FOLDABLE STRUCTURES ADAPTED FROM TESSELLATION ORIGAMI WITH HIGH DEGREE OF FREEDOM

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FOLDABLE STRUCTURES ADAPTED FROM TESSELLATION ORIGAMI WITH HIGH DEGREE OF FREEDOM

by

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LIST OF ABBREVIATIONS

CAD	Computer Aided Drawing
3D	3-Dimensional
et al.	and others
n.d	no date
CLT	Cross Laminated Timber
NURBS	Non-uniform rational B-spline
CLP	Crease Line Pattern
MVP	Mountain-Valley Pattern
M	Mountain
V	Valley
ACM	Association for Computing Machinery
SIAM	Society for Industrial and Applied Mathematics
EPFL	ÉcolePolytechniqueFédérale de Lausanne
ASME	The American Society of Mechanical Engineers
IASS	International Association for Shell and Spatial Structures
APCS	Asia-Pacific Conference on Shell and Spatial Structures
ENHSA	European Network of Heads' of Schools of Architecture

EAAE	European Association for Architectural Education
IEEE	Institute of Electrical and Electronics Engineers
ICRA	International Conference on Robotics and Automation

LIST OF SYMBOLS

°	Degree
K	Gaussian Curvature
k_1, k_2	Principal Curvature
r, R	Radius
R_{min}	Minimum radius
G_{Ω}	Global Descriptor Movement
χ	Rotation matrix
dof	Local degree-of-freedom
DOF	Global degree-of-freedom
ρ	Fold Angle
F	Constraint matrix function
R	Constraint rotation matrix function
I	Identity matrix
N_{vi}	Number of interior vertices
N_{Ei}	Number of fold lines
N_{qi}	Number of quadrilateral facets
n	Number of crease line emanating from a vertex
V	Vertex

l	Crease line
u	vector
θ	Section angle between two crease lines
π	pi
C	Control curve
S	Section curve
K	Actuator
P	Actuator point
Φ	Rotation angle

STRUKTUR LIPAT TERSUAI DARIPADA TESSELLATION ORIGAMI DENGAN DARJAH KEBEBASAN YANG TINGGI

ABSTRAK

Bentuk geometri struktur lipat berkait rapat dengan origami terutama *tessellation* origami. *Tessellation* origami yang mempunyai mekanisme lipatan yang fleksibel menyediakan konfigurasi terlipat dengan permukaan pelbagai muka daripada corak lipatan berulang. Mekanisme lipatan fleksibel adalah sistem terkurang kekangan akibat darjah kebebasan yang tinggi. Fleksibiliti mekanisma lipatan dikawal oleh sudut lipatan yang dikenakan ke atas penggerak mekanisma. Kaedah simulasi lipatan yang sedia-ada didapati tidak dapat digunakan untuk simulasi *tessellation* origami dengan darjah kebebasan yang tinggi. Justeru, kajian ini bertujuan untuk membangunkan prosedur untuk simulasi *tessellation* origami dengan darjah kebebasan tinggi yang dikawal oleh penggerak mekanisma. Objektif pertama kajian ini adalah mengenalpasti pertimbangan geometri untuk lipatan yang mempunyai mekanisma fleksibel. 40 nombor *tessellation* origami sebenar dihasilkan semula dan digunakan untuk menkaji hubungan antara ciri-ciri geometri lembah gunung dan fleksibiliti origami *tessellation* replika tersebut. Ciri-ciri geometri corak lembah gunung yang boleh membantu komponen mekanisma *tessellation* origami telah dikenal pasti dengan menyiasat darjah kebebasan tempatan *tessellation* origami tersebut. Objektif kedua adalah pendekatan simulasi alternatif menggunakan algoritma generatif. Alat pemodelan generatif yang dibangunkan dengan menggunakan editor pengaturcaraan visual Grasshopper® menyediakan alat simulasi yang lebih realistik dengan menggabungkan ciri kinematik origami tersebut. Objektif terakhir adalah penentuan perhubungan antara permukaan lipat dengan sistem kawalan dengan menggunakan penggerak dengan memeriksa kelengkungan

permukaan lipat yang dihasilkan. Enam corak *tessellation* origami yang termasuk tiga variasi Miura-ori, Illusion Optical Russo, Yoshimura dan Ron Resch telah dipilih untuk menyiasat transformasi *tessellation* origami di bawah laluan yang berbeza dengan menggunakan alat pemodelan generatif. Transformasi enam model *tessellation* origami tersebut yang dikawal oleh kombinasi yang berlainan antara jarak penggerak dan kelengkungan garisan maya yang menghubungkan semua penggerak telah menghasilkan pelbagai konfigurasi permukaan lipat dengan corak lipatan tempatan yang tidak segerak.

FOLDABLE STRUCTURES ADAPTED FROM TESSELLATION ORIGAMI WITH HIGH DEGREE OF FREEDOM

ABSTRACT

Geometrical shape of folded structure is closely related to origami especially the tessellation origami. Tessellation origami which has flexible folding mechanism provides a folded configuration with facetted surface as a result from repeated folding pattern. Flexible folding mechanism is an under constraint system due to its high degree-of-freedom. The flexibility of folding mechanism is controlled by the fold angle imposed on the actuators. Currently available method of folding simulation was found to be not able to simulate tessellation origami with high degree-of-freedom. Hence, this research is conducted with the aim to develop the procedure for simulation of tessellation origami with high-degree of freedom through the control of actuators. The first objective of this research is the identification of geometric considerations for folding pattern with flexible mechanism. 40 numbers of actual tessellation origami were first re-produced to study the relation between the geometric characteristics of the corresponding mountain valley pattern on the flexibility of the physical behaviour of the replica model. Geometric considerations affecting the folding mechanism were identified by investigating the local degree-of-freedom of the crease line pattern through the use of a procedure for the computation of local degree-of-freedom of the mechanism proposed in this research. The second objective is the development of an alternative simulation approach using generative algorithm. A generative modeling tool using visual programming editor Grasshopper® which provides a more realistic simulation tool by incorporating the kinematic property of the origami was developed. The last objective of this research is the determination of the folded configuration in relation to the control system for

actuators. Six tessellation origami models consisting of three variations of Miura-ori, Russo's Optical Illusion, Yoshimura and Ron Resch pattern were selected to investigate the transformation under different paths using the generative modeling tool. Transformations under different combination of the distance between actuator and curvature of an imaginary curve connecting all actuators have yielded a wide variety of folded configurations with non-synchonised local folding pattern for the six tessellation origami models.

CHAPTER ONE
INTRODUCTION

1.1 General

Folded structure gains its stiffness through a series of folds introduced on the surface. Fold changes the mechanical properties of the surface by increasing the second moment of inertia of the sectionto improve the ability to withstand higher load under the given span. Folded structure is well known for its ability to cover large open spaces and often used in the construction of folded roof structure and façade due to the added aesthetic value of the folded appearance.

Folded structure dated back to the 1920s where the principle of folded structure isused to design concrete bunker for Märkische Power Station boiler house by George Ehlers (Kurrer, 2012). Over almost 100 years, folded structure design has reached its peak with the varieties of construction material available and advancement in computational analysis techniques. The design for folded structure has evolved from simple corrugated line fold to facetted fold. Some examples of the folded structure with facetted fold constructed over the last decades are; Sulfur Extraction Facility in Italy (Sarah, 2012), St. Paulus Church in Germany (Polonyi, 2012), Oriente Station Platform in Portugal (Calatrava, 2015), Yokohama International Port Terminalin Japan(Osanbashi, 2014), Glass Pavilion in Spain (Rinaldi, 2013) and Pulkovo Airport in Russia (Deane, 2015)(Figure 1.1).Many architects and civil engineers look for more possible folded plate design that can serve structural purpose and at the same time aesthetics. In the past decades, the study of folded structure isoftenrelated to the paper-folding art or origami whichhas becomethesource of inspiration for many folded structure application in architectural and civil engineering such as the deployable structure, retractable structure and folded roof structure(Moussavi, 2009;Trautz and Cierniak, 2011;Šekularac et al., 2012;Woerd, 2012;Fei and Sujan, 2013;Lebee, 2015;Reis et al., 2015). Many of the studies focused on the facetted surface structure which is related to a type of origami known as the tessellation origami.

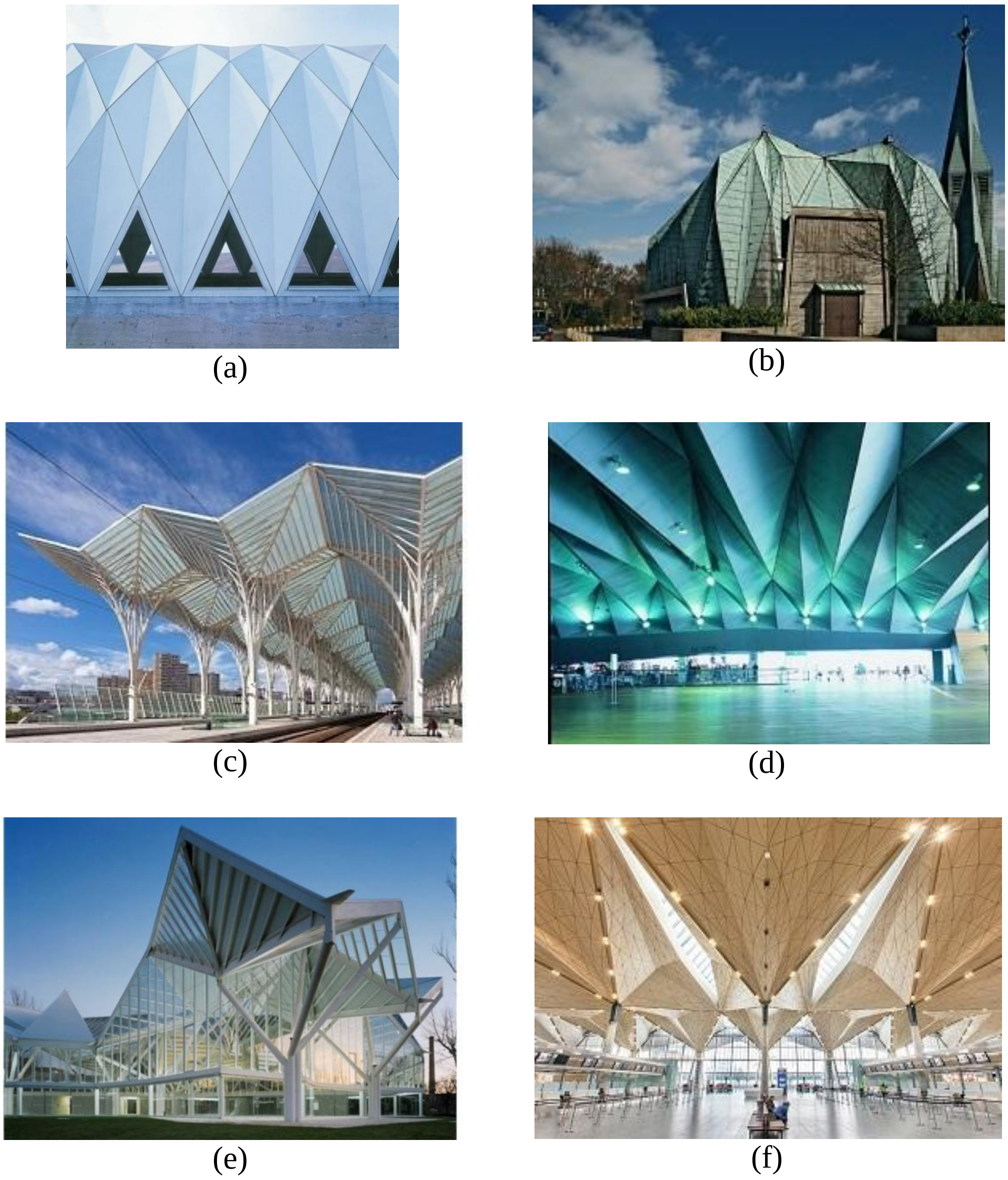


Figure 1.1 : Folded structure (a) Sulfur Extraction Facility, Italy (1966)(Sarah, 2012)(b) St. Paulus Church, Germany (1967) (Polonyi, 2012) (c) Oriente Station Platform, Portugal (1998) (Calatrava, 2015)(d) Yokohama International Port Terminal, Japan (2002)(Osanbashi, 2014) (e) Glass Pavilion at Spain (2010)(Rinaldi, 2013)and the latest (f) Pulkovo Airport, Russia(2014) (Deane, 2015)

1.2 Introduction for Tessellation Origami

The distinct feature of origami is the process of folding from a piece of paper without tearing, cutting or gluing the paper. Origami is used to produce a lot of creation from animal to geometric pattern. The extensive reviews on the type of origami used in structure are found in literature by Schenk (2012) and Dureisseix (2012). Among the type of origami described in the literature, tessellation origami is the most studied origami in folded structure for architectural and building. The extensive review on the current study of tessellation origami in folded structure are found in Sorguc et al. (2009) and Lebee (2015).

Generally, tessellation, also known as tiling, is a type of pattern covered by geometric shape in the same plane (Gjerde, 2008). In the past, origami study related to tessellation is focused on flat foldable periodic tessellation. Origami that can fold into a flat tessellation pattern is referred to as origami tessellation. Origami folding method related to tessellation is designed according to the characteristic of the initial state and final folded state of the origami (Demaine and O'Rourke, 2007). In origami folding, the configuration before the fold is referred as initial state while the configuration at the end of the fold is referred as final state. Any configuration in between is referred as intermediate state. For origami tessellation, the final folded state is a flat origami with tessellation pattern that is created from the fold. The final folded state can be either flat or non-flat origami. Kawasaki and Yoshida (1988) referred to origami tessellation as crystallographic flat origamis (Figure 1.2) and mathematically proved that a flat foldable origami tessellation can be generated by repeating the folding pattern according to any of the 17 crystallographic groups. Figure 1.3 illustrates the difference between the 17 plane crystallographic groups. Flat origami tessellation can be created by using a few simple folding steps such as twist and turn. A computer programme called TESS is created to fold an origami tessellation from a given mountain-valley pattern using the twist and turn folding steps (Bateman, 2002;TESS, n.d.).

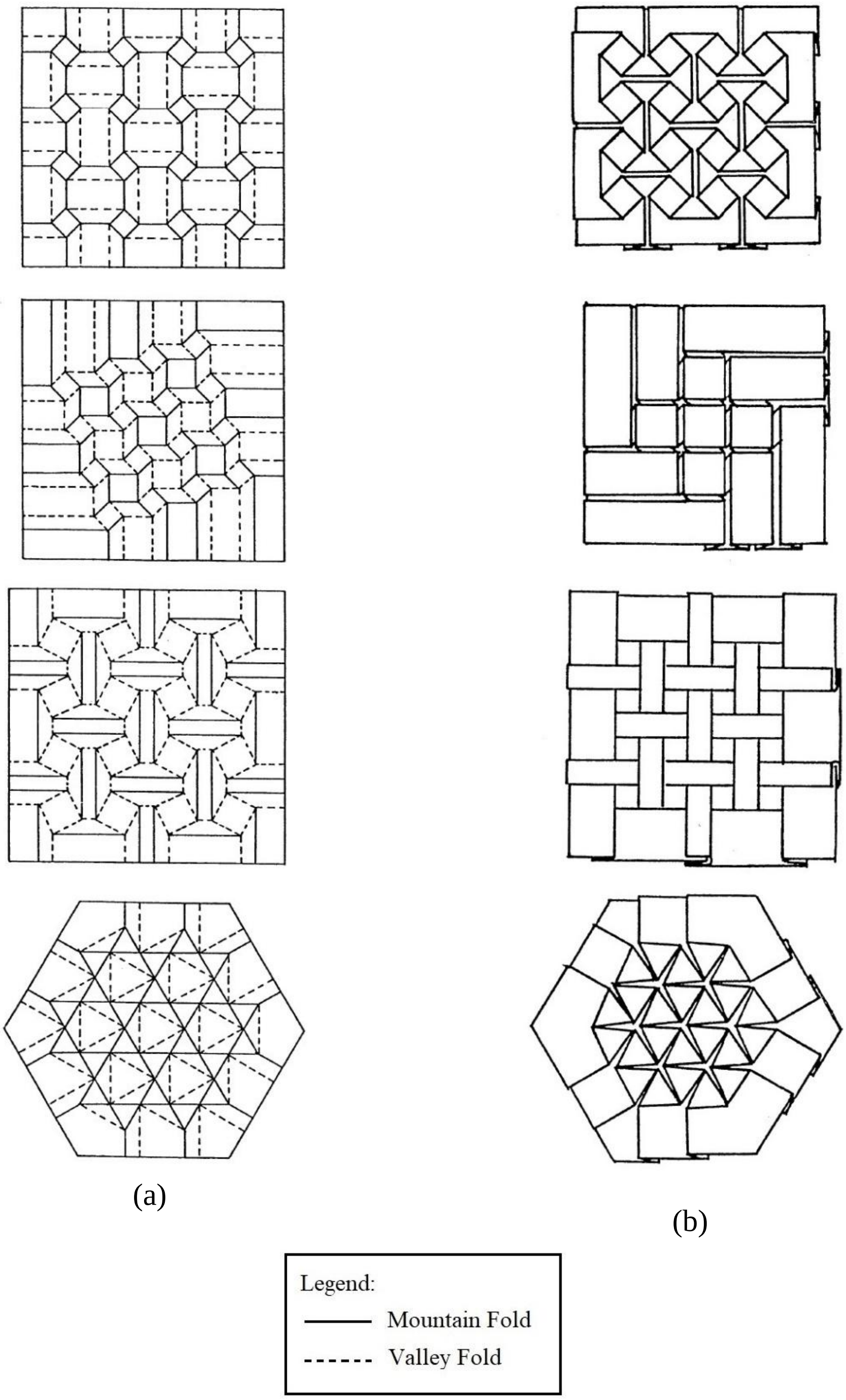


Figure 1.2 : Examples of crystallographic flat origami (a) Tessellated mountain-valley pattern (b) Final flat folded state (Kawasaki and Yoshida, 1988)

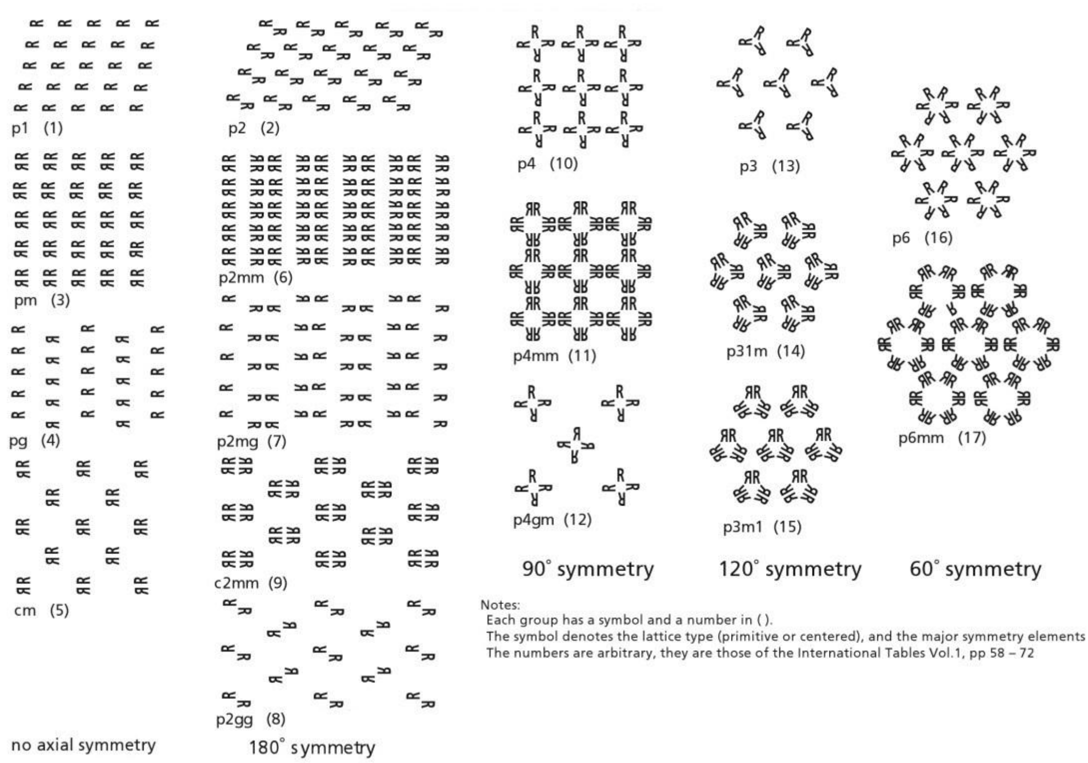


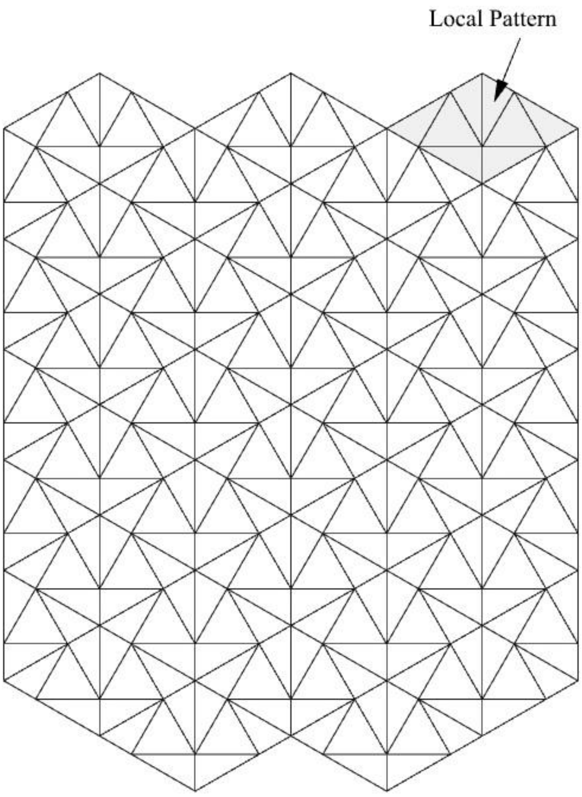
Figure 1.3 : The seventeen crystallographic group (Hammond, 2015)

On the contrary, if the folding takes a tessellation as the initial state and the final folded state is dependent on that initial state, it is refer to as tessellation origami. Tessellation origami is actively designed and folded by David A. Huffman (1925-1999) and all of Huffman's work was presented in a work by Davis et al. (2013). Davis et al. (2013) has converted all Huffman's tessellation origami into crease line pattern by reverse-engineering the folded model. Various tessellation origami that can be used as folded structure is also found in the book published by Jackson (2011).

Tessellation origami is the paper-folding art that creates afolded configuration that mimic a tessellation as shown in Figure 1.4(a). Tessellationis a geometric pattern with distinctly repeated local pattern (Figure 1.4(b)). Local pattern can be a cluster of one or more facets held together by connecting the edge of one facet to another facet without gap or overlapping of the shapes.However, in origami, local pattern is represent using vertices and refersto as local mountain-valley pattern as shown in Figure 1.5. A vertex is the point at the intersected corner of the facets. A tessellation pattern without any indication on the type of crease line refers as the crease line pattern. There are two fold characteristics of crease line. It consists of the mountain and valley fold line and is represented using a full and dashed linetype, respectively. When a characteristic of the crease line is assigned,the crease line patternrefers to as the mountain-valley pattern (Figure 1.5).The procedure to assign the pattern refers to as the mountain-valley assignment (Demaine and O'Rourke, 2007). In many literatures, tessellation origami pattern with more than one local vertex refers as multi-vertex origami.

The study on tessellation origami mainly divides according to the desired folded configuration. The act of folding the mountain-valley pattern from one configuration to another configuration is referred as transformation (Figure 1.6). In Figure 1.6, the transformation from initial state to one of the intermediate stage and then the final state is shown in left, middle and right configuration, respectively. Final state of an origami is a condition where further folding will break the fold line or facets will overlaps. There are only two types of final state, flat (Figure 1.6(a)) and non-flat origami (Figure 1.6(b)). Flat origami refers to the state where all facets are aligned in a series of parallel planes. In folded structure, tessellation origami that covers a 3-dimensional surface is desirable regardless of the final state.

(a)



(b)

Figure 1.4 : An example of Tessellation Origami – Ron Resch's Waterbomb Pattern
(a) Folded configuration (b) Crease linepattern(Resch, n.d.)

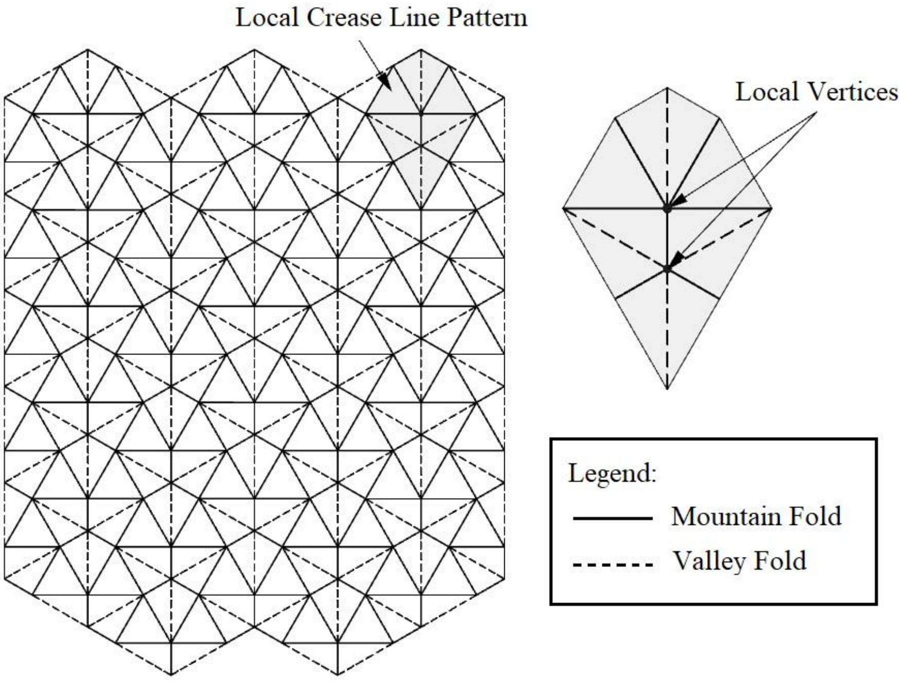
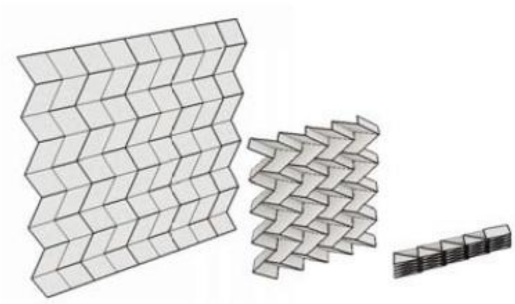
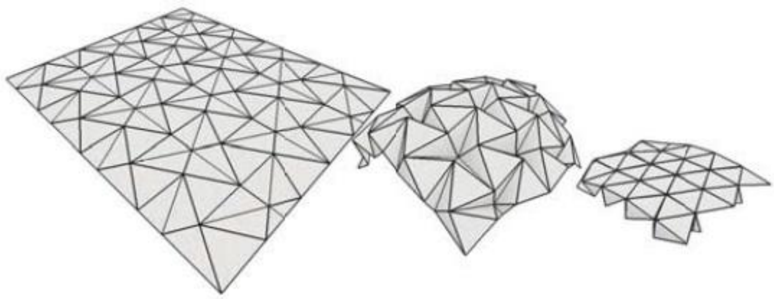


Figure 1.5 : Mountain-valley pattern and local crease line pattern for Ron Resch's Waterbomb Pattern (Resch, n.d.)



(a)



(b)

Figure 1.6 : Transformation of the tessellation origami using (a) Miura-Ori Pattern (b) Ron Resch's Waterbomb Pattern(Tachi, 2010b)

1.3 Importance of Origami Folding in Folded Structure

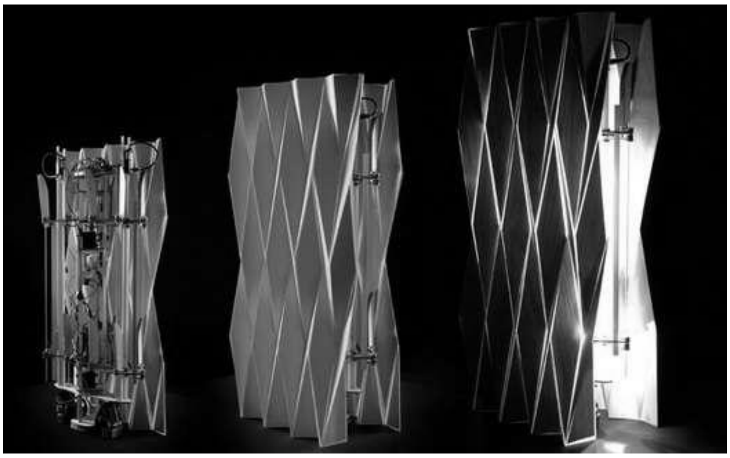
Origami folding is a powerful form finding tool for architectural and building design where feature of fold is desired. Origami such as the tessellation origami has a flexible folding mechanism which can be observed in the paper model. One example is the Ron Resch'sWaterbomb Pattern shown inFigure 1.7. Tessellation origami with high flexibility is capable to achievemany folded configurationsthat resemble a shell surface.Many of the conceptual architectural design are inspired by the origami folding and the detailed introduction to the application of origami folding in form-finding are presented byMoussavi (2009),Sorguc et al. (2009) and Šekularac et al. (2012).

Figure 1.7 : Flexible configuration of the Ron Resch's Waterbomb Pattern (Resch and Christiansen, 1973)

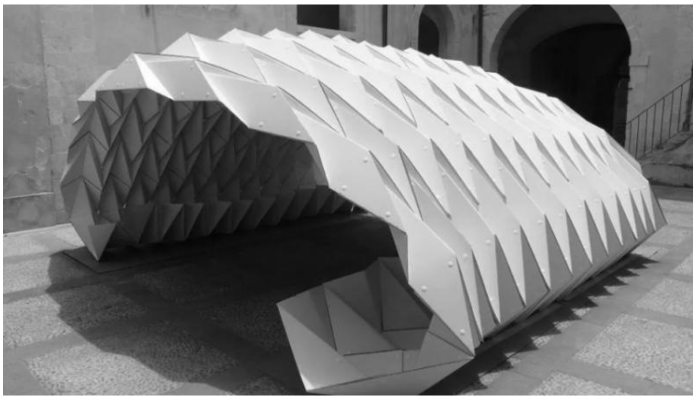
Origami folding not only providesidea for geometric design on the folded surface but also provides the kinematic mechanism that can contract and expand when needed. One example is the Cardborigami that makes use of the flat-foldable origami that acts as temporary structures designed to provide temporary shelter for homeless and victims of natural disaster(Hovsepian, 2007). Another example is in the making of an intelligent kinetic wall, WALLABOTS that can be controlled electronically(Otto-Ng, 2011) and the cardboard pavilion (Alini and Aion, 2009). All examples are shown inFigure 1.8.

One of the earliest form-finding studies using origami folding is by Chien (1999). A real life application of origami wasbuilt by Buri (2006)to serve as the temporary folded plate chapel made of wood panel. A foldable shelter was also developed using the origami that can fold flatby Temmerman et al. (2007). In a recent study, Samuelsson and Vestlund (2015) presented a parametric method to find form for folded structure using origami.Form-finding using origami folding is also used in the design for timber structure using the Cross Laminated Timber (CLT). One example is the simulation tool named as “THE FOLD” which is created to find form using origami by Meyer et al. (2014).

(a)



(b)



(c)

Figure 1.8 : Applications of tessellation origami in building structure(a) CARDBORIGAMI – Corrugated cardboard shelter for outdoor use (Hovsepian, 2007) (b) WALLBOTS - Electronic and kinetic wall system (Otto-Ng, 2011) (c) Cardboard Pavilion (Alini and Aion, 2009)

1.4 Problem Statement

Flexible tessellation origami has higher than one degree-of-freedom (*one-dof*) folding mechanism and thus has many possible transformation paths. The current studies focused more on *one-dof* tessellation origami and synchronization of the folding using a local pattern. Hence, current available computer simulation approaches are not sufficient to simulate tessellation origami with high degree-of-freedom (*high-dof*). Despite the repetitive local pattern, synchronising the origami folding with local pattern folding has limited its transformation path. *High-dof* origami is allowed to transform freely if there is no restraint imposed onto its actuators and the transformation is maximised by providing a systematic control to the actuators. However, by placing restraint onto the actuators can make the flexible mechanism becomes rigid at the desired folded state. The position of the actuators can be investigated by studying the distribution of the local degree-of-freedom. The lack of studies on *high-dof* folding mechanism has led many architects and civil engineers to find solution to model tessellation origami using geometric modeling technique. The use of this technique is normally combined with generative modeling which requires an algorithm to perform the folding. However, at current state, this technique is used to model developable tessellation origami which has repetitive local pattern with single vertex and is not suitable to model the transformation from one folded state to another. Therefore, kinematic behavior of the origami must be incorporated into the generative modeling tool. The kinematic theory of the origami can be written into an algorithm to conduct geometric folding using CAD drawing tool. Additional algorithm used to control the transformation of the *high-dof* folding mechanism must also be incorporated into the tool. The control for the transformation is different depending on the crease line pattern (CLP) for the tessellation origami. Therefore, studies are needed to investigate the transformation control system and its effect on its global folding behaviour.

1.5 Objectives

Based on the problem statement in previous section, this research is focused on the following objectives:

1. To investigate the geometric considerations for rigid tessellation origami with high degree-of-freedom using the computation of local degree-of-freedom and provide a solution to control the folding motion without restriction to synchonised folding motion using single-vertex.

2. To develop a procedure for generative algorithms to model folded configuration and introduce the independent parameter to simulate the kinematic motion for rigid tessellation origami with high degree-of-freedom and non-flat foldability.

3. To investigate the global folding behaviour of tessellation origami with high degree-of-freedom in relation to the parameters that control the transformation.

1.6 Research Scope

This study touches the discipline of origami, geometric modeling and folded structure. The subject of the study is the tessellation origami with high degree-of-freedom folding mechanism.This research did not include curve and polygon tessellated origami. This study focused onthe kinematic motion of the tessellation origami that is controlled by synchronised folding of similar actuators. The location of the actuator is investigated using the local degree-of-freedom computation. However, the investigation on how the arrangement of different types of vertex changes the global degree-of-freedom is not the interest of this research. The modeling steps to find folded configuration is proposed using the generative modeling technique. The transformation of tessellation origami is investigated under different paths. The transformation path is dependent on the location and numbers of actuator. The global form of the tessellation under different transformation path is also studied and compared with the corresponding paper model.

1.7 Thesis Layout

This thesis consists of fivechapters:

Chapter 1 gives the general overview, background and problem highlights on folded structure adapted from tessellation origami with high degree-of-freedom. The objectives and research scope are also presented.

Chapter 2 presents a review onorigami theory, tessellation pattern, the method to model and simulate the folding and construction of the tessellation origami.

Chapter 3 presents the methodology to conduct the feasibility study, kinematic of origami applied as part of algorithm for the simulation tool and finally the method used to carry out the transformations of the tessellation origami.

Chapter 4 presents the results and discussion on the feasibility study, the generative algorithm for the simulation tool, proposed method to compute the local degree-of-freedom and the case study for six selected tessellation origami.

Chapter 5 concludes the research study with conclusion andrecommendations for future study.

CHAPTER TWO
LITERATURE REVIEW

2.1 Introduction

The research focuses on formulating simulation method for tessellation origami with high degree-of-freedom. Therefore, the literatures start with the review on theory related to tessellation origami and also tessellation pattern available. The review is then followed by reviewing the current available method to model and simulate tessellation origami computationally. A review on the construction method for tessellation origami is also presented. Lastly, the issues related to the modeling and simulation approaches are presented.

2.2 Rigid Origami Theory

Rigid folding deals with rigid facets with fold lines separating the facets which act like a free hinge and allow the folding pattern to fold continuously without deformation of the hinge. The ability to fold from a given mountain-valley pattern while preserving the characteristic of a rigid origami is referred to as rigid foldability. Local foldability study evolves around flat-foldability because the final state is simply folded flat but it is difficult to predict the folded result from a non-flat foldable mountain-valley pattern.

Origami is often studied using a computer model. The branch of computer science used to study on origami related problem is called computational origami. Research study on computational origami has contributed to the growth of algorithmic origami and origami theory. Recent development on computational origami is presented by (Demaine and Demaine, 2002) and matters related to origami is extensively presented by Demaine and O'Rourke (2007).

2.2.1 Local Foldability

The study on local foldability for non-flat origami is scarce. Many studies focused on characterisation of the flat origami instead. One of the famous characterisations is the Maekawa’s theorem by Kasahara and Takahama (1987). It is proven that for a single-vertex with total section angles equal to 360°, the numbers of mountain and valley crease line is differ by two. Another famous characterization is Kawasaki’s Theorem. Kawasaki (1989) and Justin (1989a)has proved that a single-vertex with alternate section angles around a vertex sum to 180° is definitely flat-foldable. Both theorem is also revisited by Justin (1994), Hull (1994), Hull (2003), Demaine and O'Rourke (2007) and Hull (2002).

Belcastro and Hull (2002) presented the necessary conditions to mathematically model a single vertex that can fold flat. Vertices in the pattern are represented using the rotation matrix and controlled by the fold angle applied on the creases. The conditions however are not sufficient to address self-intersection problem as shown in Figure 2.1. Streinu and Whiteley (2005) presented the conditions for a single-vertex to fold with non-crossing motion provided the total angle between crease line is equal or less than 2π. In the study by Streinu and Whiteley (2005), single vertex folding is studied using the expansive motion of the spherical polygonal linkages to create a 3-dimensional folded state.

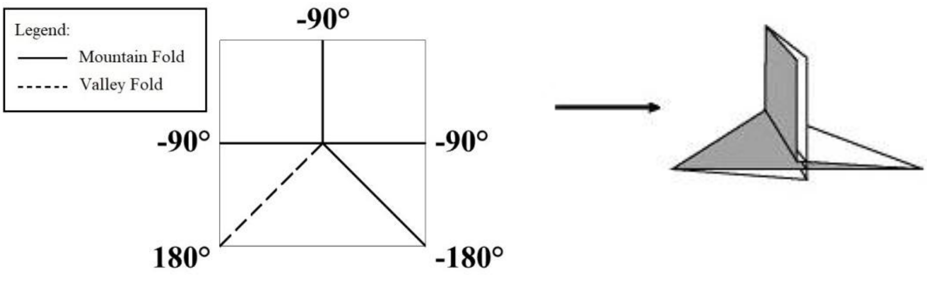


Figure 2.1 : Flat fold single-vertex with self-intersected facets (Belcastro and Hull, 2002)

The simplest case of foldable single-vertex is presented by Miura (1994). The study on single-vertex folding is based on the intrinsic Gauss's spherical representation. In the study by Miura (1994), it is proved that the simplest case of folding consisted of four crease lines emanating from a vertex with three-to-one crease lines fold properties. In other words, it can be either three consecutive mountain lines paired with one valley line or vice versa as shown in Figure 2.2.

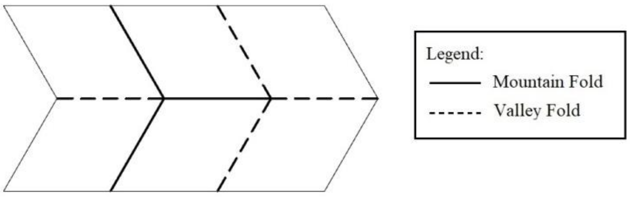


Figure 2.2 : Miura-Ori Pattern

Other studies related to single foldability is the generalisation of the single-vertex pattern. Tachi (2009a) presented the geometry conditions to generalize Miura-Ori Pattern using the quadrilateral mesh surface. New configurations for flat foldable Miura-Ori using the generalised pattern are also presented. Generalised Ron ReschWaterbomb Pattern is presented by Tachi (2010a).

2.2.2 Global Foldability

In global foldability, the folding path is intractable and finding the overlap order for the facets is proved to be an NP(nondeterministic polynomial time)-hard problem even for flat foldable pattern (Bern and Hayes, 1996). In other words, there are many numbers of possible folding paths and finding the solution for the problem requires exhaustive search in the worst case. There is no other literature that can prove otherwise. Hence, foldability issue related to global folding of multi-vertex origami is still an open question. Many of the folding simulation and geometric modeling for multi-vertex origami make use of the generalization of single-vertex by Tachi (2009a) to perform a synchronized folding mechanism. Synchronised folding is a folding that is controlled by local pattern. Change in local folding is simultaneously reflected in the other repeated local pattern. This is the main approach used in the design of simulation and modeling tools (Balkcom and Mason, 2008;Hawkes et al., 2010;Landen, 2013;Wei et al., 2013;Xi and Lien, 2015;Schenk and Guest, 2016).

Global foldability of a multi-vertex origami is related to the numbers of degree-of-freedom in the mechanism. In the folding motion study, degree-of-freedom also represents the number of actuator required to control the folding of the origami. Actuator represents the crease line in the tessellation pattern that is used to initiate the folding mechanism. The pattern is folded up when fold angle is imposed on the actuator. Fold angle is measured using the dihedral angle between two facets. The location of the actuators for tessellation origami with high degree-of-freedom can be found by computation of the local degree-of-freedom. Degree-of-freedom is studied by computing the local degree-of-freedom from one vertex to another connected vertex. The same method was presented using the same graph theory by Graver (2001) to compute the degree-of-freedom for rigid framework.

Tachi (2010b) presented the necessary considerations to find the degree-of-freedom for tessellation origami that resembles a quadrilateral and triangular mesh. The study also shows that all actuators of a triangulated pattern are located at the boundary of the tessellation pattern. Origami pattern with triangular pattern has the highest degree-of-freedom and hence very flexible; while the simplest origami pattern is the quadrilateral pattern that has only one degree-of-freedom.

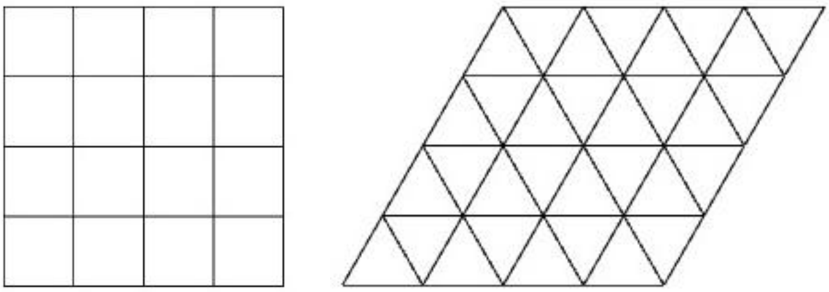
Belcastro and Hull (2002) presented the conditions for the non-flat folding of single-vertex by considering the global foldability condition using rotational matrices. Tachi (2009b) utilised the conditions by Belcastro and Hull (2002) and solved the problem of multi-vertex intersection by using the global constraint of the rotational matrices representing the multi-vertex. The resulting linear equation was solved using the pseudo-inverse matrix. However, it can only be used to find solution for a limited tessellation pattern which included the Miura-Ori Pattern, Waterbomb Pattern and pleated hypars.

2.2.3 Global Form

As mentioned earlier, global form of the tessellation origami resembles a shell surface. Therefore, the global surface is measured using the Gaussian Curvature. Schenk and Guest (2009) used the same method to study the behaviour of the folded tessellation origami and its relations to the Poisson's ratio of local single vertex that is used to control the synchonised folding. Gaussian curvature (**K**) is a function of the principal curvatures $k_1 = 1/r_1$ and $k_2 = 1/r_2$ where r_1 and r_2 is the radius of the principal curvature of the surface.

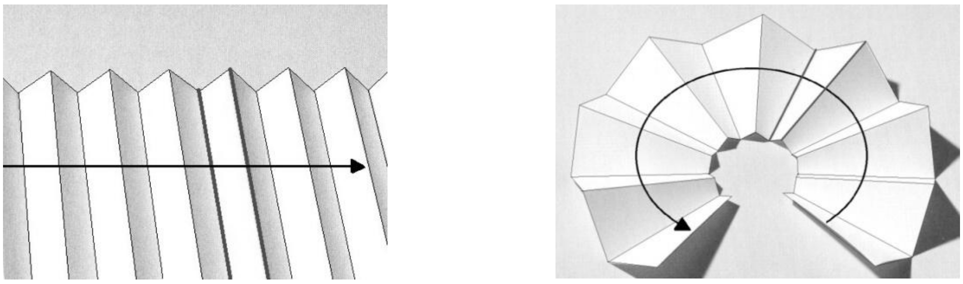
2.3 Tessellation Pattern

Tessellation origami is represented using a mountain-valley pattern. Study related to mountain-valley pattern design is focused on finding pattern that can fold flat. However, the folding pattern design for non-flat origami is obtained through paper folding. In paper-folding art, tessellation origami is produced as the result of intuitive folding using a pre-creased one unit grid. There are two types of grid observed: a square grid and equilateral triangle grid as shown in Figure 2.3(Gjerde, 2008). The simplest tessellation pattern is pleat fold which involves the arrangement of mountain and valley line in only one direction (Figure 2.4). The direction of the pleat can be either along a straight or curved line. Jackson (2011) shows a variety of simple tessellation that can be produced using four geometry transformation as shown in Figure 2.5.



(a) (b)

Figure 2.3 : Pre-creased paper folding grid (a) A unit square grid (b) Equilateral triangle grid (Gjerde, 2008)



(a) (b)

Figure 2.4 : Simple folding patterns – Pleat direction (a) Straight line (b) Curved line (Jackson, 2011)

There are five tessellation origami patterns other than Miura-ori Pattern and Waterbomb Pattern that were normally found in the related literatures. These patterns include the Yoshimura Pattern, diagonal pattern, variation of Waterbomb Pattern, Quadratic Waterbomb Pattern and Eggbox Pattern (Figure 2.6). Besides the tessellation patterns mentioned above, there are actually plenty of patterns that have not been studied computationally.

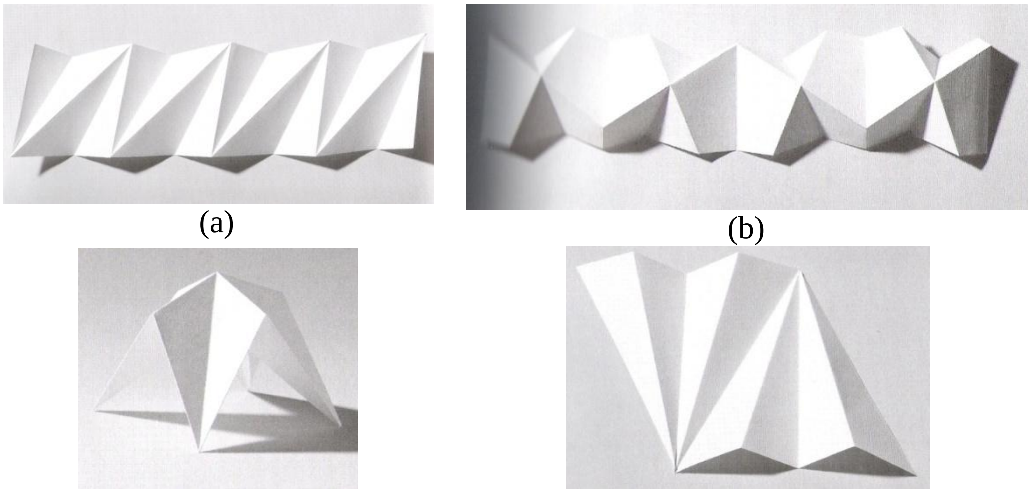
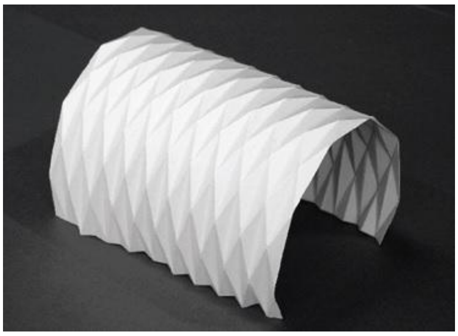


Figure 2.5 : Tranformations of origami tessellation (a) Symmetrical (b) Reflection (c) Rotation (d) Skew (Jackson, 2011)

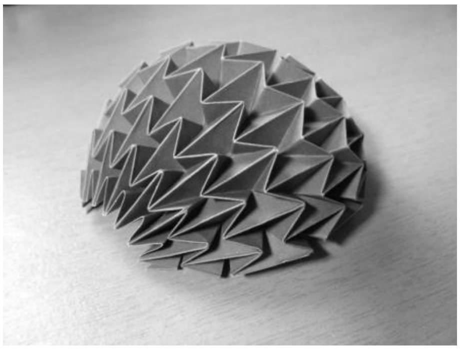
Paper artist or sculptor around the world displayed their work mostly in origami conference such as Origami in Science, Mathematics and Education (OSME) and in the photo sharing network such as Instagram and Flickr®. Among all the paper sculpture related to tessellation that has been shared in social network Flickr®, the works of one particular paper artist have been chosen as the source of patterns for investigation. The paper artist is Andrea Russo, an Italian paper artist that has produced vast amount of tessellated paper folding with curve as well as straight lines. Figure 2.7shows some examples of his work. However, the corresponding mountain-valley pattern for Russo's work is not given.

Russo's works covers a wide variety of tessellation origami. The works presented include the common pattern such as Miura-ori Pattern and Yoshimura Pattern but with different variation in local crease line pattern. More tessellation origami is also created from a hybrid of a few common patterns. Besides that, most of the works have the global form which resembles a shell structure which can serve as possible idea of folded structure in architectural and civil engineering.



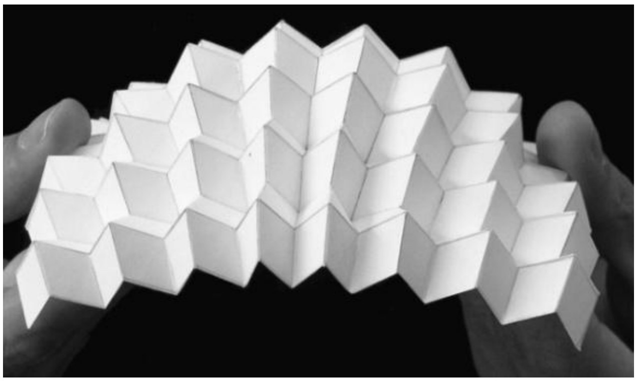
(a)

(b)



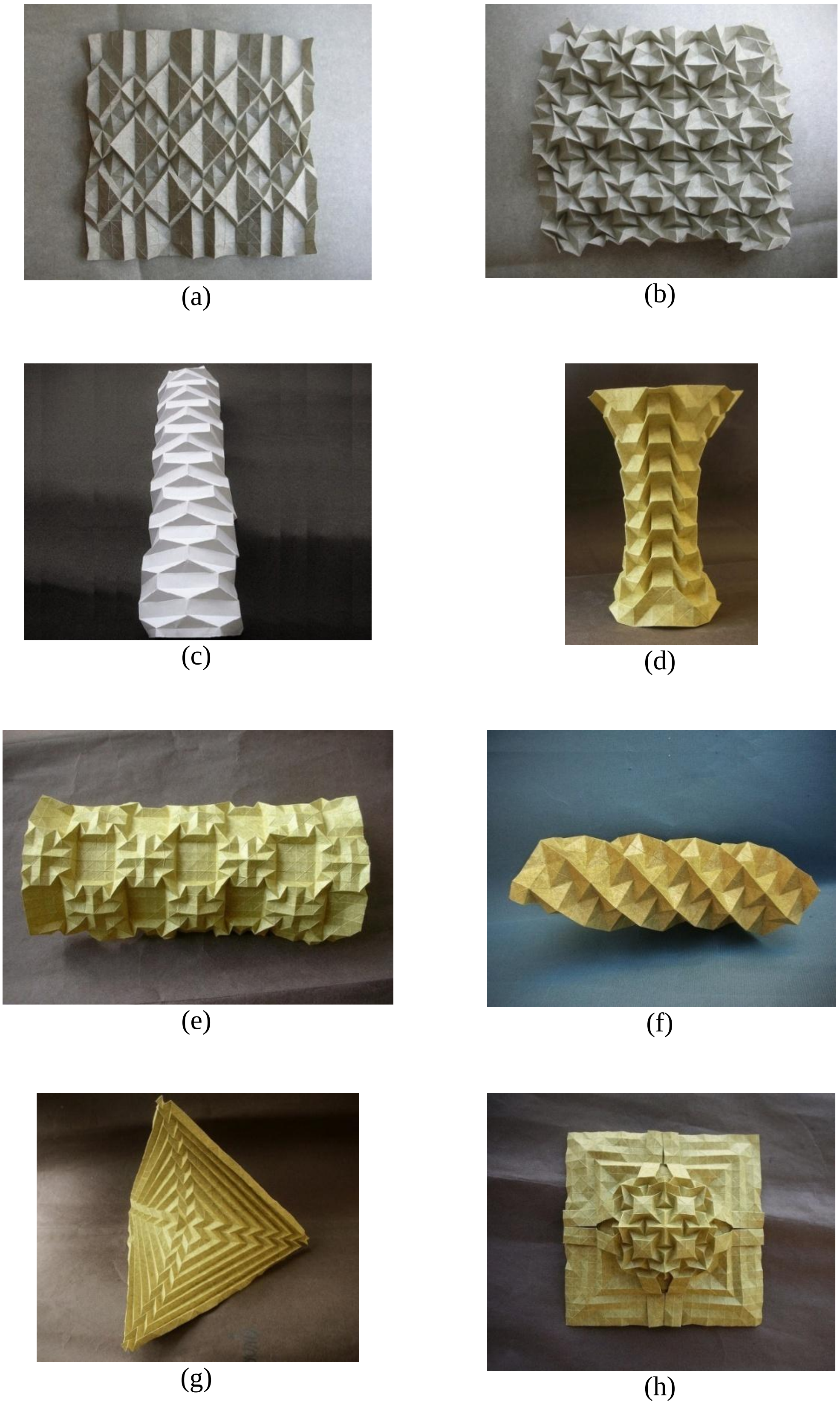
(c)

(d)



(e)

Figure 2.6 : Folding pattern (a) Yoshimura Pattern (b) Diagonal Pattern (Buri and Weinand, 2010) (c) variation of Waterbomb Pattern (Luna, 2008) (d) Quadratic Waterbomb Pattern(Jorgesainzdeaja, 2014) (e) Eggbox Pattern (Schenk and Guest, 2016)



(a) (b) (c) (d) (e) (f) (g) (h)

Figure 2.7 : Examples of tessellation origami paper folding art by Andrea Russo (a) Corrugation XII (b) Corrugation XI (c) Optical Illusion XXI VII MMVII (d) Simple Pattern Circular (e) Tessela XXV (f) Helix (g) Parabolae (h) Metope VII MMIX (Russo, n.d.)

2.4 Tessellation Origami - Geometric Modeling

Computer is an integral tool when it comes to form finding and design of tessellation origami. In architectural and civil engineering, many folded configuration is modeled using the geometric modeling technique. Tessellation origami model can be easily reproduced using computer aided drawing (CAD) tool such as AutoCAD. However, CAD tool only records the final configuration at the end of the drawing. The process to achieve the folding is not recorded and hence cannot be applied to model other tessellation origami with similar folding behaviour. Modification of the folded configuration also requires great effort for model such as the tessellation origami. This is especially true for periodic tessellation origami which has high numbers of repeated local pattern.

Hence, many tessellation origami models are represented using geometric model by combining the discipline of applied mathematics with computational geometry. Shape is represented by sequence of steps or algorithm in computational geometry. Shapes include curves, surfaces, and solids in 3D space, as well as higher-dimensional entities such as surfaces deforming in time, and solids with a spatially varying mass density(Gallier, 1999). An example is given by Gallier (1999) using a curve fitting problem (Figure 2.8). The problem is solved by using the control point d to find the curve that passing through a set of point x.

Computational geometry is widely used in the robotics folding origami where motion planner is developed to represent folding motion representing a local pattern (Hawkes et al., 2010;An et al., 2011;Xi and Lien, 2015). Besides that, computational geometry combined with programming algorithm is also widely used in the form-finding for architectural design. This type of geometric modeling technique can be