A COMPUTATIONAL STUDY ON STRUCTURAL BEHAVIOUR OF SURFACES WITH CURVED FOLD LINES

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A COMPUTATIONAL STUDY ON STRUCTURAL BEHAVIOUR OF SURFACES WITH CURVED FOLD LINES

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ABSTRAK

Permukaan dengan lipatan melengkung yang berasal dari origami boleh memberi kelebihan dalam struktur bumbung yang boleh meningkatkan kekuatan sambil mengekalkan ciri-ciri estetika. Walau bagaimanapun, masih terdapat banyak kesamaran dan kajian yang tidak mencukupi mengenai potensi penggunaan garis lipatan melengkung. Walau bagaimanapun, masih terdapat banyak kekaburan dan penyelidikan yang tidak mencukupi mengenai potensi penggunaan garis lipatan melengkung. Kajian ini bertujuan untuk merumuskan kaedah komputasi yang sistematik untuk penghasilan permukaan dengan lipatan melengkung dengan faktor yang berbeza yang menpengaruhi konfigurasi permukaan. Kajian ini juga bertujuan mengkaji kesan corak lipatan melengkung, ketebalan permukaan dan perubahan nisbah kenaikan-span pada tingkah laku struktur permukaan dengan kedutan melengkung dari segi kapasiti menanggung beban dan kekakuan. Beberapa kriteria telah ditetapkan untuk pemilihan origami dengan lipatan melengkung berdasarkan koleksi yang luas oleh penyelidik sebelumnya. Hasil kajian menunjukkan tiga model origami memiliki potensi aplikasi dalam sistem bumbung, iaitu *Non-inflated Degree-4 Vertices, Four Circular Mountain Ridge Curves* dan *Four Elliptical Mountain Ridge Curves*. Kaedah Penangkapan Imej (ICM) diadopsi untuk memlukiskan garis panduan model origami dan profil ketinggian dari bentuk kertas raster 3-D. Pembentukan permukaan diteruskan sebelum analisis struktur. Model-model origami ini kemudiannya diskalakan dengan 100 dan diperkaya dengan sifat analitik tambahan untuk menilai prestasi struktur. Analisis elemen terhingga di bawah keadaan berat sendiri hanya dijalankan sepadan dengan pelarasan, iaitu profil ketinggian garis lipatan melengkung, nisbah ketinggian-rentang keseluruhan dan ketebalan permukaan. Model yang berprestasi terbaik dari segi pengagihan tekanan yang lebih baik dan anjakan yang lebih rendah dengan pertimbangan berat sendiri kemudian akan dipilih.

Keputusan yang diperoleh daripada variasi profil ketinggian garis lipatan melengkung menunjukkan bahawa semua model di bawah kategori Model 01 (*Non-inflated Degree-4 Vertices*) dan Model 02 (*Four Circular Mountain Ridge Curves*) gagal memenuhi had tekanan. Dengan membandingkan kedua-dua jenis model ini, Model 01 menunjukkan tekanan dan anjakan yang lebih tinggi. Sebaliknya, hanya model di bawah kategori Model 03 yang memenuhi had tekanan. Ini terutamanya disebabkan oleh corak geometri model origami yang menyebabkan pengagihan tegasan yang berbeza di permukaan. Untuk pengubahsuaian yang lain, hasil FE menggambarkan bahawa semua model di bawah kategori Model 03 masih dalam had tegasan dan pesongan yang dibenarkan dengan variasi yang kecil. Oleh itu, kewujudan garis lipatan melengkung dan kedalaman efektif keseluruhan struktur mendedahkan prestasi yang luar biasa dalam kapasiti menanggung beban.

ABSTRACT

Surface with curved creases derived from origami can be advantageous in roof structures which can improve strength while preserving the aesthetic values. However, there is still a great deal of ambiguity and insufficient research on the potential use of curved fold lines. This study targets to formulate a systematic computational method for the generation of surface with curved creases with different factors governing the surface configuration. This study also aims to investigate the effects of pattern of curved crease folds, thickness of surface and the change of the rise-span ratio on structural behaviour of surface with curved crease in terms of load-carrying capacity and stiffness. Several criteria had been established for the selection of origami with curved creases based on the extensive collection by the previous researcher. The outcome shows three origami models have potential application in roofing systems, namely Non-inflated Degree-4 Vertices, Four Circular Mountain Ridge Curves and Four Elliptical Mountain Ridge Curves. Image Capturing Method (ICM) is adopted to develop the outlines of the origami models and the elevation profiles from the raster 3-D paper forms. Surface formation is then proceeded before the finite element analysis. These origami models are then scaled with 100 and assigned with additional analytical properties to evaluate the structural performance. Finite element analysis under self-weight condition only is carried out corresponding to the adjustments, i.e., elevation profiles of curved fold lines, overall rise-span ratio and surface thickness. The best-performing model in terms of better stress distribution and lower displacement with the consideration of self-weight is then selected. The results obtained from the variation of elevation profiles of curved fold lines show that all models under the category of Model 01 (Non-inflated Degree-4 Vertices) and Model 02 (Four Circular Mountain Ridge Curves) failed to meet the tensile stress limit. Comparing these two types of models, Model 01 shows relatively higher stress and

displacement. In contrast, only the models under category of Model 03 satisfy the stress limit. This is mainly due to the geometry patterns of the origami models which lead to different stress distributions over the surfaces. For the rest of the modifications, the FE results show that all models under the category Model 03 are still within the allowable stress and deflection limits with slight variation. Therefore, the presence of curved fold lines and the greater overall effective depth of the structure help to enhance the performance in load-carrying capacity.

TABLE OF CONTENTS

LIST OF TABLES

Page

LIST OF FIGURES

Page

LIST OF SYMBOLS

a	Length for semi-major axis of elliptical fold of origami model
B	Boundary Line
b	Length for semi-minor axis of elliptical fold of origami model
C	Centre of Origami Model
E	Young's Modulus or Modulus of Elasticity
e	End Point
F_d	Folding Distance
F_s	Folding Stage
f_c	Effective Strength of Concrete
f_{ctm}	Tensile Strength of Concrete
H_c	Maximum height at the center of origami model
I	Second Moment of Area
K	Gaussian Curvature or Total Curvature
κ_1	Principal Curvature 1
κ_2	Principal Curvature 2
L	Length of Square Boundary
ℓ	Model Length
ℓ_c	Length from center of origami model to the end point of major-axis of elliptical path
ℓ_M	Length between both ends of circular arc for mountain fold of origami model
ℓ_V	Length between both ends of circular arc for valley fold of origami model
M_x	Moment Resultant in the Local x-direction
M_y	Moment Resultant in the Local y-direction
R	Radius of circular arc for mountain fold of origami model
R_M	Length perpendicular to mid span of ℓ_M in defining curvature of the circular arc for mountain fold of origami model
R_V	Length perpendicular to mid span of ℓ_V in defining curvature of the circular arc for valley fold of origami model
r_1	Principal Radii of Curvature 1

r_2	Principal Radii of Curvature 2
S	Supporting Point
T	Surface Thickness of Origami Model
t	Thickness
$\Delta\theta$	Inclination angle in defining spacing between the pair of mountain fold and valley fold of origami model
θ	Inclination of mountain fold line from X-axis of origami model
δ_{all}	Allowable Deflection
δ_r	Maximum Resultant Displacement
δ_x	Maximum Displacement in Local x-Direction
δ_y	Maximum Displacement in Local y-Direction
δ_z	Maximum Displacement in Local z-Direction
γ_c	Unit Weight of Concrete
φ	Curvature of the curved fold lines
φ_M	Ratio of R_M and the ℓ_M
φ_V	Ratio of R_V and the ℓ_V
ρ	Mass Density
υ	Poisson's Ratio
σ	Stress
σ_x	Stress in the Local x-direction
σ_y	Stress in the Local y-direction

LIST OF ABBREVIATIONS

ACM	Aluminium Composite Material
ADINA	A finite element analysis application software
ANSYS	A finite element analysis application software
CAD	Computer-aid design
Divide	Command in Rhino program to divide equally into n segments
DOF	Degree of Freedom
DOGs	Discrete orthogonal geodesic nets
EdgeSrf	Command in Rhino program to create surface
FEA	Finite Element Analysis
FEM	Finite Element Modeling
ICM	Image capturing method
InterpCrv	Interpolated spline line-type in Rhino program
JoinEdge	Command in Rhino program to connect naked edges
LTR	Length–Thickness Ratio
L/T	Length to thickness ratio
LUSAS	A finite element analysis application software
MF	Mountain Fold
NURBS	Non-Uniform Rational B-Splines
Rhino	A CAD application software
ShowEdges	Command in Rhino program to show naked edges
SOFiSTiK	A finite element analysis application software used in this thesis
SSCFL	Shell surface with curve fold lines
Patch	Command in Rhino program to create surface
VF	Valley Fold

CHAPTER 1

INTRODUCTION

1.1 Background

The surface with curved fold lines is primarily derived from the Japanese art of paper-folding known as "origami". Origami is turning a single planar sheet of paper into various appealing and graceful 3-dimensional shapes without stretching, tearing or cutting (Killian et al., 2008). Traditional origami prohibits excessive deformation like stretching, tearing and stapling (Miura, 1989).

Japanese words "ori" means folding while "kami" means sheet of paper are directly linked with the term "origami" (Miura, 1989). Origami, while more closely related to Japan, has origins in China and Europe as well. Around 105 AD in China, paper was invented and *zhezhi* (folded paper) most possibly appeared soon after the invention (Dana, 2019). Paper *yuanbao* (gold nuggets) were crafted to be used as a sacred ritual and tossed into a fire is a staple for the traditional Chinese funerals by 900 AD (Peter, 2017). In Japan, paper was firstly initiated in the 6th century. Origami has been in the mainstream since Japan's Edo Period (1603–1868). Artists were allowed to cut the sheets of paper in the past. In contemporary origami, cutting the sheets of paper is no longer allowed where true origami is now sculpted entirely by only folds. In Europe, paper-folding was believed to be derived from napkin-folding in the 17th century. Napkin folding is a part of table manners practiced in German (Demaine et al., 2011; Nelson, 2018). Credit to Friedrich Fröbel, the founder of kindergartens, his effort of incorporated some hands-on activities including paper-folding enables the origami to prosper across the continent (Kelly, 2017).

1.1.1 Fold Lines in Origami

To develop an origami or a paper-folding art, crease pattern is essentially important. Crease also known as fold line is a line that is created on the paper when it has been folded or crushed. Creases usually occur in flexible materials, such as paper or cardboard and do not tear through the surface of the sheet material. The crease partitions the sheet of material into regions known as faces (An et al., 2011).

The basic of folding is derived from either straight line or curve line. Figure 1.1 shows the straight crease and curved crease. Straight crease origami also known as prismatic origami, is bending of sheet material along a straight line to develop polyhedral surface (Demaine et al., 2011). Curved crease origami is folded along curved line instead of straight line, fabricating a complex and elegant shape (Lichtblau, 2019). Without any twisting or other plastic deformation of the material, a developable surface can be formed by just simple bending of that material (Postle, 2012). By only pushing the end points of the curve crease towards the center, elastic deformation due to bending of paper can be modelled (Vergauwen et al., 2013). When two or more creases meet together, the intersection point is known as vertex. The number of creases arose from the vertex relates to the degree of vertex (Turner et al., 2016).

There are two main types of folds in origami – mountain (ridge) fold and valley (trench) fold. Mountain fold (convex) produces a ridge whereas valley fold (concave) produces a trench of paper forms (Arben, 2020). In fact, mountain fold is also a reverse valley fold (Hinders, 2019). The fold angle is deemed the supplement of the dihedral angle between two faces meeting at the crease as shown in Figure 1.2. Mountain-valley creases detected based on the sign of fold angle (An et al., 2011).

In typical diagram of origami, dashed lines represent valley-fold whereas a combination of dashed-dots line represent mountain fold (Hinders, 2019) as shown in

Figure 1.3. Both pleat and crimp have almost similar meaning which involves the repeating mountain-valley folds. The only thing that differ these two terms is crimp involves some reverse-folding (Turner et al., 2016). When the creases and mountain-valley folds meet together, unique and aesthetic forms of arts can be developed. Vertex or node is a point or end point of a line developed when two or more fold lines intersect (Norman and Arjomandi, 2017).

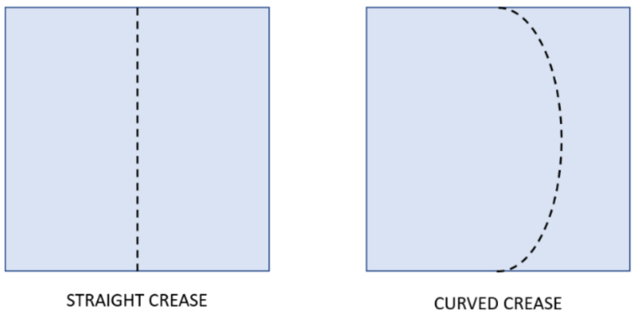


Figure 1.1 – Straight Crease and Curved Crease

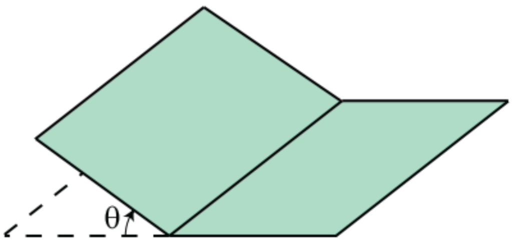


Figure 1.2 – Fold Angle at a Crease

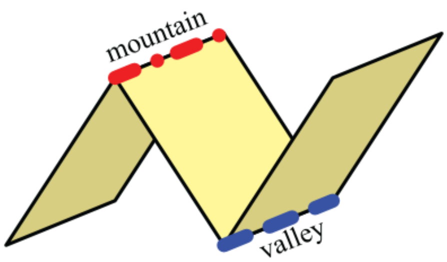


Figure 1.3 – Crease can be folded as mountain fold (left side) or valley fold (right side)

1.1.2 Evolution of Fold Lines

Before the curved crease elements breeze in the origami design, folding patterns with straight crease were being practiced (Demaine et al, 2011). According to Buri and Weinand (2008), Yoshimura (Diamond) pattern, Miura Ori pattern (Herringbone Pattern) and Diagonal pattern (parallelogram) are parts of straight crease origami that identified to be captivating to be applied in architectural and structural fields. Curved-crease sculptures are elegant artworks for almost one century ago. In 1927 to 1928, first curved fold Bauhaus model was created by students of Josef Albers with concentric circles and alternating mountain and valley folds. The design class by Albers is only emphasizes on the design through paper models without taking into account the pragmatic requirements. Irene Schawinsky, also a student from Bauhaus recreate the model with a large hole in the center later (Koschitz et al., 2008). In 70s, David A. Huffman and Ron Resch discovered the paper folding that also uses curved creases (Demaine et al, 2011). This is something distinctive with the Bauhaus model. Huffman, the pioneer of techniques for curve crease paper-folding, was also a computer scientist. Huffman is keen in mathematical analysis of curved folds whereas Resch is more interested in applying techniques to fabricate artistic sculptures. Huffman's work practiced no cut and only fold with piece of paper. Based on Huffman, "origami" is used to describe the figurative Japanese paper-folding tradition whereas "paper-folding" is used to describe the more abstract and practical Western tradition (Lichtblau, 2019).

Origami is no longer merely an ancient art form, evolution of origami designs has become an upsurge of interest in many fields (Dias et al., 2012). The inspiration of origami designs has been applied into mathematics, natural sciences, engineering, and architecture (Reis et al., 2015). Folding has an advantage in which, like the light-weight sandwich panel in Figure 1.4, it can improve stiffness with minimal weight or thickness.

The concept is often incorporated in architecture, varying from folded plate roofs to more sophisticated structures that combine an improvement in strength with aesthetic appearance (Schenk and Guest, 2011).

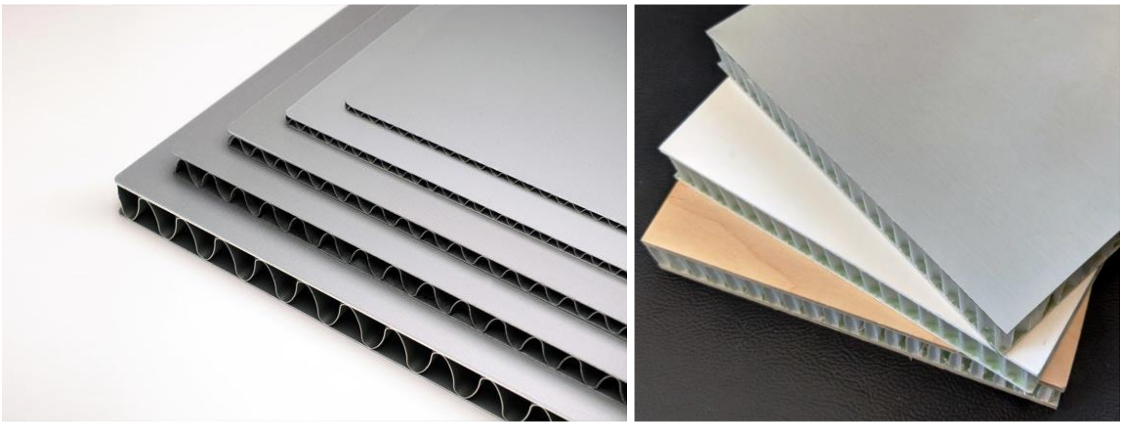


Figure 1.4 – Light-weight Sandwich Panel

In architecture domain, the concept of origami with folds has been adopted in the design of shell structures and folded plate structures. The developability and foldability properties are beneficial in designing surface structures (Demaine et al., 2011). The dissimilarity between shell structure and folded plate structure is shell structure has curvature of surface whereas folded plate structure has flat surface. They are the most often roofing method applied for spanning wide spaces without interruption in the interior section. Different options of shell structures with folded plate had been introduced for the architect or engineer for specific types of application (Ketchum and Ketchum, 1997).

Shell structures are of minimal thickness, lightweight and continuous curved surfaces that are assembled without intermediate supports to span large areas like roofs (Pereira, 2018). The shapes of the structures allow clear spans without internal supports, giving an open and unobstructed interior (InnovaConcrete, 2019). The typical shapes that had been widely used are cylindrical, hyperbolic paraboloid and ellipsoidal. They are mainly adopted for large structures such as convention centres, industrial buildings, sports complexes and multipurpose halls (Akhtar, 2016). In structural, shell structures are said to be effective because of their abilities to resist large compression loads which uniformly distributed over the surface. For the sake of minimum thickness of the surfaces, little or no tensile strength is produced. The structural efficiency is similar to that arch systems. Thus, concentrated loads should not be received directly (Pereira, 2018).

Folded plate structures are groupings of flat surfaces, leaned in various directions and joined along their longitudinal edges. Folded plate structures, owing to their structural, spatial and plastic characteristics, draw both architects and engineers (Buri et al., 2011). Folds will strengthen the thin surfaces lead to higher rigidity compared to flat surface. Folded plate structures have high capability to sustain load which makes them cost-effective over long spans that need column-free for internal space.

Indeed, the merging of aesthetics and functionality has already a trend for a few decades ago. Curved fold line offers new creativity for the architects and engineers in designing roof structures. The curved-crease of the surface can improve the stiffness & load-bearing capacity effectively with relatively thin section. The smooth corrugated profile of the roof surfaces has the aesthetic attribute which can be employed in the future. The curved-crease of the surface can improve the stiffness and load-bearing capacity effectively.

In 2014, One Fold in Figure 1.5 is an experimental project that seeks design possibilities by folding just once. The free-standing structure was created from a single sheet of stainless steel by Patkau Architects. A simple but elegant type of structure is conceived by merely using bending and folding (One Fold, 2014). An origami work of paper inspires this project. To build a freestanding piece, a sheet of paper folded once can then be forced to buckle around the crease. The attempt is then turned into a larger scale with sheet of stainless steel for a landscape shelter. One Fold is a thin-shell enclosure with lightweight, durable, demountable and recyclable properties (Canadian Architect, 2015).

Figure 1.5 – One Fold, Vancouver, BC, Canada (One Fold, 2014; Canadian Architect, 2015)

1.2 Problem Statement

In this modern era, a combination of aesthetic appearance and strength augmentation due to the minimal thickness of surface structures with folding can effectively benefit architects and engineers in designing roof surfaces or open space covering structures. There is a resemblance in term of concept as origami (Ng, 2018). A thin sheet of paper's foldability characteristics can help to improve the stiffness when it is being folded. Figure 1.6 explained that strength and load-carrying capacity could be gained when a thin material becomes corrugated with folds.

Besides that, it is hard to find actual buildings being constructed with curved fold lines on the roof surface through literature search. The only available structures with folds are of straight creases. Altering from the conventional straight fold into curved fold produces a neoteric creativity form that can hardly be described using simple parameters. Therefore, surface with curved folds is quite complicated and needs to be further studied (Lichtblau, 2019).

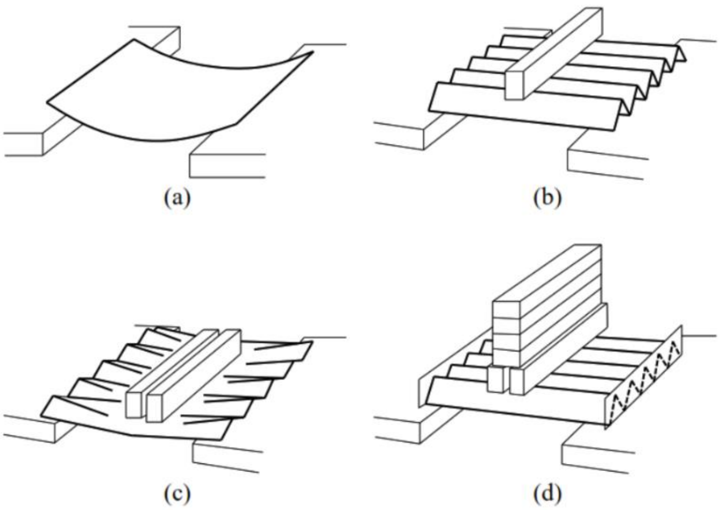


Figure 1.6 – The Stiffness and Load Carrying Capacity of Folded Paper (Ng, 2018)

In addition, catching sight of the "One Fold" actual model in Figure 1.5 (One Fold, 2014), there are some reasons and motivations emerged to introduce curve fold lines in roof surfaces. Sophisticated yet stunning appearance can be developed if the curved crease element is adopted. There is a potential to apply this concept for future application in the field of architecture. Referring to the classification of curved patterns by Ng (2018), it shows that a wide range of curved crease can be produced in circles, ellipses, and parabolas as shown in Figure 1.7 and examples of curved crease origami derived from these patterns are shown in Figure 1.8. When applying idea of curve folds in roof structures, the potential of stiff and durable roofs could be developed. Such kind of structures behaved like shell structure which had the high load capacity with minimal thickness.

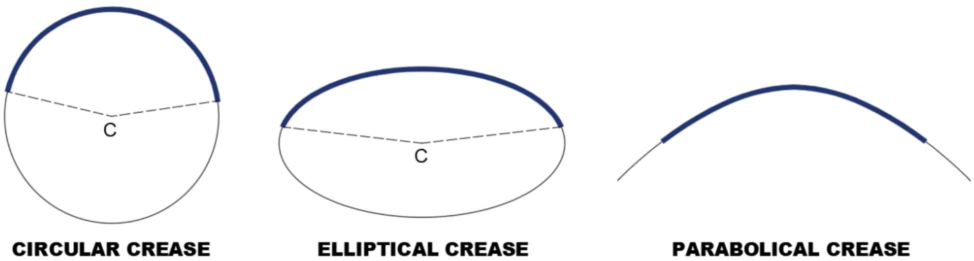


Figure 1.7 – Curve Crease Patterns

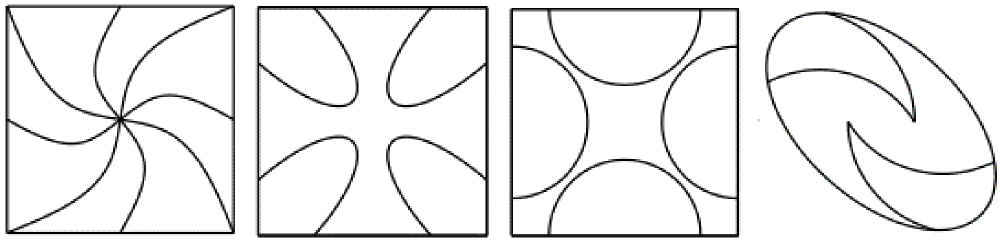


Figure 1.8 – Examples of Curved Crease Origami (Ng, 2018)

From the past researches, curved crease origami is extensively being explored. However, the study focuses more on the mathematical description of the geometry of curved creases (Huffman, 1976; Miura, 1989) and the constructive method either using computational software or fabricating the real physical models (Koschitz et al., 2008, Hemmerling and Mazzucchi, 2016; Demaine et al., 2016). To implement this concept in roof structures, the curved fold lines' behaviour along the surfaces in terms of structural need to be identified. Yet, there is still a lack in the studies of the structural behaviour of surface with curved fold lines. The effect of surface parameters such as surface thickness, rise-span ratio, pattern of curved crease on the structural behaviour need to be further studied (Rohim et al., 2013; Ng et al., 2014; Ng, 2018).

In short, owing to much uncertainty about implementing this idea in actual construction, this research can be part of the pathway towards transforming paper model with curved fold line into actual physical structures that have the feasibility to impress the world. The primary role of this research is to formulate a systematic computational method for the generation of surface with curved creases with different factors governing the surface configuration. This work can be achieved by generating models using Rhino software for surface geometry modelling and using SOFiSTiK software for structural analysis. This research aims to evaluate the structural behaviour of surfaces with curved fold lines. To satisfy this objective, the effects of surface parameters, the thickness of surface, and the change of the rise-span ratio towards the load-carrying capacity, stiffness, and deformation need to be investigated.

1.3 Objectives

This research aims to evaluate the structural behaviour of surface with curved fold lines. The particular objectives of this study are as follows:

1. To formulate a systematic computational method for the generation of surface with curved creases with different factors governing the surface configuration.
2. To investigate the effects of pattern of curved crease folds, thickness of surface, and the change of the rise-span ratio on structural behaviour of surface with curved crease in terms of load-carrying capacity and stiffness.

1.4 Scope of Work

This research is an interdisciplinary study involving the fields of structural engineering and architecture. It covers the disciplines of origami, roof structures or open space covering structures and computational analysis. The motivation of this study was the Japanese art of paper folding and the enhancement of aesthetic values by transforming straight crease into curved crease in designs. The research focusses on surface geometry of origami sculptures with only curved creases and the effect of surface configuration on the structural behaviour of surfaces with curved creases. Selection is performed based on several criteria, i.e. the number of curved fold lines, the number of supporting points, boundary condition, and the folding mechanism. Selected origami-inspired models of curved fold lines are then regenerated using Rhino software for surface geometry modelling. The effects of surface geometry of the origami models in terms of surface parameters, thickness of surface and also the rise-span ratio on the structural behaviour are studied using finite element analysis via SOFiSTiK software. The structural behaviour to be investigated involves the load-carrying capacity and stiffness developed by models with curved fold lines. This study's ultimate goal is to establish a suitable model with curved crease that has the capability to be applied in surface structures.

1.5 Significance of Study

In architectural design, origami or paper folding is not simply an inspiration source. Also, the developability and foldability characteristics of origami is beneficial in roof structure designs. Since a few centuries ago, straight fold lines are commonly being employed in designs for many fields. A fusion of folding and bending a sheet of material relates to curved folding. The surface is made up of curved fold lines and smooth developable surface patches. In relation, pure folding produces prismatic origami The smooth developable surface is developed from pure bending of a sheet of material.

Many artists had been practiced this approach (a hybrid of folding and bending) to create magnificent models with sheet of paper. The sculptures fabricated from only thin, flat sheet materials not only visually extraordinary; they also show favourable structural outcomes. Thereby, folded structures have become an ideal implementation for light-weight and geometrically stiff roof surfaces in architecture and engineering (Hemmerling and Mazzucchi, 2016).

Shifting from the conventional straight form of folds into the curved fold in real physical structures might be complex and required further studies. Nevertheless, the fascinating and elegant appearance of structures due to the curved crease is believed to have aesthetical attributes. Compared to those with straight creases, curved creases worked on the roof surface can enhance the stiffness and strength. Therefore, this research will help pave the way for the designers to implement such enticing properties in roof design.

1.6 Organization of Dissertation

This dissertation composes of five chapters: Introduction, Literature Review, Methodology, Results and Discussion, as well as Conclusions and Recommendations. The highlight of every chapter is shown below:

Chapter 1 addressed the background of the research study. An extensive interpretation of origami with curved fold lines was presented. It was then followed by the problem statement that clarified the problem or motivation of this study, objectives to be accomplished, scope of work, significance of the study and organization of dissertation.

Chapter 2 presented the literature review on the subjects associated to surfaces with curved fold lines. Past research findings corresponding to origami with curved crease and behaviour of surface with curved fold lines were reviewed. Research progress and recent work on surfaces with curved fold lines were also included.

Chapter 3 showed a series of procedures carried out for this study. Selection of models with curved fold lines were presented. Regeneration of selected origami sculptures was conducted using Rhino software. The step was proceeded with the modifications of the regenerated models by varying several parameters. Lastly, finite element modelling and analysis were carried out via SOFiSTiK software.

Chapter 4 illustrated the outcomes of this research study. Results of finite element analysis that investigated the structural behaviour of geometrical patterns of curved creases for the selected origami models were presented. A thorough discussion of the results obtained through the analysis was included as well.

Lastly, a conclusion and a list of recommendations for future research works were made in Chapter 5 to wrap up the dissertation. The best performance model that had the potential to be applied in roof structures is concluded.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Roof surfaces or surface structures are the main component or skeleton of a building, which shield the internal against rain, snow, sunlight, extremes of temperature, and wind. This research studies the surface geometry with curved fold lines, which has the potential to be adopted in surface structures or roof structures. Therefore, the studies begin with understanding how curved fold lines lying on a thin paper's surface could bear the loads effectively. Such structures are related to form-resistant structures. Form-resistant structures, as shown in Figure 2.1, are a form of material shaped based on the design loads to gain strength (Muljadinata and Darmawan, 2016). The concept has been discussed in the previous section about the stiffness generation through folds (Figure 1.6 in Chapter 1).

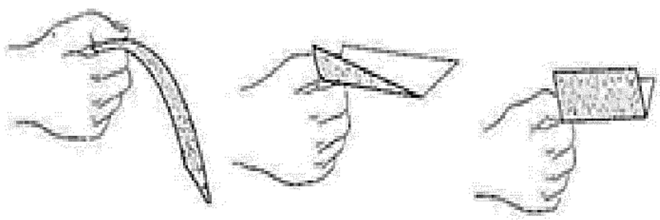


Figure 2.1 – Form-resistant Structure (Muljadinata and Darmawan, 2016)

In this study, the geometrical features of curved creases are being investigated. This chapter will present a comprehensive review of literature related to the topic of the surface with curve fold lines which covered the principles, the folding behaviour, formation of developable surfaces, current application, implementation of curved crease folding system, structural behaviour of surface structures and the past research studies corresponding to the surfaces with curved fold lines.

2.2 Curved-Crease Origami

In a simple word, origami (with one typical example of crane as shown in Figure 2.2) is known as paper-folding. Based on a simple technique, an incredible formal richness and variety of forms are produced from an open-state to a closed-state (Buri and Weinand, 2008). Two types of crease patterns can be identified during the folding process, i.e., straight-crease and curved-crease. Rigid origami or straight-crease origami forms strictly planar faces with one degree-of-freedom (DOF) mechanism (Vergauwen et al., 2013). Complex geometry like the curved crease origami is generated by manipulating techniques on the sheet of paper material in terms of bending (elastic deformation) and folding (plastic deformation) (Buri and Weinand, 2008; Vergauwen et al., 2013; Rabinovich et al., 2019). According to Lee et al. (2018) and Lee et al. (2020), origami with curved-crease is a branch of origami with a non-rigid folding motion to generate a 3-dimensional non-zero principal curvature of the model.

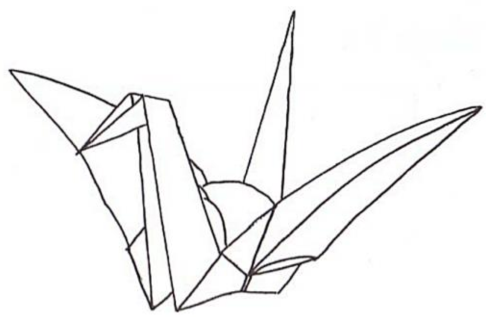
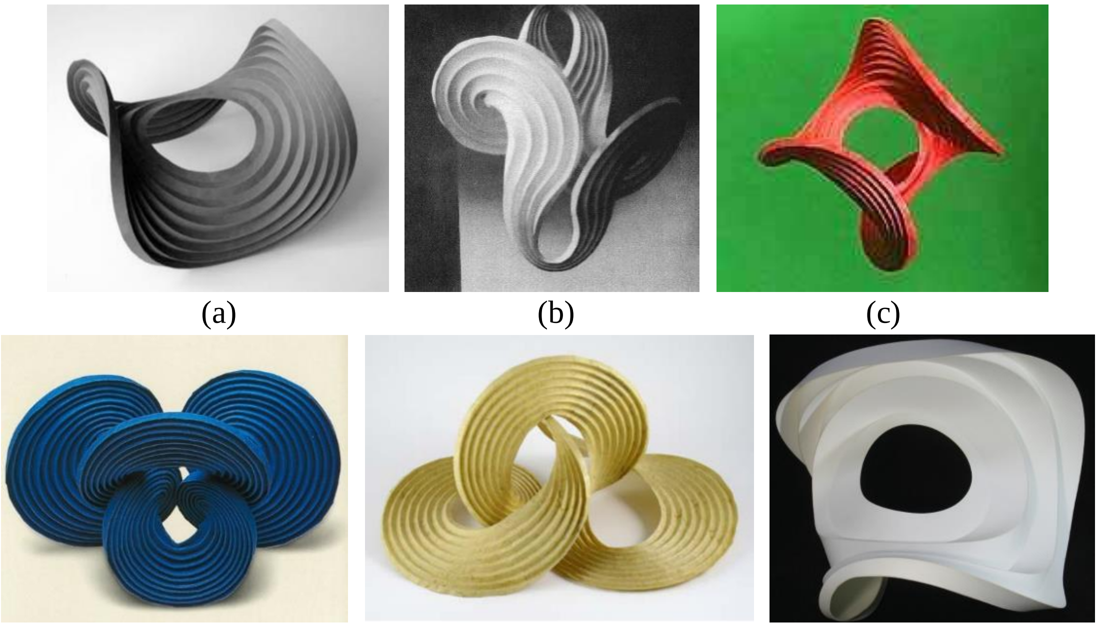


Figure 2.2 – Traditional Origami Crane (Macri, 2015)

Over the last 30 years, the art of origami has undergone a revolution where new designs of accretive complexity have been evolved. There is no coincidence that the increased sophistication of origami designs happens simultaneously with scientists, mathematicians, and origami artists discover more of the mathematical laws that dictate how paper folds (Hull, 2015).

The metamorphosis of the Bauhaus model since the 1920s (Figure 2.3) follows by the striking elegance of Huffman's model (Figure 2.4) and Ronald Resch's model

(Figure 2.5) in the 1970s until a more complicated contemporary art (Figure 2.6), has proved that the complexity of origami designs is continually being explored and transformed from time to time. The advanced evolution of origami design techniques is therefore springing out some curiosity and spirit among researchers to further study the transition of origami towards mathematical ways.



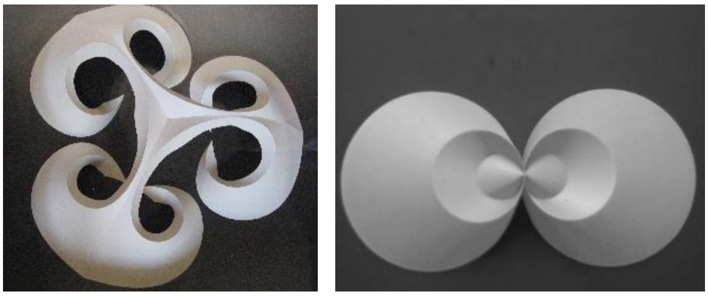
(a) (b) (c)

(d) (e) (f)

Figure 2.3 – Examples of Variation of Bauhaus Models: (a) First Bauhaus Model by student of Josef Albers (Demaine et al., 2011); (b) Variation of Bauhaus Model with larger concentric circular hole by Irene Schawinsky (erikdemaine.org, n.d.); (c) “Before the Big Bang” by Thoki Yenn (erikdemaine.org, n.d.); (d) Variation of Bauhaus Model by Kunihiko Kasahara (erikdemaine.org, n.d.); (e) Bauhaus Model with multiple circular pieces of paper by Erik Demaine and Martin Demaine (Demaine et al., 2011); and (f) Bauhaus Model with Circular Boundary by Koschitz et al. (Koschitz et al., 2008)

(a) (b) (c)

Figure 2.4 – Examples of Huffman’s Model with Curved Crease: (a) Hexagonal column with cusps (Demaine et al., 2011); (b) Rotational Tiling (Demaine et al., 2015); and (c) Two degree-2 vertices (Geometric Paper Folding, 1996)



(a) (b)

Figure 2.5 – Examples of Ronald Resch's Model with Curved Crease: (a) Space Curve (erikdemaine.org, n.d.); (b) Yellow Kissing Cones (Bhooshan, 2015)

(a) (b) (c) (d)

Figure 2.6 – Examples of Contemporary Art: (a) Design by Richard Sweeney (Demaine et al., 2011); (b) Design by Yuko Nishimura (Demaine et al., 2011); and (c) Design by T.Roy Iwaki (Demaine et al., 2011); (d) Design by Hoang Tien Quyet (Stewart, 2017)

2.2.1 Fundamental Concepts of Folding Present in Origami

To adopt paper-folding techniques in the design of surface structures or roof structures, the transformation or folding process and principles need to be taken into account. Folds that can be distinguished during the development of origami are mountain folds and valley folds. A combination of mountain-valley folds forms the reverse fold or the pleated fold. Robert Lang, an origami artist, states that origami's fundamentals are related to mathematical laws. Origami is not only a papercraft. It is also math due to the existence of creases pattern when the sheet material is folded (Peter, 2017). Four fundamental concepts in terms of mathematical relationships have been used to describe the fold pattern (Norman and Arjomandi, 2017; Peter, 2017):

i. At any vertex in a fold pattern, the difference between the amount of mountain (ridge) folds and valley (trench) folds must be two in either direction (Maekawa’s Theorem).

ii. The sum of alternating angles between fold lines surrounding any vertex is 180 degrees (Kawasaki’s Theorem).

iii. A sheet cannot penetrate a crease at overlaps in the pattern no matter how many layers of folds and sheets are stacked together.

iv. Two-colorability applies to any fold pattern where the same colour would never meet along a crease if the entire pattern were coloured with only two colours (Two-colorability Law).

Artistic origami uses elastic material like paper to create different papercrafts. Paper is not only the restricted folding material that can be used in fabricating model; any kind of thin sheet material which can withstand folding and bending can be utilised, for instance, metal, cardboard or polymer sheets (Vergauwen et al., 2014).

Corrugation of paper is a folding process that reaches its yield point to make it permanently (plastic) deforms. Plastic deformation means that the paper is unable to return to its initial state. Physics plays a vital role in finding static equilibrium among the forces created by the folds on the surface (Turner et al., 2016).

Besides, paper is used to visualize the forms of patterns with the crease on a smaller scale and determine the behaviour acting on it before it is modelled on a larger scale with other materials. The strength of a sheet of paper can be gained by folding the paper; thus, the weight of the object will eventually reduce (Yu-Ruei, 2016).

Paper-folding can also be related to morphing structures (Schenk and Guest, 2011). Morphing structures are load-bearing systems that capable to transform into various modes of forms to adapt new functions. As a rule, the ability of structures to take load depends on their stiffness, strength, and geometry stability. Buildings are therefore designed in a rigid manner to be stable against any loadings. Morphing structures rebel the traditional sense by optimizing the design with sufficient flexibility and stiffness to bear the loads (Vasista et al., 2019).

2.2.2 Folding Behaviour of Origami Model

Folding behaviour is an important subject that needs to be understood to fabricate a sheet material into 3-D form. The main components involved in developing an origami model are the boundary condition of sheet material (square, circle, ellipse, etc.), the crease patterns (straight-crease or curved-crease and mountain fold or valley fold), the actuation points, and the support conditions (pinned, roller or fixed support). Figure 2.7 shows the example of the main components involved when folding a Huffman model, non-inflated degree-4 vertex. Blue dot-dashed lines represent the mountain folds, whereas red dashed lines imply the valley folds. The black arrows indicate the actuation points. Apart from this, there are still some elements required to determine the folding behaviour, such as folding angle and folding stage.

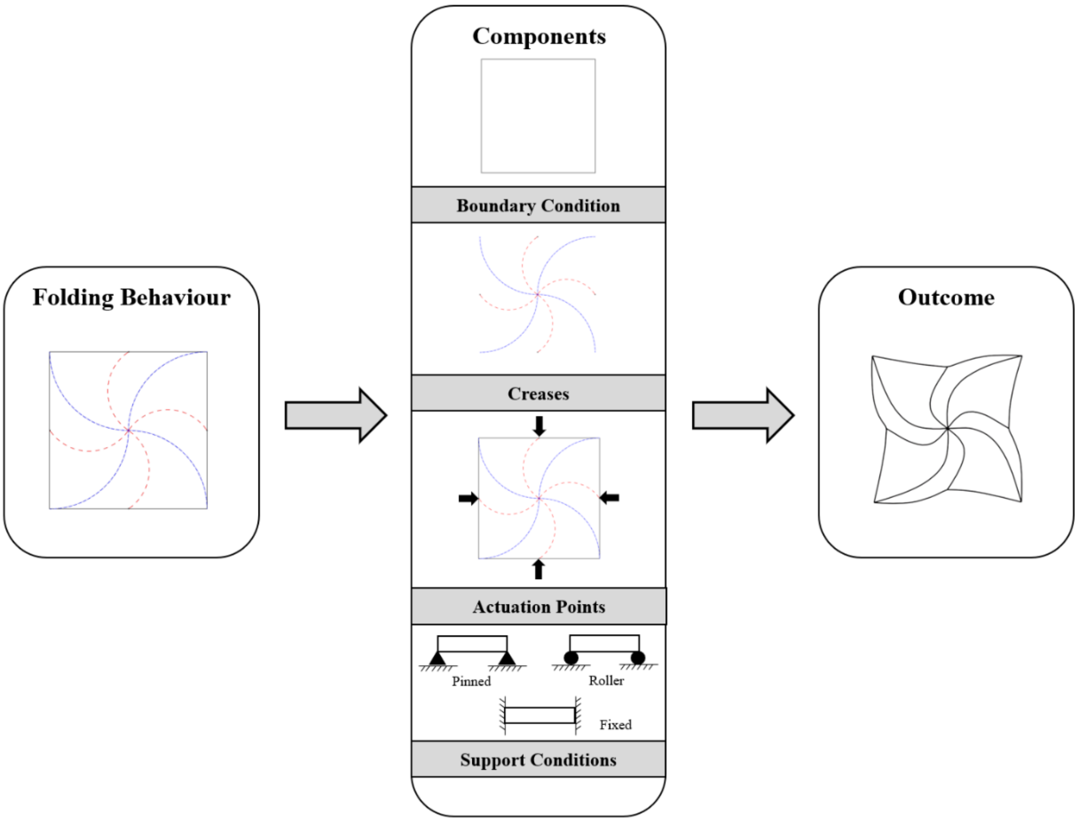


Figure 2.7 – Main components involved in developing origami model

Through folding, a crease line is formed and separates the sheet material into surfaces known as faces in general or developable surfaces in a specific manner. When one of the element's surface areas is elastically deformed, the kinetic connection along the crease line leads the neighbouring surface areas to warp as well (Vergauwen et al., 2013; Vergauwen et al., 2014; Vergauwen et al., 2017). This formation happened due to the internal forces and moments are transmitted through the curved fold lines leading to the adjacent surface to fold or bend. Vergauwen et al. (2013) have mentioned that the number of actuation points, actuation forces, and required displacements between actuation points are the key factors that govern the actuation characteristics. Figure 2.8 gives an overview of the actuation characteristic for one, two, three and four fold lines present on the paper element. The green arrows mean the direction of actuation. Figure 2.9 illustrates the actuation process that occurred when the four endpoints of the legs are pushed towards the center.

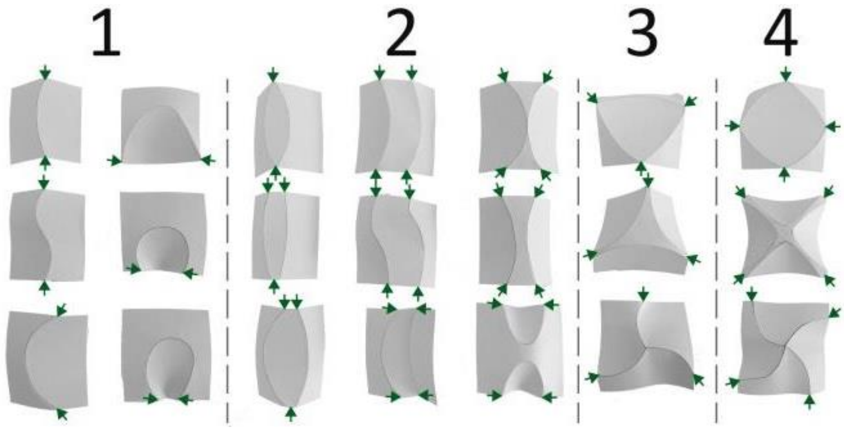


Figure 2.8 – Overview of curved-crease folding elements based on a composition of one, two, three or four creases. (Vergauwen et al., 2013)

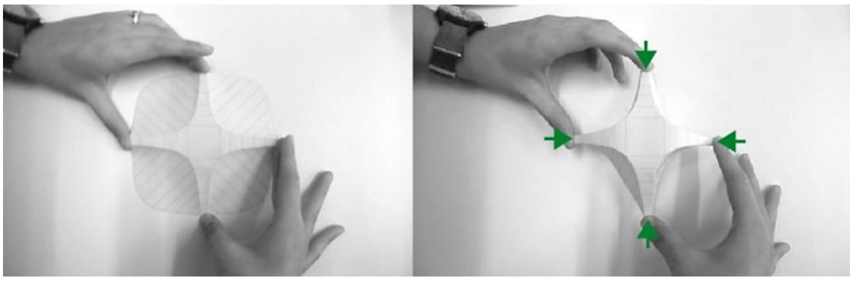


Figure 2.9 – Actuation Process of the Paper Model (Vergauwen et al., 2014)

2.2.3 Formation of Developable Surfaces

In this context of study, the form-finding mechanism initiated by folding is also worth being explored. Gaussian curvature or total curvature, *K*, a curvature intrinsic at any point on the surface, is the product of the principal curvatures, κ_1 and κ_2 at that point as defined in Equation 2.1 (Huffman, 1976; Morgan, 2016; Lee, 2019).

$$K = \kappa_1 \times \kappa_2 \quad (2.1)$$

The reciprocal of the corresponding principal radii of curvature, r_1 and r_2, are the principal curvatures, κ_1 and κ_2, respectively (refer Equation 2.2). A concave form has a positive curvature, a convex form has a negative curvature, and a planar form has zero curvature.

$$K = \frac{1}{r_1} \times \frac{1}{r_2} \quad (2.2)$$

The sign of the Gaussian curvature can be used to evaluate the characteristic of a smooth surface, as seen in Figure 2.10. If $K > 0$, a dome-like configuration is developed where both principal curvatures are in the similar orientation. A smooth developable surface can be developed with one principal curvature being zero and the other principal curvature being non-zero if $K = 0$. If $K < 0$, a saddle-like surface with opposing principal curvatures is developed.

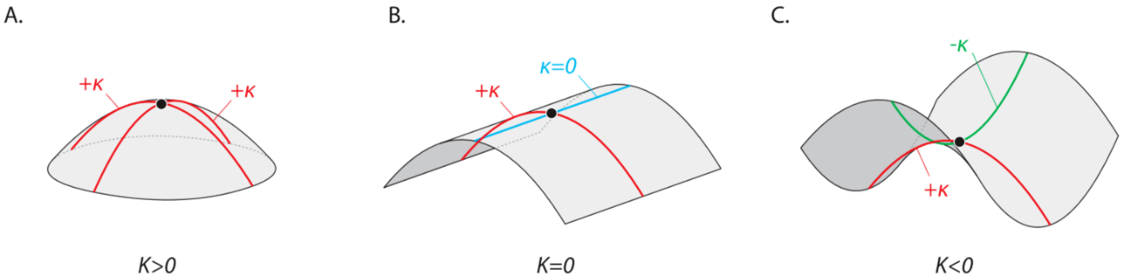


Figure 2.10 – Characteristic of Smooth Surfaces correspond to Gaussian Curvature: (A) $K > 0$: Dome-like surface; (B) $K = 0$: Developable surface; and (C) $K < 0$: Saddle-like surface. (Lee, 2019)

In a geometric sense, multiple interconnected developable surfaces are formed when a planar sheet of material is folded along the curved fold lines without stretching and tearing along the creases (Killian et al., 2008). In other words, origami surfaces are made up of at least two developable surfaces subjected to geometric constraints that the developability along curved fold lines are preserved (Lee, 2019). Huffman (1976), Killian et al. (2008), Lawrence (2011), Hemmerling and Mazzucchi (2016), Vergauwen et al. (2017), Nelson (2018), Tachi (2019), and Butler (2020) show up the characteristics of developable surfaces as listed below.

i. Able to unfold or unroll into flattening condition (unwrapped state) whilst preserve the original length of fold lines and angles on the surface.

ii. Composed of planar patches of ruled surfaces where all points on the rulings are located at the same tangent plane.

iii. Able to discretised according to the pre-identified and fixed rulings.

iv. Rulings that are either parallel (cylindrical), meet at a fixed point (conical), or tangent to the regression curve or space curve (tangent developable), respectively. (refer Figure 2.11)

v. Have zero Gaussian curvature, which is also known as singly curved surface.

vi. Various methods to unroll a 3D-geometry with curved folding exist, composed by single-curved surfaces into planar elements.

Clearly, rulings on the developable surfaces are significant in the geometric modelling of surfaces with curved fold lines. This can be explained where an infinite number of pairs of developable surfaces can be developed for one unique curved crease, as illustrated in Figure 2.12 (Vergauwen et al. 2017). Refer to Figure 2.13, Watanabe and Mitani (2019) also clarify the transformation of ruling patterns according to the folding motion.

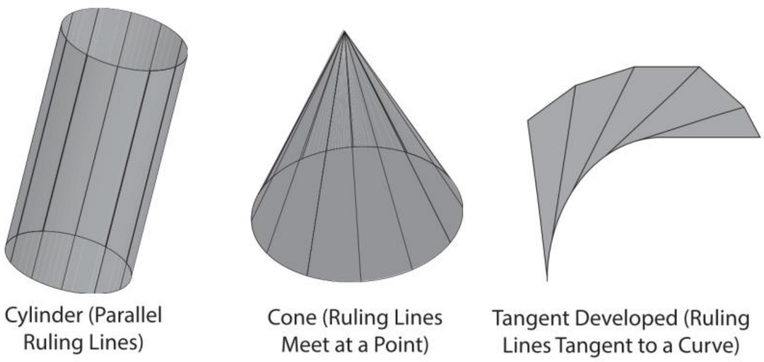


Figure 2.11 – Three Classes of Developable Surface: Cylinder (left), Cone (middle), and Tangent Developed (right). (Nelson, 2018)

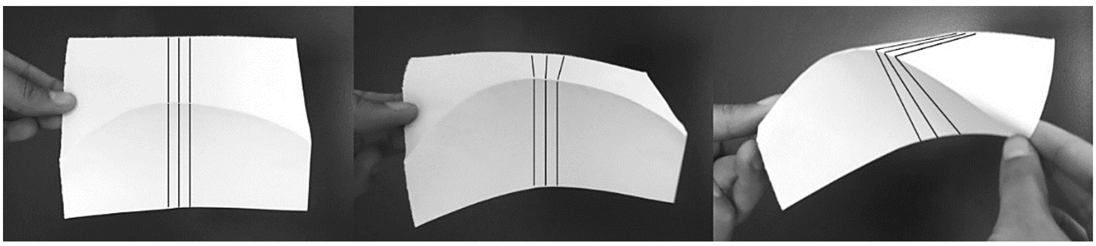


Figure 2.12 – Variation of Basic Shape of Paper: Initial Condition of Paper (Left), Bending of Paper along Curved Crease (Middle) and Physical Interaction of Paper through Twisting (Right). (Vergauwen et al. 2017)

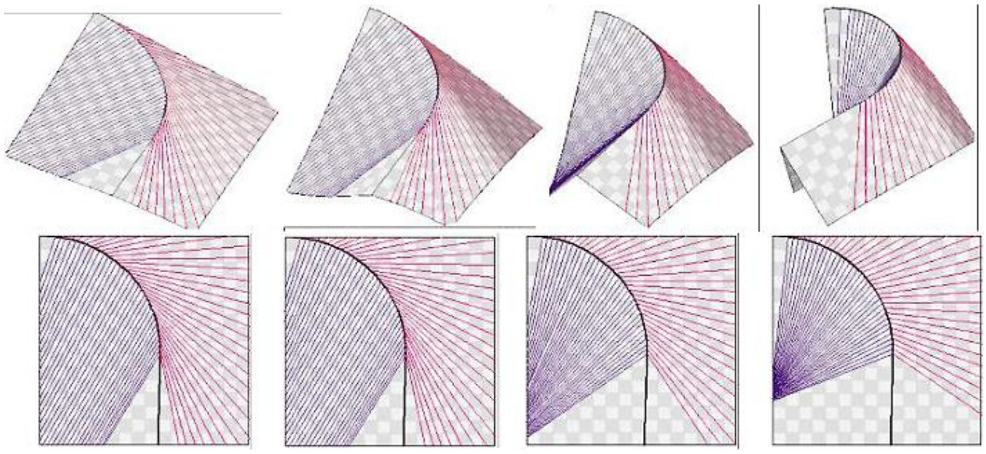


Figure 2.13 – Transition of Ruling Patterns according to Folding Motions (Watanabe and Mitani, 2019)

2.2.4 Current Applications of Origami-Inspired Design with Curved Fold Lines

The concepts and the aesthetic patterns of origami have been well-known and garnered attention in mathematics, engineering designs and applications (Reis et al., 2015; Norman and Arjomandi, 2017; Rabinovich et al., 2019; Tachi, 2019). Generally, those applications are in folded forms of static 3-D solutions where plastic (permanent) deformation occurs along the crease lines (Vergauwen et al., 2017). Presently, the inspiration of design based on origami is extensively being implemented in sculptural art, robotics, biomedical functions, product packaging design, solar energy practices, aircraft, and temporary structure design.

LE KLINT was founded in 1943 as a lightning company. The lamp designs are still handcrafted today. 'Pendant 172' in Figure 2.14(a) is one of the lampshade models with curved creases built in the form of continuous surfaces by Poul Christiansen (Demaine et al., 2011; Koschitz, 2014; Demaine et al., 2015). Figure 2.14(b) depicts 'ARUM', a partnership between Zaha Hadid Architects and Robofold. The design incorporates lightweight shells and tensile structures in which thin sheets of metal are pleated with curved-creases by industrial robots (Etherington, 2012; Gerfen, 2012;

Vergauwen et al., 2014; Adriaenssens et al., 2016; Vergauwen et al., 2017). For the purpose of furniture use, Andreas Lund designs a collection of moulded plywood stools with curved folds known as ‘Sit’ illustrated in Figure 2.14(c) (Vergauwen et al., 2014; Vergauwen et al., 2017). With the use of Robofold as well, a coffee table is shown in Figure 2.14(d) made out of curved-folded metal is designed by Gregory Epps (Vergauwen et al., 2014). In terms of structural element design, Haresh Lalvani creates the metal column covers of 8-12 feet tall in folded sheet metal shown in Figure 2.14(e) (Demaine et al., 2011). In Figure 2.14(f), Tachi designs a deployable rigid-foldable structure with curved folds. The curved folded arches are assembled to form a cellular structure (Demaine et al., 2015).

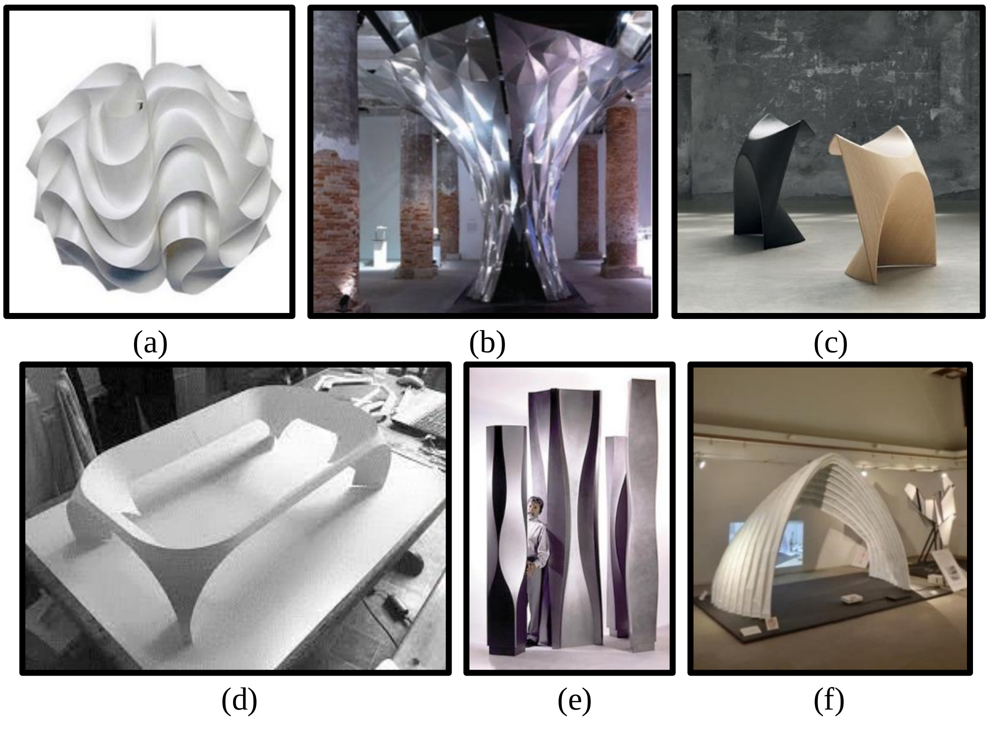


Figure 2.14 – Design Implementations of Curved Fold Lines: (a)‘Pendant 172’ by Poul Christiansen (Demaine et al., 2011); (b) “ARUM” by Zaha Hadid Architects (Adriaenssens et al., 2016); (c) ‘Sit” by Andreas Lund (Vergauwen et al., 2017); (d) Coffee Table by Gregory Epps (Vergauwen et al., 2014); (e) Metal Column Covers by Haresh Lalvani; and (f) Rigid-Foldable Curved Deployable Structure by Tachi (Demaine et al., 2015)

Besides that, Figure 2.15 shows the Undulatus Asperatus cloud pavilion made by Brancart et al. (2015) and is hung at the IASS2015 expo. A total of 99 components are