

UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 1998/99

April 1999

JIM 414 - Pentaabiran Statistik

Masa: [3 jam]

ARAHAN KEPADA CALON:

- Sila pastikan bahawa kertas peperiksaan ini mengandungi DUAPULUH TIGA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.
 - Jawab SEMUA soalan. Setiap soalan bernilai 100 markah dan markah subsoalan diperlihatkan di penghujung subsoalan itu.
 - Setiap jawapan mesti dijawab di dalam buku jawapan yang disediakan.
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1. (a) X_1, X_2, \dots, X_n adalah sampel rawak daripada taburan $N(\mu, \sigma^2)$.

$$\text{Andaikan } S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 \text{ di mana } \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i.$$

Dapatkan $\text{Var}(S^2)$.

(25 markah)

- (b) X_1, X_2, \dots, X_n adalah sampel rawak daripada taburan $N(0, \sigma^2)$. Dapatkan penganggar kebolehjadian maksimum buat σ^2 .

(25 markah)

- (c) X_1, X_2, \dots, X_n adalah sampel rawak daripada taburan $N(1, \sigma^2)$. Binakan selang keyakinan 95% bagi σ^2 dengan menggunakan kaedah kuantiti pangsaan.

(25 markah)

- (d) Diberikan $n = 10, s = 0.04$. Katakan kita ingin menguji $H_0 : \sigma^2 = 0.01$ menentang $H_A : \sigma^2 < 0.01$. Dapatkan rantau genting ujian ini jika statistik ujian ialah

$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}.$$

(25 markah)

2. (a) Takrifkan statistik tertib.

(20 markah)

- (b) Ia digunakan di dalam kaedah kebolehjadian maksimum untuk mencari penganggar. Berikan suatu contoh tentang hal ini.

(30 markah)

- (c) X_1, X_2, \dots, X_n adalah sampel rawak daripada taburan seragam $\left(\theta - \frac{1}{2}, \theta + \frac{1}{2}\right)$. Andaikan $Y_1 < Y_2 < \dots < Y_n$ menandai statistik tertib bagi sampel rawak tersebut. Katakan $\frac{Y_1 + Y_2}{2}$ adalah penganggar bagi θ . Semak kesaksamaan penganggar ini. (50 markah)
3. (a) Nyatakan Teorem Lehmann-Scheffe'. (20 markah)
- (b) X_1, X_2, \dots, X_n adalah sampel rawak daripada populasi bertaburan seragam $(0, \theta)$, $\theta > 0$. Tunjukkan $Y_n = \text{maksimum}(X_1, X_2, \dots, X_n)$ adalah statistik cukup dan lengkap. (30 markah)
- (c) Seterusnya cari PSVMS bagi θ di dalam (b). (50 markah)
4. X_1, X_2, \dots, X_n adalah sampel rawak daripada populasi yang mempunyai fungsi ketumpatan kebarangkalian $f(x; \theta) = \frac{1}{\theta} e^{-x/\theta}$, $\theta > 0, x > 0$.
- (a) Dapatkan penganggar kebolehjadian maksimum bagi θ . (25 markah)
- (b) Dapatkan maklumat Fisher bagi θ . (25 markah)
- (c) Seterusnya nyatakan taburan asimptot bagi (a). (20 markah)
- (d) Dengan menggunakan (c) binakan selang keyakinan hampiran 95% bagi θ . (30 markah)

5. (a) Takrifkan Ujian Paling Berkuasa Secara Seragam.

(20 markah)

(b) Andaikan X_1, X_2, \dots, X_5 adalah sampel rawak daripada taburan Bernoulli.

$f(x; \theta) = \theta^x(1 - \theta)^{1-x}$, $x = 0, 1$, $0 < \theta < 1$. Katakan kita hendak menguji

$H_0: \theta = \frac{1}{2}$ menentang $H_A: \theta < \frac{1}{2}$. Berdasarkan sampel tersebut, tolak H_0 jika

$\sum_{i=1}^5 X_i < c$. Tunjukkan ini adalah ujian paling berkuasa secara seragam.

(50 markah)

(c) Dapatkan aras keertian apabila $c = 1$.

(30 markah)

Lampiran

Bab 5

1. $f_{X_1, X_2, \dots, X_n}^{(x_1, x_2, \dots, x_n)} = \prod_{i=1}^n f(x_i)$
2. $M_r = \frac{1}{n} \sum_{i=1}^n X_i^r, \quad r = 1, 2, 3, \dots$
3. $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$
4. $M'_r = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^r, \quad r = 1, 2, 3, \dots$
5. $S_n^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$
6. $\tilde{S}_n^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$
7. $E[M^r] = E[X^r]$
8. $E[\bar{X}_n] = \mu$
9. $\text{Var}(\bar{X}_n) = \sigma^2/n$
10. $\sum_{i=1}^n (X_i - \mu)^2 = \sum_{i=1}^n (X_i - \bar{X})^2 + n(\bar{X} - \mu)^2.$
11. $E[S_n^2] = \sigma^2.$
12. $M_{\bar{X}}(t) = [M(t/n)]^n$
13. $\text{had}_{n \rightarrow \infty} P(|\bar{X}_n - \mu| < \varepsilon) = 1$
14. $\text{had}_{n \rightarrow \infty} F_n = F$

$$15. Z_n = \frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma}$$

$$16. \text{had } F_n(z) = \Phi(z)$$

$$17. (X_1 - \bar{X})^2 = \left[\sum_{i=2}^n (X_i - \bar{X}) \right]^2$$

$$18. \sum_{i=1}^n \frac{(X_i - \bar{X})^2}{\sigma^2} = (n-1) \frac{S_n^2}{\sigma^2}$$

$$19. G_1(y) = 1 - [1 - F(y)]^n$$

$$20. G_n(y) = [F(y)]^n$$

$$21. g_1(y) = n[1 - F(y)]^{n-1} f(y)$$

$$22. g_n(y) = n [F(y)]^{n-1} f(y)$$

$$23. G_\alpha(y) = \sum_{j=\alpha}^n \binom{n}{j} [F(y)]^j [1 - F(y)]^{n-j}$$

$$24. g_\alpha(y) = \frac{n!}{(\alpha-1)!(n-\alpha)!} [F(y)]^{\alpha-1} f(y) [1-F(y)]^{n-\alpha}$$

$$25. g_{\alpha,\beta}(x,y) = \frac{n!}{(\alpha-1)!(\beta-\alpha-1)!(n-\beta)!} [F(x)]^{\alpha-1} f(x) [F(y) - F(x)]^{\beta-\alpha-1} f(y) [1-F(y)]^{n-\beta}, \alpha < \beta$$

$$26. g(y_1, y_2, \dots, y_n) = n! \prod_{i=1}^n f(y_i)$$

$$27. \text{Median sampel} = \begin{cases} \frac{1}{2}(Y_{n/2} + Y_{(n+2)/2}), & \text{jika } n \text{ genap} \\ Y_{(n+1)/2} & , \text{jika } n \text{ ganjil} \end{cases}$$

$$28. \text{Julat sampel} = Y_n - Y_1$$

29. Tengah julat sampel = $\frac{1}{2}(Y_1 + Y_n)$
30. $\lim_{n \rightarrow \infty} F_n(x) = \begin{cases} 0, & x < c \\ 1, & x \geq c \end{cases}$
31. $P(|X_n - c| < \varepsilon) = 1, \varepsilon > 0$
32. $\lim_{n \rightarrow \infty} M_n(t) = M(t)$
33. $e^x = 1 + x + \frac{x^2}{2!} + \frac{e^z x^3}{3!}, 0 < z < x$
34. $\lim_{n \rightarrow \infty} \left[1 + \frac{a}{n} + \frac{\psi(n)}{n} \right]^n = \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a$, jika $\lim_{n \rightarrow \infty} \psi(n) = 0$

Bab 6

1. $L(\theta; x_1, x_2, \dots, x_n) = \prod_{i=1}^n f(x_i, \theta)$
2. $L(\theta_1, \theta_2, \dots, \theta_k) = \prod_{i=1}^n f(x_i, \theta_1, \theta_2, \dots, \theta_k)$
3. $\chi^2 = \sum_{i=1}^k \frac{[N_i - np_i(\theta)]^2}{np_i(\theta)}$
4. $E[T] = \tau(\theta)$
5. $E_{\theta}[\{T - \tau(\theta)\}^2] = \text{Var}(T) + \{E[T] - \tau(\theta)\}^2$
6. $\text{Var}(T) \geq \frac{[\tau'(\theta)]^2}{n E \left[\left\{ \frac{\partial}{\partial \theta} \log f(X; \theta) \right\}^2 \right]}$

7. $E\left[\left\{\frac{\partial}{\partial\theta}\log f(X;\theta)\right\}^2\right] = -E\left[\frac{\partial^2}{\partial\theta^2}\log f(X;\theta)\right]$
8. $\lim_{n\rightarrow\infty} P_\theta [|T_n - \tau(\theta)| < \varepsilon] = 1, \varepsilon > 0$
9. $\lim_{n\rightarrow\infty} E_\theta \left[\{T_n - \tau(\theta)\}^2 \right] = 0$
10. $f(x_1, x_2, \dots, x_n; \theta) = g(t; \theta) h(x_1, x_2, \dots, x_n)$
11. $f(x_1, x_2, \dots, x_n; \theta) = g(t_1, t_2, \dots, t_r; \theta) h(x_1, x_2, \dots, x_n)$
12. $L(\theta; x_1, \dots, x_n) = g(t; \theta) h(x_1, x_2, \dots, x_n)$
13. $E[X] = E[E[X | Y = y]] = E[E[X | Y]]$
14. $\text{Var}(X | Y = y) = E[(X - E[X | y])^2 | y]$
15. $\text{Var}(X) = \text{Var}(E[X | Y]) + E[\text{Var}(X | Y)]$
16. $E[z(T)] = 0 \Rightarrow P[z(T) = 0] = 1$
17. $f(x; \theta) = a(\theta) b(x) \exp [c(\theta) d(x)]$
18. $f(x; \theta_1, \dots, \theta_k) = a(\theta_1, \dots, \theta_k) b(x) \exp [c_1(\theta_1, \dots, \theta_k) d_1(x) + \dots + c_k(\theta_1, \dots, \theta_k) d_k(x)]$
19. $f(x; \theta) = h(x - \theta)$
20. $f(x; \theta) = \frac{1}{\theta} h(x/\theta)$
21. $u(x_1 + c, x_2 + c, \dots, x_n + c) = u(x_1, x_2, \dots, x_n) + c.$

$$22. u(cx_1, cx_2, \dots, cx_n) = cu(x_1, x_2, \dots, x_n)$$

$$23. u(X_1, X_2, \dots, X_n) = \frac{\int \theta \prod_{i=1}^n f(X_i; \theta) d\theta}{\int \prod_{i=1}^n f(X_i; \theta) d\theta}$$

Rumus-Rumus JIM 312 - Teori Kebarangkalian

Modul 1

Pelajaran 1

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
2. $P(A) = P(A \cap \bar{B}) + P(A \cap B)$
3. $P(\bar{A}) = 1 - P(A)$
4. ${}^n P_r = \frac{n!}{(n-r)!}$
5. $\binom{n}{r} = \frac{n!}{r!(n-r)!}$
6. $N = \frac{n!}{n_1! n_2! \dots n_k!}$

Pelajaran 2

1. $P(A | B) = \frac{P(A \cap B)}{P(B)}$
2. $P(A \cap B) = P(A)P(B)$
3. $P(A) = P(A | B) P(B) + P(A | \bar{B}) P(\bar{B})$
4. $P(B_i | A) = \frac{P(A \cap B_i)}{\sum_{j=1}^n P(A \cap B_j) P(B_j)}$

Pelajaran 3

1. $P(a \leq X \leq b) = \int_a^b f(x) dx$
2. $P(a < X < b) = \sum_{a < x < b} p(x)$
3. $F(t) = P(X \leq t)$
4. $P(a < X \leq b) = F(b) - F(a)$
5. $\frac{d}{dt} F(t) = f(t)$
6. $F_Y(t) = F_X(g^{-1}(t))$
7. $F_Y(t) = 1 - F_X(g^{-1}(t))$
8. $f_Y(t) = f_X(g^{-1}(t)) |J|$
9. $J = \frac{dg^{-1}(t)}{dt}$
10. $f_Y(t) = \sum_{i=1}^k f_X(g_i^{-1}(t)) |J_i|$
11. $J_i = \frac{d}{dt} g_i^{-1}(t)$
12. $P_Y(y) = \sum_{x \in A} P_X(x)$

Modul 2**Pelajaran 1**

1. $E(X) = \sum_{x \in \text{Julat } X} xp(x)$
2. $1 + x + x^2 + \dots + x^n + \dots = \frac{1}{1-x}, |x| < 1$
3. $1 + 2x + \dots + nx^{n-1} + \dots = \frac{1}{(1-x)^2}, |x| < 1$
4. $E(X) = \int_{-\infty}^{\infty} xf(x) dx$

$$5. E(X) = \int_0^{\infty} [1 - f(x)] dx - \int_{-\infty}^0 F(x) dx$$

$$6. E[G(X)] = \sum_{x \in \text{Julat } X} G(x) p(x)$$

$$7. E[G(X)] = \int_{-\infty}^{\infty} G(x) f(x) dx$$

$$8. E[c] = c$$

$$9. E[cX] = c E[X]$$

$$10. E[X + c] = E[X] + c$$

$$11. \text{Var}(X) = E[X - E[X]]^2$$

$$12. \text{Var}(X) = E[X^2] - \mu_X^2$$

$$13. \text{Var}(X) = \sum_{x \in \text{Julat } X} x^2 p(x) - \mu_X^2$$

$$14. \text{Var}(X) = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu_X^2$$

$$15. \text{Var}(a) = 0$$

$$16. \text{Var}(aX + b) = a^2 \text{Var}(X)$$

$$17. F_X(t_k) = k, 0 < k < 1$$

Pelajaran 2

$$1. m_k = E[X^k]$$

$$2. m_k = \sum_{x \in \text{Julat } X} x^k p(x)$$

$$3. m_k = \int_{-\infty}^{\infty} x^k f(x) dx$$

$$4. \mu_k = E[(X - \mu_X)^k]$$

$$5. \gamma_1 = \mu_3 / \sigma_X^3$$

$$6. \quad \gamma_2 = \frac{\mu_4}{\sigma_x^4} - 3.$$

$$7. \quad \mu_{[k]} = E[X(X-1)(X-2) \dots (X-k+1)]$$

$$8. \quad m(t) = E[e^{tX}]$$

$$9. \quad m(t) = \sum_{x \in \text{Julat } X} e^{tx} p(x)$$

$$10. \quad m(t) = \int_{-\infty}^{\infty} e^{tx} f(x) dx$$

$$11. \quad m_Y(t) = E[e^{tg(X)}]$$

$$12. \quad m_Y(t) = \sum_{x \in \text{Julat } X} e^{tg(x)} p(x)$$

$$13. \quad m_Y(t) = \int_{-\infty}^{\infty} e^{tg(x)} f(x) dx$$

$$14. \quad m_Y(t) = e^{bt} m_X(at)$$

$$15. \quad m^{(i)}(0) = m_i$$

$$16. \quad k(t) = \ln m(t)$$

$$17. \quad \psi(t) = E[t^X]$$

$$18. \quad f(t) = \sum_{i=0}^{\infty} \frac{f^{(i)}(a)}{i!} (t-a)^i$$

$$19. \quad \psi^{(i)}(0) = i! p(i)$$

$$20. \quad P(|X| \geq a) < \frac{1}{a^2} E[X^2]$$

$$21. \quad P(|X - \mu| \geq a\sigma) \leq \frac{1}{a^2}$$

$$22. \quad P(|X - \mu| < a\sigma) \geq 1 - \frac{1}{a^2}$$

$$23. P(X \geq a) \leq \frac{E[X]}{a}$$

$$24. E[X^n] = \int_0^{\infty} nx^{n-1} (1 - F(x)) dx$$

Pelajaran 3

$$1. (i) p(x) = \begin{cases} q, & x = 0 \\ p, & x = 1 \\ 0, & \text{di tempat lain} \end{cases} \quad X \sim \text{Bernoulli } (p)$$

$$(ii) E[X] = p$$

$$(iii) \text{Var } (X) = pq$$

$$(iv) m(t) = q + pe^t$$

$$2. (i) p(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & x=0, 1, 2, \dots, n \\ 0, & \text{di tempat lain} \end{cases} \quad X \sim \text{Binomial } (n, p)$$

$$(ii) E[X] = np$$

$$(iii) \text{Var } (X) = npq$$

$$(iv) m(t) = (q + pe^t)^n$$

$$3. (i) p(x) = \begin{cases} \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}, & x=0, 1, 2, \dots, n \\ 0, & \text{di tempat lain} \end{cases} \quad X \sim \text{hipergeometri } (N, k, n)$$

$$(ii) E[X] = \frac{nK}{N}$$

$$(iii) \text{Var } (X) = \frac{nK(N-K)(N-n)}{N^2(N-1)}$$

$$4. (a + b)^n = \sum_{i=0}^n \binom{n}{i} a^i b^{n-i}$$

5. (i) $p(x) = \begin{cases} q^{x-1}p, & x = 1, 2, 3, \dots \\ 0, & \text{di tempat lain} \end{cases}$ X ~ geometri (p)
- (ii) $E[X] = 1/p$
- (iii) $\text{Var}(X) = q/p^2$
- (iv) $m(t) = \frac{pe^t}{1 - qe^t}$
-
6. (i) $p(x) = \begin{cases} \binom{x-1}{r-1} p^r q^{x-r}, & x=r, r+1, r+2 \\ & r=2, 3, 4, \dots \\ 0, & \text{di tempat lain} \end{cases}$ X ~ negatif binomial (r, p)
- (ii) $E[X] = r/p$
- (iii) $\text{Var}(X) = rq/p^2$
- (iv) $m(t) = \left[\frac{pe^t}{1 - qe^t} \right]^r$
-
7. (i) $p(x) = \begin{cases} e^{-\lambda} \frac{\lambda^x}{x!}, & x = 0, 1, 2, \dots \\ 0, & \text{di tempat lain} \end{cases}$ X ~ Poisson (l)
- (ii) $E[X] = \lambda$
- (iii) $\text{Var}(X) = \lambda$
- (iv) $m(t) = e^{\lambda(e^t-1)}$
-
8. $\lim_{x \rightarrow 0} (1+x)^{1/x} = e$
9. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$
10. $\lim_{x \rightarrow 0} (1+ax)^{1/x} = e^a$

PELAJARAN 4

1. (i) $f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{di tempat lain} \end{cases}$ X ~ seragam (a, b)

(ii) $E[X] = \frac{a+b}{2}$

(iii) $\text{Var}(X) = \frac{(b-a)^2}{12}$

(iv) $m(t) = \frac{e^{bt} - e^{at}}{t(b-a)}$

2. (i) $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, -\infty < x < \infty$ X ~ N(μ , σ^2)

(ii) $E[X] = \mu$

(iii) $\text{Var}(X) = \sigma^2$

(iv) $m(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

3. $\lim_{n \rightarrow \infty} P \left[a \leq \frac{S_n - np}{\sqrt{npq}} \leq b \right] \rightarrow P(Z \geq a) - P(Z > b)$

4. $\lim_{\lambda \rightarrow \infty} P \left[a \leq \frac{X - \lambda}{\sqrt{\lambda}} < b \right] \rightarrow P(Z > a) - P(Z \geq b)$

5. (i) $f(x) = \begin{cases} \lambda e^{-\lambda}, & x \geq 0 \\ 0, & \text{di tempat lain} \end{cases}$ X ~ eksponen (λ)

(ii) $E[X] = 1/\lambda$

(iii) $\text{Var}(X) = 1/\lambda^2$

(iv) $m(t) = \frac{\lambda}{\lambda - t}$

$$6. \quad \Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$$

$$7. \quad \Gamma(n) = (n-1) \Gamma(n-1)$$

$$8. \quad \Gamma(n) = (n-1)!$$

$$9. \quad (i) \quad f(x) = \begin{cases} \frac{\lambda^n x^{n-1}}{\Gamma(n)} e^{-\lambda x}, & x > 0 \\ 0, & \text{di tempat lain} \end{cases} \quad X \sim \text{Gamma}(n, 1)$$

$$(ii) \quad E[X] = n/\lambda$$

$$(iii) \quad \text{Var}(X) = n/\lambda^2$$

$$(iv) \quad m(t) = \left(\frac{\lambda}{\lambda - t} \right)^n$$

$$10. \quad (i) \quad f(x) = \begin{cases} \frac{1}{2^{v/2} \Gamma\left(\frac{v}{2}\right)} x^{v/2-1} e^{-x/2}, & x > 0 \\ 0, & \text{di tempat lain} \end{cases} \quad X \sim \chi_v^2$$

$$(ii) \quad E[X] = v$$

$$(iii) \quad \text{Var}(X) = 2v$$

$$(iv) \quad m(t) = \left(\frac{1}{1-2t} \right)^{v/2}$$

$$11. \quad B(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt$$

$$12. \quad B(x, y) = \int_0^{\infty} \frac{t^{x-1}}{(1+t)^{x+y}} dt$$

$$13. \quad B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$$14. \text{ (i) } f(x) = \begin{cases} \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1}, & 0 < x < 1 \\ 0, & \text{di tempat lain} \end{cases} \quad X \sim \text{Beta}(a, b)$$

$$\text{(ii) } F_X(p) = \sum_{x=a}^n \binom{n}{x} p^x (1-p)^{n-x}$$

$$\text{(iii) } E[X] = \frac{a}{a+b}$$

$$\text{(iv) } \text{Var}(X) = \frac{ab}{(a+b+1)(a+b)^2}$$

Modul 3

Pelajaran 1

$$1. P(X \leq x, Y \leq y) = \sum_{t_1 \leq x} \sum_{t_2 \leq y} p(t_1, t_2)$$

$$2. P(X \leq x, Y \leq y) = \int_{-\infty}^x \int_{-\infty}^y f(t_1, t_2) dt_1 dt_2$$

$$3. F(x, y) = P(X \leq x, Y \leq y)$$

$$4. f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y}$$

Pelajaran 2

$$1. p(x) = \sum_y p(x, y)$$

$$2. p(y) = \sum_x p(x, y)$$

$$3. f(x) = \int_{-\infty}^{\infty} f(x, y) dy$$

$$4. f(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

$$5. F(x) = F(x, \bullet)$$

$$6. F(y) = F(\bullet, y)$$

$$7. f(x) = \frac{\partial F(x, \bullet)}{\partial x}$$

$$8. f(y) = \frac{\partial F(\bullet, y)}{\partial y}$$

$$9. p(x | y) = \frac{p(x, y)}{p(y)}$$

$$10. f(x | y) = \frac{f(x, y)}{f(y)}$$

$$11. p(x, y) = p(x) p(y)$$

$$12. f(x, y) = f(x) f(y)$$

Pelajaran 3

$$1. E[g(X, Y)] = \sum_x \sum_y g(x, y) p(x, y)$$

$$2. E[g(X, Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) f(x, y) dx dy$$

$$3. E[g_1(X, Y) + g_2(X, Y)] = E[g_1(X, Y)] + E[g_2(X, Y)]$$

$$4. E[h_1(X) h_2(Y)] = E[h_1(X)] E[h_2(Y)]$$

$$5. (i) \text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$$

$$(ii) \text{Cov}(X, Y) = E[XY] - \mu_X \mu_Y$$

$$6. \text{Cov}(aX, bY) = ab \text{Cov}(X, Y)$$

$$7. \text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2 \text{Cov}(X, Y)$$

$$8. \text{Var} \left(\sum_{i=1}^n X_i \right) = \sum_{i=1}^n \text{Var} (X_i) + 2 \sum_{i < j} \text{Cov} (X, Y)$$

$$9. \rho(X, Y) = \frac{\text{Cov} (X, Y)}{\sigma_X \sigma_Y}$$

$$10. E[g(X, Y) | Y = y] = \sum_x g(x, y) p(x | y)$$

$$11. E[g(X, Y) | Y = y] = \int_{-\infty}^{\infty} g(x, y) f(x | y) dx$$

$$12. E[E[X | Y = y]] = E[X]$$

$$13. E[E[Y | X = x]] = E[Y]$$

$$14. E[E[g(X) | Y = y]] = E[g(X)]$$

$$15. E[E[g(Y) | X = x]] = E[g(Y)]$$

$$16. \text{Var} (X | Y = y) = E[X^2 | Y = y] - (E[X | Y = y])^2$$

$$17. m(t_1, t_2) = E[e^{t_1 X_1 + t_2 X_2}]$$

$$18. m(t_1, t_2, \dots, t_n) = E \left[e^{\sum_{i=1}^n t_i X_i} \right]$$

$$19. m(t_1) = \lim_{t_2 \rightarrow 0} m(t_1, t_2)$$

$$20. m(t_1, t_2, \dots, t_n) = m(t_1) m(t_2) \dots m(t_n)$$

Pelajaran 4

$$1. (i) p(x_1, x_2, \dots, x_k) = \frac{n!}{x_1! x_2! \dots x_k!} p_1^{x_1} p_2^{x_2} \dots p_k^{x_k}$$

$$(ii) p(x_i) = \binom{n}{x_i} p_i^{x_i} (1 - p_i)^{n - x_i}$$

$$(iii) p(x_i, x_j) = \frac{n!}{x_i! x_j! (n - x_i - x_j)!} p_i^{x_i} p_j^{x_j} (1 - p_i - p_j)^{n - x_i - x_j}$$

$$(iv) E[X_i X_j] = n(n - 1) p_i p_j$$

$$(v) \text{Cov} (X_i, X_j) = -n p_i p_j$$

$$2. (i) f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[\left(\frac{x-\mu_x}{\sigma_x} \right)^2 - 2\rho \left(\frac{x-\mu_x}{\sigma_x} \right) \left(\frac{y-\mu_y}{\sigma_y} \right) + \left(\frac{y-\mu_y}{\sigma_y} \right)^2 \right] \right\},$$

$$-\infty < x < \infty, -\infty < y < \infty$$

$$(ii) f(x|y) = \frac{1}{\sigma_x\sqrt{2\pi(1-\rho^2)}} \exp \left\{ -\frac{1}{2(1-\rho^2)\sigma_x^2} \left[x - \mu_x - \rho \frac{\sigma_x}{\sigma_y} (y - \mu_y) \right]^2 \right\}$$

$$-\infty < x < \infty$$

$$(iii) m(t_1, t_2) = \exp \left[t_1\mu_x + t_2\mu_y + \frac{1}{2} (t_1^2\sigma_x^2 + 2\rho t_1 t_2\sigma_x\sigma_y + t_2^2\sigma_y^2) \right]$$

$$(iv) E[XY] = \mu_x\mu_y + \rho\sigma_x\sigma_y$$

$$(v) \text{Cov}(X, Y) = \rho\sigma_x\sigma_y$$

Modul 4

Pelajaran 1

$$1. M_k = \frac{1}{n} \sum_{i=1}^n X_i^k$$

$$2. E[M_k] = m_k$$

$$3. \text{Var}(M_k) = \frac{1}{n} [m_{2k} - m_k^2]$$

$$4. E[\bar{X}] = \mu$$

$$5. \text{Var}(\bar{X}) = \frac{1}{n} \sigma^2$$

$$6. S^2 = \frac{1}{(n-1)} \sum_{i=1}^n (X_i - \bar{X})^2$$

7. $E[S^2] = \sigma^2$

8. $\text{Var}(S^2) = \frac{1}{n} \left(\mu_4 - \frac{(n-3)}{(n-1)} \sigma^4 \right)$

9. $\sum_{i=1}^n (X_i - \mu)^2 = \sum_{i=1}^n (X_i - \bar{X})^2 + n(\bar{X} - \mu)^2$

10. $\bar{X} - \mu = \frac{1}{n} \sum_{i=1}^n (X_i - \mu)$

Pelajaran 2

1. $p(u, v) = p_{X,Y} \left(g_1^{-1}(u, v), g_2^{-1}(u, v) \right)$

2. $f(u, v) = f_{X,Y} \left(g_1^{-1}(u, v), g_2^{-1}(u, v) \right) |J|$

3. $J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$

4. $f(u, v) = \sum_{i=1}^m |J_i| f_{X,Y} \left(g_i^{-1}(u, v), h_i^{-1}(u, v) \right)$

5. $J_i = \begin{vmatrix} \frac{\partial g_i^{-1}(u, v)}{\partial u} & \frac{\partial g_i^{-1}(u, v)}{\partial v} \\ \frac{\partial h_i^{-1}(u, v)}{\partial u} & \frac{\partial h_i^{-1}(u, v)}{\partial v} \end{vmatrix}$

6. $m_{u,v}(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{t_1 g(x,y) + t_2 h(x,y)} f(x,y) dx dy$

7. $m_u(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{t g(x,y)} f(x,y) dx dy$

$$8. \quad (i) \quad f_{u=x+y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(x, u-x) dx$$

$$(ii) \quad f_{u=x+y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(u-y, y) dy$$

$$9. \quad (i) \quad f_{u=x-y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(x, x-u) dx$$

$$(ii) \quad f_{u=x-y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(u+y, y) dy$$

$$10. \quad (i) \quad f_{u=xY}(u) = \int_{-\infty}^{\infty} \frac{1}{|x|} f_{X,Y}(x, u/x) dx$$

$$(ii) \quad f_{u=xY}(u) = \int_{-\infty}^{\infty} \frac{1}{|y|} f_{X,Y}(u/y, y) dy$$

$$11. \quad f_{u=xY}(u) = \int_{-\infty}^{\infty} |y| f_{X,Y}(uy, y) dy$$

Pelajaran 3

$$1. \quad (i) \quad f(x) = \frac{\Gamma[(n+1)/2]}{\Gamma(n/2)\sqrt{\pi n}} \left(1 + \frac{x^2}{n}\right)^{-(n+1)/2}, \quad -\infty < x < \infty \quad X \sim t_n$$

$$(ii) \quad T = \frac{Z}{\sqrt{V/n}}$$

$$(iii) \quad E[X] = 0$$

$$(iv) \quad \text{Var}[X] = \frac{n}{n-2}$$

$$2. \quad (i) \quad f(x) = \begin{cases} \frac{\Gamma[(m+n)/2]}{\Gamma(m/2)\Gamma(n/2)} \left(\frac{m}{n}\right)^{m/2} \frac{x^{(m-2)/2}}{[1+(m/n)x]^{(m+n)/2}}, & x > 0 \\ 0 & , \text{ di tempat lain} \end{cases} \quad X \sim F_{m,n}$$

$$(ii) \quad F = \frac{U/m}{V/m}$$

$$(iii) \quad E[X] = \frac{n}{n-2}$$

$$(iv) \quad \text{Var}(X) = \frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$$

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