducing finite difference approximations between the points.

In order to solve the heat transfer problems numerically, the system is subdivided into a number of small but finite sub volumes and each of then is assigned with a reference number. Each sub volume is assumed at the temperature corresponding to its center, or nodal point of the sub volume. If N points are selected, a set of N algebraic equations is obtained. It can be solved by matrix inversion or a numerical method for the values of unknown at the N points.

In this Chapter, the Finite Volume Method [8] is used to develop the solution for the one-dimensional unsteady-state heat conduction through a multi-layer composite wall with convective and periodic temperature boundary conditions. As for validation of the code, numerical solutions for transient heat transfer through one-layer wall with step change in surfaces temperature and with convective surface are developed and the results are compared with the analytical solutions taken from reference [5, 6, 7].

3.2 Finite Volume Method

The finite volume method was originally developed as a special finite difference formulation. For reader information, the method is used to develop the main commercially available CFD codes such as FLUENT, FLOW 3D, PHOENIICS and CFX. The finite volume algorithm consists of 3 main steps:

1. Divide the solution domain into finite number of control volumes and mesh grid points.
2. Integrate the governing equations over all the control volumes and apply the initial and boundary conditions.
3. Solve the algebraic equations to find the dependent variable in all the grid points in the solution domain.

The numerical solution for the one-dimensional unsteady-state heat conduction through a one-layer plane wall is as follows:

The governing differential equation is as in Equation (2.4). Rearrange the equation, gives

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) \tag{3.1}$$

For convenience, $\rho\, c_p$ is assumed to be constant.

Since time is a one-way coordinate, the solution is given by marching in time from a given initial distribution of temperature. Thus, in a typical "time step" the task is: Given the grid-point values of T at time t, find the values of T at time $t + \Delta t$. At time t, the values of T at the grid points are superscripted with "o" and the values of T at time $t + \Delta t$ is not superscripted. The schematic diagram of grid spacing and control volumes along the thickness of the wall is shown in Figure 3.1.

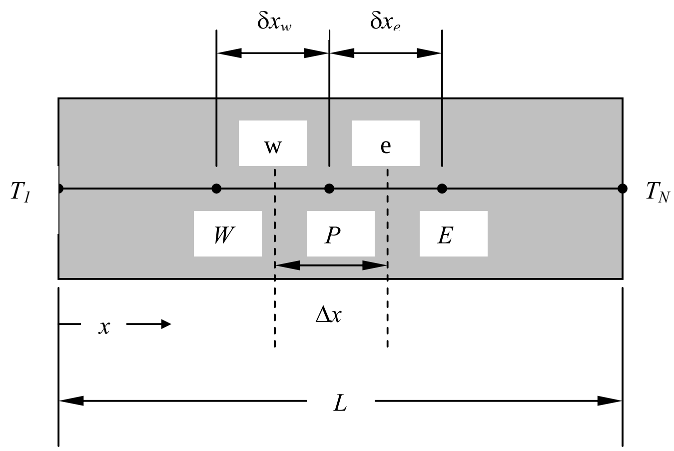

Figure 3.1 The schematic diagram for grid spacing and control volume.

The discretization equation is now derived by integrating Equation (3.1) over the control volume shown in Figure 3.1 and over the time interval from t to $t + \Delta t$. Thus,

$$\rho c_p \int_w^e \int_t^{t+\Delta t} \frac{\partial T}{\partial t} dt\, dx = \int_t^{t+\Delta t} \int_w^e k \frac{\partial T}{\partial x} dx\, dt \tag{3.2}$$

where the order of the integrations is chosen according to the nature of the term. For the representation of the term $\frac{\partial T}{\partial t}$, the grid-point value of T is assumed to prevail throughout the control volume. Then,

$$\rho c_p \int_w^e \int_t^{t+\Delta t} \frac{\partial T}{\partial t} dt\, dx = \rho c_p \Delta x \left(T_P - T_p^o \right) \tag{3.3}$$

The right term of Equation (3.2) can be written as:

$$\int_t^{t+\Delta t} \int_w^e k \frac{\partial T}{\partial x} dx\, dt = \int_t^{t+\Delta t} \left[\frac{k_e \left(T_E - T_P \right)}{(\delta x)_e} - \frac{k_w \left(T_p - T_w \right)}{(\delta x)_w} \right] dt \tag{3.4}$$

At this point, an assumption about how T_P, T_E, and T_W vary with time from t to $t + \Delta t$. The assumptions can be generalized by proposing

$$\int_t^{t+\Delta t} T_P \, dt = \left[f\, T_P + (1-f) T_P^o\right] \Delta t \tag{3.5}$$

where f is a weighting factor between 0 and 1. Using similar formulas for the integrals of T_E and T_W, gives

$$\rho c_p \frac{\Delta x}{\Delta t}\left(T_P - T_P^o\right) = f\left[\frac{k_e\left(T_E - T_P\right)}{(\delta x)_e} - \frac{k_w\left(T_P - T_W\right)}{(\delta x)_w}\right] + (1-f)\left[\frac{k_e\left(T_E^o - T_P^o\right)}{(\delta x)_e} - \frac{k_w\left(T_P^o - T_W^o\right)}{(\delta x)_w}\right] \tag{3.6}$$

The fully implicit scheme (f = 1) is used due to its stability for any time and space steps. Therefore Equation (3.6) can be written as :

$$A_P T_P = A_E T_E + A_W T_W + S \tag{3.7}$$

where $A_E = \dfrac{k_e}{(\delta x)_e}$

$A_W = \dfrac{k_w}{(\delta x)_w}$

$A_P^o = \rho c_p \dfrac{\Delta x}{\Delta t}$

$S = A_P^o T_P^o$

$A_P = A_E + A_W + A_P^o$

The Equation (3.7) is the discretization equation that requires an initial temperature at t = 0 for all the nodes and two boundary conditions at x = 0 and $x = L$ as shown in Figure (3.1).