

**COMPRESSED SENSING IMPLEMENTATIONS
FOR SPARSE CHANNEL ESTIMATION IN OFDM
SYSTEMS**

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**COMPRESSED SENSING IMPLEMENTATIONS FOR SPARSE CHANNEL
ESTIMATION IN OFDM SYSTEMS**

by

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LIST OF ABBREVIATIONS

ADC	Analog-to-Digital Converter
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BP	Basis Pursuit
CS	Compressed Sensing
CSI	Channel State Information
CFR	Channel Frequency Response
CIR	Channel Impulse Response
CE	Channel Estimation
CoFA	Collaborative Framework of Algorithms
CoSaMP	Compressive Sampling Matching Pursuit
CDF	Cumulative Density Function
CDS	Cyclic Difference Set
CP	Cyclic Prefix
DDCE	Decision Directed Channel Estimation
DAB	Digital Audio Broadcasting
DVB	Digital Video Broadcasting
DFT	Discrete Fourier Transform
FACS	Fusion of Algorithms for Compressed Sensing

FFT	Fast Fourier Transform
gOMP	Generalized Orthogonal Matching Pursuit
GP	Greedy Pursuit
GI	Guard Interval
ICI	Inter-carrier Interference
ISI	Inter-symbol Interference
i.i.d.	independent and identically distributed
IDFT	Inverse Discrete Fourier Transform
IFFT	Inverse Fast Fourier Transform
LS	Least Squares
MATLAB	Matrix Laboratory
ML	Maximum Likelihood
MSE	Mean Square Error
MIP	Mutual Incoherence Property
M-QAM	Multilevel Quadrature Amplitude Modulation
NSP	Null Space Property
OFDM	Orthogonal Frequency Division Multiplexing
OMP	Orthogonal Matching Pursuit
PACE	Pilot Assisted Channel Estimation
PDF	Probability Density Function
PSAM	Pilot Symbol Assisted Modulation

QPSK	Quadrature phase shift keying
RF	Radio Frequency
ROMP	Regularized Orthogonal Matching Pursuit
RIP	Restricted Isometry Property
SAMP	Sampling Adaptive Matching Pursuit
SdMP	Stage-determined Matching Pursuit
SP	Subspace Pursuit
WCS	Wireless Communications Service
WSSUS	Wide Sense Stationary-uncorrelated Scattering
WLAN	Wireless Local Area Networks

LIST OF SYMBOLS

\mathbf{A}^H	Complex conjugate and transpose (hermitian) of the matrix \mathbf{A}
\mathbf{A}^T	Transpose of the matrix \mathbf{A}
arg	Argument
j	Imaginary unit: $j = \sqrt{-1}$
π	Pi: $\pi = 3.1416$
min	Minimum
max	Maximum
$\ \mathbf{a}\ _0$	l_0 norm of the vector \mathbf{a} : $\ \mathbf{a}\ _0 \triangleq \{\neq 0\} $
$\ \mathbf{a}\ _1$	l_1 norm of the vector \mathbf{a} : $\ \mathbf{a}\ _1 \triangleq \sum_{i=0}^n x_i $
$\ \mathbf{a}\ _2$	l_2 norm of the vector \mathbf{a} : $\ \mathbf{a}\ _2 \triangleq \sqrt{\mathbf{a}^H \mathbf{a}}$
$\ \mathbf{a}\ _p$	l_p norm of the vector \mathbf{a} : $\ \mathbf{a}\ _p \triangleq (\sum_{i=1}^m a ^p)^{\frac{1}{p}}$
k	sparsity number i.e., number of non-zero elements
$ \cdot $	Cardinality operator
$\lceil \cdot \rceil$	the ceiling function
$\#\{\cdot\}$	Number of elements
L	Channel length
\mathbf{h}	Channel impulse response
N	Total number of subcarriers
\mathbf{y}	Received CS measurements

\mathbf{A}	Measurement matrix
$\delta(t)$	Dirac delta function.
\mathbb{R}, \mathbb{C}	The set of real and complex numbers.
\mathbb{F}	Any Field.
$x \in \mathbb{F}$	A scalar in the field \mathbb{F} .
$x \in \mathbb{F}^n$	A column vector with n entries in \mathbb{F} .
x_i	The i th entry of the vector \mathbf{x} .
$\mathbf{0}, \mathbf{1}$	The vectors of all zeroes and all ones, respectively
x_k	The best k -sparse approximation to x .
$A \in \mathbb{F}^{m \times n}$	An $m \times n$ matrix with entries from \mathbb{F}
$A_{i,j}$	The (i, j) -th entry of the matrix \mathbf{A}
\mathbf{A}^\dagger	Moore-Penrose pseudo-inverse of \mathbf{A}
\mathbf{A}_Λ	The matrix formed by taking the columns of \mathbf{A} specified by Λ
σ_v	Singular values of v
\mathbf{I}	The $n \times n$ identity matrix
$(\cdot)^T$	transpose
$\text{diag}\{\cdot\}$	diagonal Matrix
\hat{T}_Δ	the set of indices contained in \check{T} that is pruned by the algorithm.
I_N	Identity matrix of size N
$\mathbb{R}(\cdot)$	real field with dimension (\cdot)
\mathcal{CN}	complex Gaussian distribution

\emptyset	empty set
$\mathbf{A}_{\check{T}}$	column submatrix of \mathbf{A} , with column indices equal to elements in the set \check{T}
$\text{supp}(x)$	the support of a vector x
x_p	sub-vector formed by the elements of vector x listed in the set p .
T^c	complements of the set T
\setminus	difference set
\tilde{T} , \check{T} and \hat{T}	estimated support set
$\tilde{\mathbf{h}}$, and $\hat{\mathbf{h}}$	CIR estimates
\mathbf{A}^*	Conjugate transpose or Hermitian transpose of \mathbf{A}

PELAKSANAAN PENDERIAAN TERMAMPAT BAGI ANGGARAN SALURAN YANG JARANG DI DALAM SISTEM OFDM

ABSTRAK

Permintaan yang meningkat bagi perhubungan berkadar data tinggi di atas saluran perlunturan tanpa-wayar bebilang-jalan, biasanya memerlukan maklumat terkini tentang saluran tersebut diketahui pada penerima. Hal ini diperolehi melalui pengetahuan terkini tentang Maklumat Keadaan Saluran (CSI) di penerima dalam menghasilkan pembinaan semula sambutan dedenyut saluran dari isyarat penerima. Bagi sistem OFDM berpandukan pengesanan jelas, anggaran saluran bagi reka bentuk penerima perlu tepat supaya prestasi CSI boleh ditingkatkan. Walau bagaimanapun, maklumat tersebut jarang ada *priori* dan perlu dianggarkan. Isyarat termampat menggunakan maklumat terkini yang menyatakan kebanyakan isyarat fizikal adalah jarang dan memerlukan beberapa ukuran untuk memperolehinya. Oleh itu, cabaran utama bagi CE berasaskan CS di dalam sistem OFDM adalah; pertama, reka bentuk untuk ukuran matriks yang tepat, mengeksploitasikan struktur isyarat jarang bagi sesetengah jelmaan asas. Kedua, berpandukan kepada maklumat terkini tentang ukuran vektor dan matriks, untuk mencari sokongan bagi vektor isyarat tidak diketahui dengan tepat daripada beberapa ukuran yang hingar. Pengoptimuman nilai simbol perintis dan penempatannya sebagai masalah pengoptimuman tak-bersambung berkemungkinannya tidak menunjukkan CE termampat yang jelas. Oleh itu, simbol perintis bersambung dan skim penempatannya dicadangkan untuk mengoptimumkan kedua-dua nilai simbol perintis dan penempatannya sebagai satu masalah pengoptimuman reka bentuk. Hasil simulasi menunjukkan bahawa skim yang dicadangkan adalah berkesan dan memberi keputusan dari segi

CE yang lebih baik daripada skim yang lain, serta boleh menghasilkan 18.75% penambahbaikan pada kecekapan jalur lebar dengan prestasi CE yang sama berbanding dengan CE persegi kurang (LS). Gabungan beberapa algoritma pembinaan semula boleh menyebabkan kemungkinannya tergabung beberapa indeks anggaran yang tidak tepat di saluran yang hingar. Maka, rangka kerja gabungan yang baru, dikenali sebagai Rangka Kerja Kerjasama bagi Algoritma (CoFA) dicadangkan untuk mendapatkan pemulihan isyarat jarang yang tepat daripada beberapa ukuran linear. Tambahan lagi, bagi aplikasi pendam lemah, algoritma bernama Pengejaran Tahap Ditetapkan (SdMP) dicadangkan untuk menghasilkan pembinaan semula isyarat yang cepat dan mudah dikendalikan. Dengan menggunakan sifat isometri terhadap, analisis teori bagi algoritma CoFA dan SdMP yang dicadangkan untuk CE memperolehi lebih kurang 11.1%, 18.3%, 28.9% dan 42.8% serta 5.6%, 13.9%, 22.8% dan 33.3% penambahbaikan pada nilai 2×10^{-3} MSE apabila dibandingkan dengan algoritma FACS, gOMP, OMP dan ROMP. Tambahan lagi, pada nilai 2×10^{-3} BER, algoritma CoFA dan SdMP yang dicadangkan memperolehi 9%, 14%, 19.5% dan 25%, serta 5%, 10%, 14% dan 22.5% penambahbaikan apabila dibandingkan dengan algoritma FACS, gOMP, OMP dan ROMP. Kesimpulannya, algoritma CoFA dan SdMP boleh dianggap sebagai algoritma yang paling cekap tenaga apabila dibandingkan dengan algoritma FACS, gOMP, OMP dan ROMP.

COMPRESSED SENSING IMPLEMENTATIONS FOR SPARSE CHANNEL ESTIMATION IN OFDM SYSTEMS

ABSTRACT

The ever-increasing demand for high-data-rate communication over a wireless multipath fading channel usually necessitates that at the receiver, prior knowledge about the channel is known. This is often achieved using knowledge of current Channel State Information (CSI) to produce at the receiver channel impulse response reconstruction obtained from the received signals. For coherent detection based OFDM system, CE is critical for the receiver design as accurate CSI can remarkably improve performance. However, such information is seldom available *a priori* and needs to be estimated. CS uses the prior knowledge that many physical signals are sparse and acquire them with few measurements. Therefore, the main challenge in CS-based CE in OFDM system is two-fold: firstly, the design of proper measurements matrix, exploiting signal sparsity structure over certain transform basis. Secondly, based on prior knowledge of the measurement vector and measurement matrix, to accurately find the support of the unknown signal-vector from very few noisy measurements. The optimization of pilot symbols values and their placement as a disjoint optimization problem may not necessarily exhibit low coherence compressed CE. Hence, a joint pilot symbol and placement scheme is proposed that optimizes over both the pilot symbol values and their placements as a single design optimization problem. Simulation results demonstrate that the proposed scheme is effective and offer a better CE performance compared to other schemes, and can realize 18.75% improvement in bandwidth efficiency with the same CE performance compared to the Least Squares (LS) CE. Fus-

ing different reconstruction algorithms may result in the probability of fusing several incorrectly estimated indices over noisy channels. Hence, a new fusion framework namely, Collaborative Framework of Algorithms (CoFA) is proposed, to pursue accurate recovery of the sparse signals from few linear measurements. Additionally, for low latency applications an algorithm namely, Stage-determined Matching Pursuit (SdMP) is proposed to provide tractable and fast signal reconstruction. By using the restricted isometry property, the theoretical analysis of both CoFA and SdMP algorithms and the sufficient conditions for realizing an improved reconstruction performance were presented. Simulation results demonstrate that the proposed CoFA and SdMP algorithms for CE have around 11.1%, 18.3%, 28.9% and 42.8% and around 5.6%, 13.9%, 22.8% and 33.3% performance improvement at MSE value of 2×10^{-3} when compared to FACS, gOMP, OMP and ROMP algorithms, respectively. Additionally, at BER value of 2×10^{-3} the proposed CoFA and SdMP algorithms for CE have around 9%, 14%, 19.5% and 25% and around 5%, 10%, 14% and 22.5% performance improvement when compared to FACS, gOMP, OMP and ROMP algorithms, respectively.

CHAPTER ONE

INTRODUCTION

1.1 Background

Advancements in wireless communication systems translates into advancements in human civilization. Presently, cost-efficient Wireless Communications Service (WCS) that operates in the 2.3 GHz portion of the Radio Frequency (RF) spectrum are now available to billions of people living in the world today (Gozalvez, 2011). The continued demand within the ever-growing digital world for data services confers no indication of ever slowing down. With such needs growing exponentially, significant pressure has remained fixed on the evolution of new and continually enhancing technology. One such measure is the construction of a new sensing/sampling paradigm (Donoho, 2006), which deals with a particular class of sparse signal modeling problem known as Compressed Sensing (CS). Nonetheless, wireless communication system has to withstand transmission and propagation delays which are considerably much hostile with several design challenges as compared to the wired system (Zou et al., 2016).

In a distinctive scattering environment, a radio signal can be reflected, refracted, or scattered as it propagates the wireless channel environment from the transmitter to the receiver (Tse and Viswanath, 2005). This effect creates multiple transverse paths of the transmitted signal, causing the receiver to perceive a superposition of multiple copies of the transmitted information traversing separate paths (Tse and Viswanath, 2005). Hence, each signal copy will encounter dissimilarities in delay, attenuation, as well as a shift in phase, while propagating from source to destination as shown in

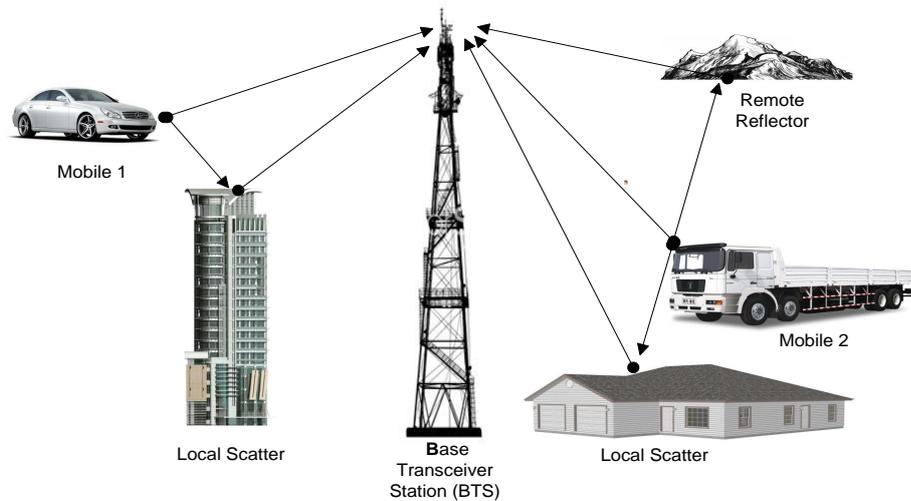


Figure 1.1: Wireless channel multipath propagation.

Figure 1.1. This consequently attenuates or amplifies the amplitude of the transmitted signal (Rappaport et al., 1996). The effect of attenuation causes fading which reduces the resultant/received signal amplitude and, consequently, alters the rate and reliability of the transmitted signal in the wireless channel environment. The effect of amplification, on the other hand, increases diversity (Krikidis et al., 2010) –the number of independent propagation paths, in which the wireless channels transverse without violating constraints which consequently improves the reliability of the wireless system. The impact of fading can greatly be suppressed by providing the communication system receivers with known channel state information of the signal transmitted. Invariably, it could be said that the Channel State Information (CSI) actually describes to the communication system receivers how the received signal at the channel output has been attenuated (Ji et al., 2017). The presence of the CSI at the transmitter block induces multiplexing gain and reliability while if present at the receiver block induces

the receiver's immunity against fading.

High speed applications now, require high sampling rate, which generates a high data “flood” that critically overburdens the role of Analog-to-Digital Converter (ADC) (i.e., the data-acquisition) in signal processing (Bhadoria et al., 2014; Eldar and Kutyniok, 2012; Patel and Chellappa, 2013). Moreover, after signal acquisition, the acquired data is compressed for an efficient transmission. Hence, a significant amount of the acquired data, the least “significant information” content is “thrown away” (Palangi et al., 2017), for lossless compression. The aspect of universal compressibility, however, uplifts some very logical questions as Donoho (2006) puts it “why go to so much effort to acquire all the data when most of what we get will be thrown away? Can we not just directly measure (take random linear combinations of the entries of the transmitted signal vector) the significant information content that will not end up being thrown away?”. Compressed Sensing (CS) theory asserts that this is indeed achievable. By using CS, a signal can be reconstructed from a randomly chosen small set of possibly noisy measurements, provided that the transmitted signal is sparse with respect to some basis (Donoho, 2006). In many applications, data acquisition (measurement) for use is critical, one of such system is the Orthogonal Frequency Division Multiplexing (OFDM) system (Liu et al., 2014).

OFDM has been broadly utilized in modern and emerging wireless communication systems mainly due to its efficient spectrum utilization, high-data-rate transmissions, and ability to manage multipath fading channels (Liu et al., 2014; Mohammadian et al., 2017a). It has been used in Digital Video/ Audio Broadcasting (DVB/DAB) (Sheng et al., 2017) and in the Wireless Local Area Networks (WLANs) standards such as

the IEEE 802.11a (D'Amico et al., 2017), which operates in the 5GHz portion of the unlicensed band and the newly introduced IEEE 802.11g extension (Au, 2016), which uses the frequency band of 2.4GHz. Nonetheless, Channel Estimation (CE) is critical for coherent detection based OFDM systems, as accurate Channel State Information (CSI) can notably improve performance (Mohammadian et al., 2017b; Shu et al., 2017; Zhang et al., 2016a). In OFDM system, a reliable way to obtain a successful CSI at the receiver is primarily realized through the use of pilot tones and several pilot-symbol-aided CE schemes have been investigated (Abdelkader et al., 2010; Mohammadian et al., 2017b; Shu et al., 2017). Nonetheless, the assumption that the underlying statistical models for wireless channels are rich multipath, and apply a huge number of pilot signal to achieve a better CSI accuracy will probably result in a lower system spectral efficiency (Bajwa et al., 2010).

Interestingly, recent research has shown that the wireless communication channel is indeed characterized as a channel that possesses only a few dominant paths in propagation; this is as a result of the signal well approximated, as a linear combination of a few coefficients taken from a known basis, e.g., the Fourier basis or wavelet basis (Wang, 2012). These paths which are considered dominant are in time, largely separated, and thus, exhibit within the Channel Impulse Response (CIR) a sparse nature. This has now given rise to a new paradigm in the field of CS that is focused on acquiring the sparse signal at a rate appreciably below the Nyquist rate, which is, however, contrary to the rich multipath channel assumptions that the most existing models of wireless channels acquire (Bajwa et al., 2010; Eldar and Kutyniok, 2012). CS has revealed itself as a new signals sampling paradigm for acquiring high-dimensional signals that are compressible, and has dragged a lot of recognition with a wide variety

of applications (Bajwa et al., 2010; Donoho et al., 2006). Typically, if the signal is sparse and incoherent on a known basis, a perfect reconstruction of the underlying signal can be achieved through optimization techniques from far fewer measurements than what is generally considered necessary (Blanchard et al., 2011a; Eldar and Kutyniok, 2012). Since, CS simultaneously accomplishes data acquisition (i.e., sensing) and compression, then there will be a significant reduction in the number of linear measurements that is required to be processed. Research has, however, revealed that the Restricted Isometry Property (RIP) is a sufficient condition for sparse signal recovery from noisy measurements which employs random matrices (Blanchard et al., 2011a). Based on RIP, it has been demonstrated that random pilot locations will produce distinctive CS measurements and can guarantee the sparse recovery with a high probability (He et al., 2016; Ren et al., 2015). This, however, suggests that optimal pilot pattern is obtained by the use of uniformly random pilot allocation. But, randomly generating pilot patterns is in general, computationally complex in practice (Candes et al., 2006; Pejoski and Kafedziski, 2015). This, however, necessitates the fixing of the pilot pattern through means of deterministic allocation (Kamali et al., 2014; Masoumian and Tazehkand, 2015; Qi et al., 2015b). However, no results have since been established on how the pilot pattern are to be fixed for an improve performance in CS based sparse CE in OFDM system. Since different pilot allocation scheme gives rise to different CS measurements (Ambat et al., 2013; Hurley and Rickard, 2009), a well-designed measurements matrix design scheme is then desirable for a good CS behaviour. Therefore, research is needed for the development of new sampling technique (pilot structure) and CS reconstruction algorithms for an enhanced sparse CE performance in OFDM systems.

1.2 Problem Statement

Signal reconstruction from random incomplete measurements plays a significant role in the task of signal processing, where it belongs to a general class of sparse channel (signal) estimation problem. By considering the signal sparse property, the problem of sparse-signal recovery from noisy measurements can be overcome, whereby requiring the developments of algorithms for signal reconstruction from noisy, incomplete linear CS measurements. CS is a very efficient sub-Nyquist signal sampling scheme that allows, under certain assumptions, the accurate recovery of signals that are sparse or nearly sparse in a known representation (Donoho, 2006; Needell and Vershynin, 2010). The efficiency of the CS scheme relies on the *measurement scheme* employed with respect to this representation and the *reconstruction algorithm* used for support identification (i.e., to identify the sparse set of representation coefficients). Typically, given a sparse signal $\mathbf{h} \in \mathbb{R}^N$ that is k -sparse (i.e., with at most k non-zero entries), observed via a measurement matrix \mathbf{A} (where \mathbf{A} denotes a matrix with dimensionality reduction $m \times N$ since, it maps \mathbb{R}^N into \mathbb{R}^m , with N relatively larger than m , i.e., $m \ll N$), yields a set of very few linear measurements, $\mathbf{A}\mathbf{h} = \mathbf{y} \in \mathbb{R}^m$. Let the i th column of \mathbf{A} be represented by c_i , where $i \in [N]$ such that $[N] := \{1, 2, \dots, N\}$. As the entries in \mathbf{y} is a linear combination of k columns of \mathbf{A} , it then means that the reconstruction of the CIR \mathbf{h} can be formulated as the problem of identifying the locations of $\{c_1, c_2, \dots, c_k\}$ in which the target signal lie as depicted in Figure 1.2. Through CS, the dimensionality reduction in the number of measurements give rise to the reduction in the number of samples below the Nyquist rate and translates into a reduction in the number of pilot overhead, which enhances the spectrum utilization efficiency of the system; although, the performance of the sparse recovery block may degrade. Nonetheless, by exploit-

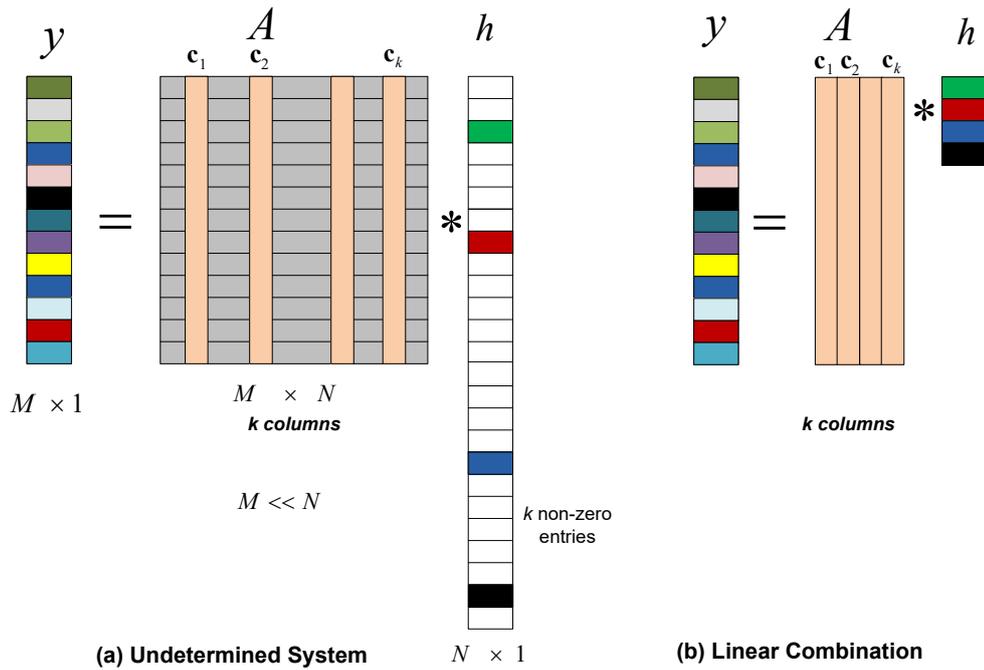


Figure 1.2: Underdetermined system as a system of linear equations (Zhang et al., 2014; Zhang and Drew, 2015).

ing the sparsity of the CIR, it is still possible to capture the necessary information in the frequency domain in the fewer number of pilots but requires optimal pilot patterns for estimating the sparse channel. On the assumption that \mathbf{h} is k -sparse, leads to two obvious and important questions:

First, what is the easiest possible model that adequately describes the observation signal vector that does not unnecessary adds complexity? In other words, how would the measurement matrix, \mathbf{A} , be constructed to accurately describe the observations \mathbf{y} ?" Hence, a good construction of \mathbf{A} is desirable to adequately describe the observation \mathbf{y} which will possibly lead to an accurate estimation of \mathbf{h} . Most existing works on the deterministic pilot pattern design for sparse Channel Estimation (CE) in Orthogonal Frequency Division Multiplexing (OFDM) system are based on the assumption that the pilot symbols are equally-powered (Chen et al., 2013; Mohammadian et al.,

2017a). This assumption may not necessarily exhibit low coherence compressed CE. This, therefore, calls for the optimization of pilot symbols and their placement which in existing works are considered as a disjoint optimization problem. Hence, the joint pilot placement and symbol design optimization problem for sparse CE in OFDM systems is considered based on minimizing the mutual coherence of the Fourier submatrix associated with the pilot subcarriers. In order to avoid the disjoint optimization of the pilot placements and pilot symbol design sub-problems, a joint pilot symbol and placement scheme is required that will optimize over both symbol values and placement as a single design optimization problem to minimize the mutual coherence of the measurement matrix.

Second, based on prior knowledge of the measurement vector \mathbf{y} and measurement matrix \mathbf{A} , how can one accurately estimate \mathbf{h} from $\mathbf{y} = \mathbf{A}\mathbf{h}$? In other words, what are efficient reconstruction algorithms? Hence, the ability of a basis to provide an exact reconstruction of the transmitted signal depends on how well the reconstruction algorithm is designed. For noisy measurements, the cardinality of the support set, say $|\Gamma|$, is often far larger than k (i.e., $|\Gamma| \gg k$) and may require additional rightly chosen support indices (Ambat et al., 2013). According to data fusion principle, fusing completely the estimated support set of different reconstruction algorithms can improve signal recovery performance. It can, however, lead to the increased probability of estimating incorrect support indices, and thus degrades the signal reconstruction accuracy. Hence, since it is evident that the estimate collected by each individual algorithm usually contains partially correct information regarding the sparse signal, exploiting the partial information can possibly lead to a better sparse signal estimate. Therefore, this possibility will be explored and then propose a new scheme (that potentially corrects

erroneously estimated atoms chosen from the sensing basis) to improve the performance of arbitrary sparse signal reconstruction algorithms.

Additionally, since low latency applications requires computationally fast reconstruction algorithms, the Orthogonal Matching Pursuit (OMP) algorithm (Cai and Wang, 2011) therefore requires further acceleration. Existing algorithms such as Generalized OMP (gOMP) (Wang et al., 2012), Regularized OMP (ROMP) (Needell and Vershynin, 2009), Compressive Sampling Matching Pursuit (CoSaMP) (Needell and Tropp, 2009), and Subspace Pursuit (SP) (Dai and Milenkovic, 2009) have tried to speed up the OMP algorithm. Thus, causing a significant compromise in the accuracy of sparse signals from noisy CS measurements. Therefore, without significantly compromising accuracy, an algorithm is required that can provide fast execution time as compared to the existing algorithms.

1.3 Aim and Objectives

The main aim of this thesis is to improve the CS reconstruction methods for Sparse Channel Estimation in OFDM Systems. The key objectives of this research include the following:

1. To propose a new deterministic method that jointly optimizes pilot symbol and placement in OFDM systems, which can improve the accuracy of sparse signal reconstruction from a limited number of noisy compressed measurements.
2. To propose a new fusion framework of algorithms which can improve the accuracy of sparse signal reconstruction in OFDM systems from a limited number of noisy compressed measurements.

3. To propose a new sparse signal recovery algorithm for OFDM systems, for pursuing efficiency (in terms of accuracy and complexity) in the reconstruction of sparse signals.

1.4 Research Contributions

The thesis contribution context diagram is presented in Figure 1.3. There are three main contributions of this thesis which are presented as follows.

1. To develop a new joint pilot symbol and placement scheme that optimizes over both the pilot symbol values and their placement as a single design optimization problem, to minimize the mutual coherence of the measurement matrix associate with the pilot subcarrier. The proposed method avoids the disjoint optimization

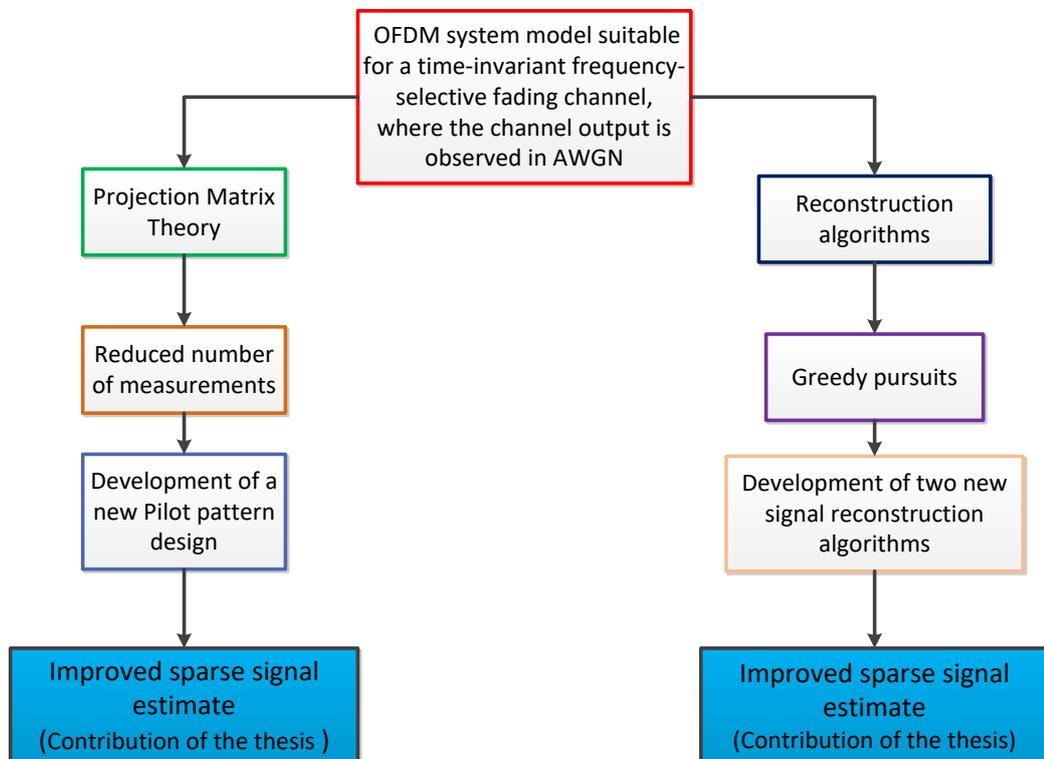


Figure 1.3: The thesis contribution context diagram.

- of pilot placements and pilot symbol as two separate optimization subproblems.
2. To develop a new fusion framework, namely Collaborative Framework of Algorithms (CoFA), to improve the accuracy of sparse signals reconstruction in OFDM systems. The CoFA scheme exploits both the estimated support-set and sparse coefficients of algorithms to estimate the support-set of the target signal. By using the Restricted Isometry Property, the theoretical analysis of the CoFA scheme and the sufficient conditions (guarantees) for realizing an improved reconstruction performance are presented.
 3. To develop a new sparse signal recovery algorithm, namely Stage-determined Matching Pursuit (SdMP), for pursuing efficiency (in terms of accuracy and complexity) in the reconstruction of sparse signals in OFDM systems. By using the restricted isometry property, the theoretical analysis of the SdMP algorithm and the sufficient conditions (guarantees) for realizing an improved reconstruction performance are presented.

1.5 Scope of Work

The scope of this research is limited to the mathematical formulation, algorithm development, and implementation of the overall strategy on a Matlab Software Platform. Therefore, the proposed methods are analyzed mathematically. The mathematical models in the form of equations represent the input-output relationship of the wireless communication system. These equations are the representation of the system being modeled. Next, the thesis verifies the correctness of the simulated proposed reconstruction algorithms with the mathematical expressions. The results for verification of the mathematical models and simulations are accurate to satisfy the level of prototyping

and testing. Thus, hardware implementation is not within the scope of this thesis. However, the thesis relied on the processing and generation of random signals as discussed in Appendix C.

1.6 Organization of the Thesis

The remainder of this thesis is organized as follows:

Chapter 2 presents an overview of the fundamental theories that are used in this work and also prior work in the literature. It begins by discussing the fading phenomena in wireless communication channels. Subsequently, it introduces the wireless channel impulse response statistics, followed by the OFDM system fundamentals. Then, the underlying idea behind the CS theory is introduced. This is accomplished in a more conceptually based approach to gradually introduce and reveal the fundamental problem of CS in its simplest mathematical form. The chapter ends with some reviews of prior work on sparse CE in OFDM systems and the specific research problems identified in the literature.

Next, Chapter 3 introduces the overall research methodology. It begins by designing a joint pilot symbol and placement scheme using the method of deterministic pilot allocation. Subsequently, a new fusion framework namely, Collaborative Framework of Algorithms (CoFA) was developed, to improve the accuracy of sparse signal reconstruction in OFDM systems. Then, a new algorithm namely, Stage-determined Matching Pursuit (SdMP) was developed, for pursuing efficiency in the reconstruction of sparse signals in OFDM systems.

In Chapter 4, the complexity analysis and simulation results of the proposed joint pilot symbol and placement scheme, is first presented. Next, the theoretical analysis and the sufficient conditions (guarantees) for realizing an improved reconstruction performance using the proposed CoFA scheme is presented, followed by a discussion of numerical experiments using the CoFA scheme. Finally, the chapter then presents a discussion on the theoretical analysis of the proposed SdMP algorithm and the sufficient conditions (guarantees) for realizing an improved reconstruction performance, followed by the complexity analysis and simulation results of the proposed SdMP algorithm. Chapter 5 concludes the thesis and discusses future work.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter presents a detailed discussion of the principles and recent research activities regarding fading channels, OFDM systems, Compressed Sensing (CS) and sparse Channel Estimation (CE) techniques. The chapter is organized into seven main sections. The first section presents an introduction to wireless communications, setting the basic background to comprehend the subsequent sections. The second section discusses the wireless Channel Impulse Response (CIR) statistics and Orthogonal Frequency Division Multiplexing (OFDM). The third section presents a review on CS and sparse signal reconstruction algorithms, which shows that a low coherence compressed CE (i.e., a good measurement matrix design) and a suitable nonlinear reconstruction algorithm are desirable for an enhanced CE performance. The fourth section presents the mathematical model of CS based sparse CE in OFDM systems. The fifth section presents a review on the measurement matrix design. While the fifth and sixth sections presents the CS reconstruction algorithms, and a comprehensive review of the literature revealing the performance gap that currently exists, respectively. The last section summarizes the chapter.

2.2 Fading

Wireless communication is one of the most ubiquitous of modern technologies and had been limited however by fading. Typically, fading in wireless communications

occur as a result of multipath propagation (also regarded as multipath induced fading), or due to shadowing caused by obstacles (Panic et al., 2013). It is experienced when the radio signal arrives at the receiver at slightly different time delays or at slightly different frequency shift due to the effects of electromagnetic wave scattering in the environment. Hence, these random variations (attenuation) in the received signal level is referred to as multipath fading (Fan and Tsai, 2015). In order to compensate for the distortions and attenuations that occurs during transmission, the wireless channels have been widely studied (Barbu et al., 2017; Fan and Tsai, 2015; Liu et al., 2014), and different channel models have been introduced (Liu et al., 2014). These effects are mostly summarized as reflection, shadowing, path loss and scattering that attenuates the unpredictability of existing paths between the transmitter and the receiver, which can typically be classified into large-scale fading (i.e., slowly varying) and small-scale fading or multipath fading (i.e., rapidly fluctuating signal envelop) (Fan and Tsai, 2015). Large-scale fading is caused by path loss and large obstacles such as buildings and hills and it is characterized as a function of the received signal strength as a function of distance (Dey and Rossi, 2017; Kumar, 2015). Hence, the fluctuations in signal strength due to the effect of large-scale fading are slowly varying. Small-scale fading is due to the amplification (constructive) or attenuation (destructive) of the power of the transmitted signal that is received from different multipath component (Dey and Rossi, 2017). In general, it can be concluded that large-scale fading is more relevant to the design of optimization models for cell site planning, whereas the small-scale multipath fading is more relevant to the design of optimization models for reliable and efficient communication systems (i.e., to track the channel impulse response for reliable CE), hence, the focus of this thesis.

Basically, small-scale fading is guided by the nature of the transmitted signal and radio environments (Panic et al., 2013; Zhang et al., 2017). Small-scale fading is available in two types which result in the spreading of the transmitted signal in time which translates into Intersymbol Interference (ISI) (Kumar, 2015; Li et al., 2017; Zhang et al., 2017), denoted as multipath delay spread, while the frequency-variation of the channel results in frequency spreading which translates into Doppler frequency spread as shown in Figure 2.1. Hence, wireless channels exhibit time-variant characteristics due to multipath fading and frequency selectivity characteristics due to the Doppler spread. Frequency non-selective (flat fading, i.e., $B_c \gg B_s$, where B_s and B_c denote the transmitted signal bandwidth and the channel coherence bandwidth, respectively) channel or frequency selective channel (i.e., $B_c < B_s$) are two categories of multipath channels with time delay spread (multipath delay spread) (Kumar, 2015). In the flat fading channel, the channel bandwidth is much larger than the bandwidth of the transmitted signal while in the frequency selective fading channel, the channel is smaller than the bandwidth of the signal (Kumar, 2015; Li et al., 2017). Additionally, the frequency selective fading channel experience a decorrelated fading caused by the different frequency components of the signal which are affected independently, and therefore, a deep fade does not simultaneously affect all parts of the signal but, suffers from

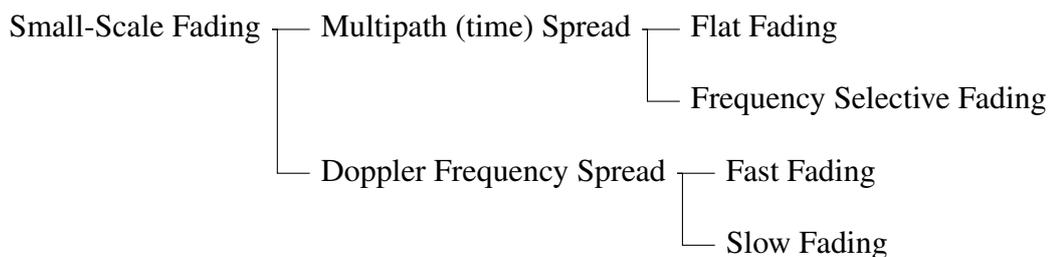


Figure 2.1: Small-Scale Fading

ISI (which limits the data rate) which must be mitigated (Kumar, 2015).

2.3 Channel Impulse Response Statistics

The Channel Impulse Response (CIR) representation of a complex baseband mobile wireless communication channel can be expressed in complex notation as (Chiueh and Tsai, 2008; Trivedi, 2013)

$$\mathbf{h}(t, \tau) = \sum_{l=0}^{L-1} \alpha_l(t) \delta(\tau - \tau_l), \quad (2.1)$$

where $\mathbf{h}(t, \tau)$ is the response at delay τ to an impulse at time t , where $\delta(\tau)$ denotes the Dirac-delta function. The $\alpha_l(t)$ are the time-varying complex amplitude (magnitude and phase) of tap l , with delay τ_l . The number of resolvable multipath components is L . The $\alpha_l(t)$ may aggregate many more unresolvable multipath components, typically resulting in Ricean or Rayleigh statistics for these parameters. As a result of vehicular motion, the complex amplitudes experience a wide-sense stationary (i.e., a random process is known to be wide-sense stationary (WSS) complex process if its mean function and its correlation function do not change by shifts in time) that is narrow band and are on different paths independent. From Equation (2.1), the time-varying Channel Frequency Response (CFR) of the channel can be expressed in time t as (Malepati, 2010)

$$\begin{aligned} \mathbf{h}(t, f) &= \int_{-\infty}^{\infty} h(t, \tau) e^{-j2\pi f\tau} d\tau \\ &= \sum_{l=0}^{L-1} \alpha_l(t) e^{-j2\pi f\tau_l}. \end{aligned} \quad (2.2)$$

Therefore, at different time and frequency, the frequency response correlation function can be presented as (Malepati, 2010)

$$\begin{aligned}
r_H(\Delta t, \Delta f) &= E\{H(t + \Delta t, f + \Delta f)H^*(t, f)\} \\
&= \sum_{l=0}^{L-1} r_{\alpha_l}(\Delta t) e^{-j2\pi\Delta f\tau_l} \\
&= r_t(\Delta t) \left(\sum_{l=0}^{L-1} \delta_l^2 e^{-j2\pi\Delta f\tau_l} \right).
\end{aligned} \tag{2.3}$$

Hence, from Equation (2.3), the frequency correlation function can be defined as (Malepati, 2010)

$$r_f(\Delta f) \triangleq \sum_{l=0}^{L-1} \delta_l^2 e^{-j2\pi\Delta f\tau_l}, \tag{2.4}$$

while the time correlation function as

$$\tau_t(\Delta t) \triangleq r_t(\Delta t). \tag{2.5}$$

Subsequently, the channel is then normalized and the average impulse power becomes $\sum_{l=0}^{L-1} \delta_l^2 = 1$. Therefore, the correlation function can be re-expressed as

$$r_H(\Delta t, \Delta f) = r_t(\Delta t)r_f(\Delta f). \tag{2.6}$$

The correlation function in Equation (2.6) is thus, a product of both the time domain correlation function $r_t(\Delta t)$ which is motion (vehicular) dependent and the frequency domain correlation function $r_f(\Delta f)$ which is multipath delay spread dependent.

2.3.1 Orthogonal Frequency Division Multiplexing Fundamentals

OFDM system is a multicarrier modulation technique. OFDM manages to transform the frequency-selective fading channel into several flat fading subchannels with independent additive noise vectors (Pathak and Sharma, 2013), resulting in a significant reduction in complexity in the receiver design (Shakeri and Bajwa, 2015). In wireless communication systems, the transmitted information reaches the receiver through a radio channel (Malepati, 2010; Shakeri and Bajwa, 2015). Hence, the effect of the channel on the transmitted signal, for coherent receivers must be estimated to recover the transmitted information (Ozdemir and Arslan, 2007). In other words, since the receiver accurately estimates how the channel modifies the transmitted signal, it can recover the transmitted information (Ozdemir and Arslan, 2007). While CE technique can be avoided by employing differential modulation, the differential modulation technique suffers from 3 dB SNR penalty when compared to the coherent modulation technique (Ozdemir and Arslan, 2007). Hence, CE is critical in OFDM systems (Pejoski and Kafedziski, 2015), as accurate CSI can notably improve performance. The technique, however, remains very attractive for use for the following reasons; high spectral efficiency, immunity to interference caused by impulse noise, ability to combat multipath fading channels (Pathak and Sharma, 2013; Pejoski and Kafedziski, 2015). Hence, OFDM can effectively mitigate ISI introduced by channel variations (Ma et al., 2014; Qing et al., 2014; Sou and Wang, 2014). OFDM technique still suffers a major drawback from its inability to sufficiently and adequately provide accurate estimates of CSI indispensable to coherent receiving and demodulation at the receiver which is crucial for achieving reliable communication (Larsson et al., 2014; Prasad et al., 2014; Zhang et al., 2013).

The block diagram of an OFDM system transceiver is shown in Figure 2.2. Weinstein and Ebert (1971) introduced a major contribution for the implementation of OFDM systems. They applied the Discrete Fourier Transform (DFT) to implement baseband modulation and demodulation in OFDM while focusing on efficient processing. In order to further reduce the complexity and improve reliability of OFDM system, two essential strategies have been proposed by the researchers and are presented as follows:

1. Modulated symbols are converted from their frequency domain components to their time domain components with the help of an Inverse Fast Fourier Transform (IFFT) and inversely using the Fast Fourier Transform (FFT). Typically, the Fourier transform is an extremely powerful mathematical tool used to inspect signals in another domain, among which several severe "enigmas" (problems) become quite easy to analyze (Huang et al., 2008). Hence, if the signal that is to be sampled is discrete-time, the Inverse Discrete Fourier Transform (IDFT) is employed to convert modulated symbols to their discrete time components and

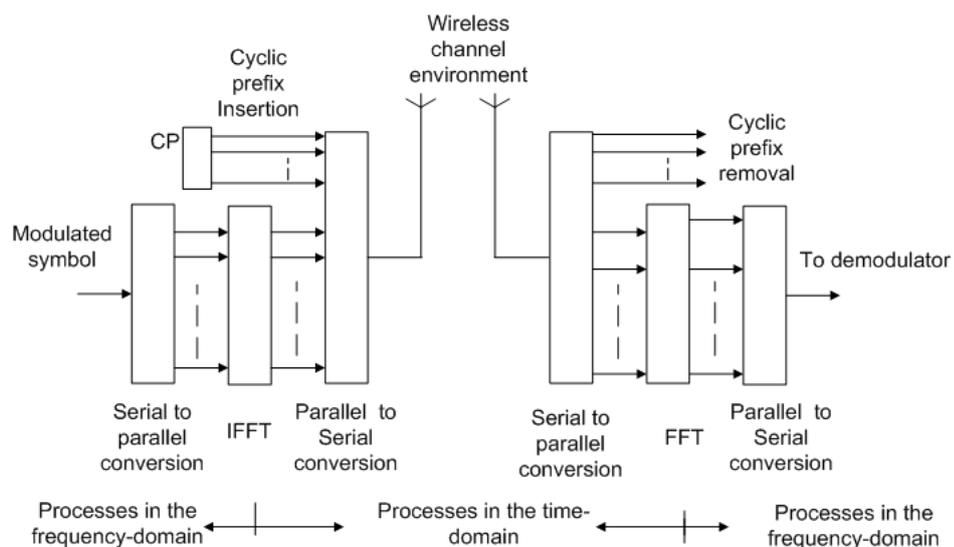


Figure 2.2: OFDM System Block Diagram (Kiayani et al., 2012)

inversely using the Discrete Fourier Transform (DFT). Nonetheless, the direct calculation of the IDFT and the DFT is usually computationally intensive and needs N^2 complex multiplications and $N(N - 1)$ complex additions to calculate each system update (Ho et al., 2009). Hence, the FFT and IFFT are employed to overcome the mathematical operations employed in the calculation of DFT and IDFT, respectively.

2. The insertion of a Guard Interval (GI) to initially eliminate the Intersymbol Interference (ISI) and Inter-carrier Interference (ICI), whose length needs to be larger than the maximum excess delay (i.e., the time difference between arrival of the first and last propagation path) of the channel (Zhang et al., 2016a).

2.3.2 OFDM System Pilot Arrangements

OFDM signals can be demodulated either coherently or differentially (Chiueh and Tsai, 2008). The most significant benefit of the differential detection is that they do not need to acquire Channel State Information (CSI), however, a 3 – 4 dB loss in signal-to-noise ratio (SNR) (Liu et al., 2014) will result, making the receiver relatively simple to implement (Liu et al., 2014). Furthermore, the differential detection technique cannot be implemented in practice in multi-level modulation schemes (Xiong et al., 2000). Hence, the coherent detection technique is preferred to achieve higher data rates and improved spectral efficiency for multi-level modulation schemes (Liu et al., 2014). For coherent demodulation, CE techniques based on pilot insertion can be grouped into two types of pilot arrangements (Hu and Chen, 2008; Kumar and Grover, 2012; Niranjane and Bhoyar, 2011), such as the block-type and the comb-type pilot arrangements as illustrated in Figure 2.3. The block-type pilot arrangement is employed for

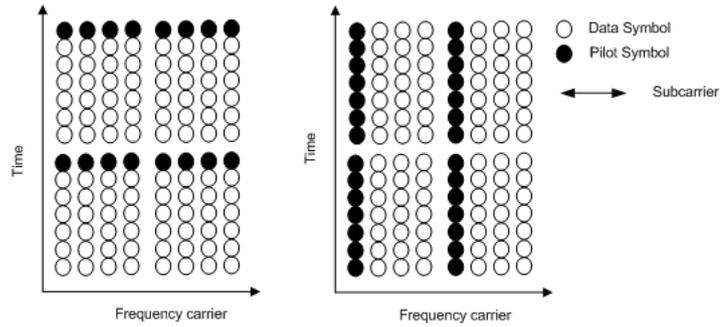


Figure 2.3: Pilot Tone arrangements in OFDM channel estimation Block-Type pilot (Left) and Comb-Type pilot (Right) (Hsieh and Wei, 1998)

slow-fading channels or quasi-static channel that is considered stationary within a certain period of OFDM symbols (Kumar and Grover, 2012). In this technique, all the pilot symbols are arranged in one OFDM block and are transmitted periodically in the time-domain. As the training block contains all frequencies, channel interpolation is not required (Bogdanović, 2014). When the channel encounters fast fading, there is an absolute loss in the parameter of the estimated channel (Abdelkader et al., 2010). The comb-type pilot arrangements is employed to manage channel variations that change within one OFDM symbol and it requires interpolation of the channel (Hsieh and Wei, 1998). In comb-type pilot arrangements, the pilots are placed in the subcarriers at fixed interval in one OFDM symbol (Hsieh and Wei, 1998; Pakrooh et al., 2012). Therefore, Channel Frequency Response (CFR) at these subcarriers can be estimated by applying classical CE methods such as Least Square (LS) or Minimum Mean Square Error (MMSE) (Pakrooh et al., 2012). Consequently, in estimating the CFR at non-pilot subcarriers, interpolation-based techniques for deriving channel estimates at non-pilot subcarriers are employed (Hsieh and Wei, 1998). If the interval between the adjacent pilot sub-carriers in the interpolation based technique is decreased for an improved CE performance, it leads to the increased number of pilot overhead required for CE (Hsieh and Wei, 1998). Thus, the pilot signals are placed at equidistant subcarriers to render

uniformity.

Fortunately, the wireless multipath channels encountered in practice exhibit a sparse structure, where only a few channel paths are significant and other channel coefficients are zero or close to zero (Bajwa et al., 2010). This implies that the delay spread of the channel is very large as compared to the number of significant propagation paths (Bajwa et al., 2010). However, the signal recovery performance is independent of the position and value of the channel taps; i.e., unlike the conventional interpolation-based CE techniques, the number of required pilot subcarriers is independent of the degree of frequency selectivity and the delay spread of the wireless multipath channel. Thus, it is possible to capture the necessary information in the frequency domain in the fewer number of pilots, thereby reducing pilot overhead.

2.4 Compressed Sensing and Sparse Signal Reconstruction

Compressed sensing, also termed compressive sensing or compressive sampling, is a signal processing technique that requires far fewer measurements or data samples than what the Nyquist/Shannon sampling theory states (Donoho, 2006; Eldar and Kutyniok, 2012). Hence, this new rapidly growing research field is based on the revelation that, a small group of nonadaptive linear projections of a sparse signal carries sufficient information for successful reconstruction. In other words, the unknown signal to be recovered must be sparse in a known basis or transform domain (Eldar and Kutyniok, 2012; Qi and Wu, 2012a; Qi et al., 2015a).

2.4.1 Sparse signal models

Signals can often be well-approximated as a linear combination of only a few significant elements from a known basis, frame or dictionary (Eldar and Kutyniok, 2012). When this representation is exact, the signal is said to be sparse (Eldar and Kutyniok, 2012). Sparse signal models provide a mathematical framework for capturing the fact that many naturally occurring high-dimensional signals contain relatively little information compared to their ambient dimension (Eldar and Kutyniok, 2012; Ma et al., 2014; Qi et al., 2015b). Mathematically, a signal \mathbf{h} is said to be k -sparse if it contains at most k nonzeros elements in its representation (Eldar and Kutyniok, 2012)

$$\Sigma_k = \{\mathbf{h} : \|\mathbf{h}\|_0 \leq k\} \quad (2.7)$$

In practice, however, some signals may not be sparse but may admit a sparse representation (compressible) with reference to some properly chosen basis functions (Eldar and Kutyniok, 2012; Qi et al., 2015b).

2.4.2 Compressed Sensing Signal Model

Compressed sensing is a very efficient sub-Nyquist signal sampling scheme that allows the accurate recovery of signals that are sparse or nearly sparse in a known representation—under the assumption that the measurement projections are selected independently at random (Donoho, 2006; Eldar and Kutyniok, 2012). The efficiency of the CS scheme relies on the measurement technique employed with respect to this representation and the reconstruction algorithm used for support identification i.e., to identify the sparse set of representation coefficients. If $\Psi = \{\boldsymbol{\psi}_1, \boldsymbol{\psi}_1, \dots, \boldsymbol{\psi}_N\}$, with