THREE-DIMENSIONAL ELASTIC-PLASTIC CRACK TIP STRESS FIELDS

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THREE-DIMENSIONAL ELASTIC-PLASTIC CRACK TIP STRESS FIELDS

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DECLARATION

This work has not previously been accepted in substance for any degree and is not being concurrently submitted in candidate for any degree.

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Date ………………………………

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This thesis is the result of my own investigation, excepts where otherwise stated.

Other sources are acknowledged by giving explicit references. Bibliography/references are appended

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LIST OF SYMBOLS

A_2	Higher-order terms in-plane strain crack configurations
A_P	In-plane and out-of-plane constraint
B	Thickness
CTOD	Crack-tip opening displacement
E	Young's Modulus
G	Energy release rate
HRR	Hutchinson-Rice-Rosengren
J	J-integral
J-Δσ	
K	Stress intensity factor
LEFM	Linear elastic fracture mechanic
n	Strain hardening coefficient
Q	In-plane constraint parameter
r	Distance
SH	Sham-Hancock
T	Non-singular T-stress
T_Z	Out-of-plane constraint
z	Distance measured normal from the free surface of a cracked-plane
ε_0	Yield strain
σ_0	Yield strength
δ	Displacement at the crack tip
σ_{ij}	Stress components in cylindrical
$\sigma_{\theta\theta}$	Hoop stress
σ_{rr}	Radial stress
$\sigma_{r\theta}$	Shear stress
σ_m	Mean stress
θ	Angle
γ	Sensitivity constant constraint
$\Delta\sigma$	Change in stress
3-D	Three-dimensional

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ABSTRAK

Kesan tegasan adalah faktor terpenting dalam penilaian integriti struktur berdasarkan fraktur mekanik di hujung retak untuk memahami keadaan tegasan bahan yang mengalami retak. Penyelidikan berdasarkan kepada perubahan medan tegasan sepanjang ketebalan dan sekitar sudut hujung retak. Pendekatan parameter dua dimensi boleh dilanjutkan untuk menilai medan hujung retak plastik anjal tiga dimensi dengan mengubah suai formula yang diterbitkan oleh Hutchinson Rice, Rosengren, dan F. Yusof. Perantaraan tegasan diterapkan untuk menunjukkan keadaan tegasan di sepanjang hujung retak melalui parameter kehilangan tegasan , α. Keputusan daripada teknik HRR-J-$\Delta\sigma$ ditunjukkan di sepanjang hujung retak, tegasan gelung dan jejari berbeza di sepanjang hujung retak antara $0\leq \alpha \leq 1$ dan dalam sudut $0°\leq\theta\leq 180°$. Satu siri ungkapan analitik telah diterapkan untuk memahami ciri medan hujung retak keteganagn-plastik 3-D. Ini membolehkan penyelidik untuk dapat memetakan medan tegasan hujung retak dalam struktur dan menganjurkan penilaian langsung ke atas integriti struktur dalam pendekatan yang cekap dan pantas.

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ABSTRACT

The constraint impact is the most important factor in structural integrity evaluations aspects on crack tip fracture mechanics to accurately anticipate the stresses condition of material under the influence of crack. This thesis investigates a stress field changes along the thickness and around the angle of the crack-tip. Through modification of the analytical formulas proposed using HRR-J-$\Delta\sigma$, the two-dimensional parameter method can be extended to analyse the three-dimensional elastic-plastic crack-tip field. The intermediate stresses are developed to shows the stresses state along the crack tip through the constraint loss parameter, α. The results from HRR-J-$\Delta\sigma$ technique is shown along the crack tip, the hoop and radial stresses vary along the crack tip between $0 \leq \alpha \leq 1$ and in the angular range $0° \leq \theta \leq 180°$. A series of analytical expression have been developed to characterize 3-D elastic-plastic crack-tip fields. This will allow researcher to be able to map crack-tip stress field in structures and allow a direct assessed of structural integrity in an efficient and quick approach.

CHAPTER 1

INTRODUCTION

1.1 Overview

The advancement of technology has improved the quality of human life in many fields, such as transportation and building, but it has also raised the possibility of structural failure. Fortunately, developments in fracture mechanics have reduced some of the risks that could arise from growing technological complexity. Almost all engineered design structure, including buildings, ships, and gears, ought to be designated for safety and functionality. A failure in even the most insignificant of the system's components might have catastrophic results. There are numerous ways in which an engineering structure can fail in service, and an engineer's task is to recognise these possibilities and ensure to prevent it. Additionally, the study estimated that existing technology and fracture mechanics research could minimise loss.

The mechanics of fracture presented using mathematical analysis to show the condition prevailing near a crack tip. Consider an in-plane loaded cracked plate of thickness B. The plate would be under plane stress if there were no crack. The amount of triaxiality is high inside the plate, as seen in Figure 1.1 [1].

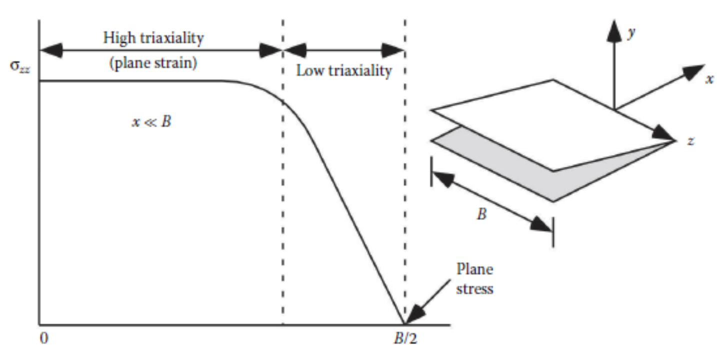

Figure 1.1. Stress and strain variation through the thickness

At distances from the crack tip that are close to the plate thickness, the stress condition in this centre area is plane strain. The stress triaxiality is lower at the area close to the free surface; however, the free surface is the only location where a situation of pure plane stress may occur. The plastic zone shape and size from Von Mises yield criterion as shown Figure 1.2 [2] and Figure 1.3 [3]. The ratio of plastic zone to thickness is a crucial coefficient for determining the stress state at the crack tip. As the fracture tip stress field changes from a plane stress state to a plane strain state, the size of the plastic zone gradually decreases from the specimen's surface to its centre. The overall plastic zone of the specimen is smaller than the surface plastic zone radius.

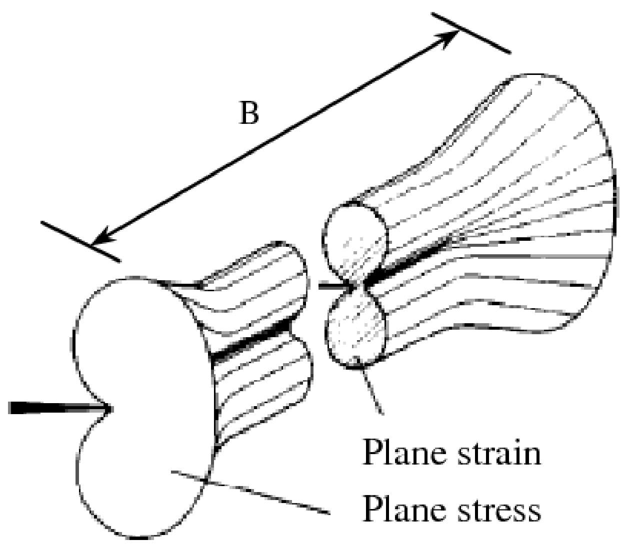

Figure 1.2 Schematic representation of three-dimensional of plastic zone.

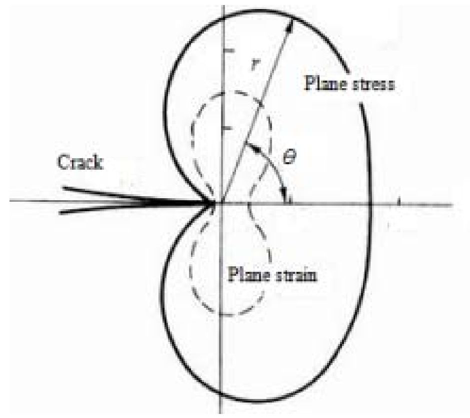

Figure 1.3 Plastic zone size

The field of fracture mechanics provides method for evaluating the severity of structural defects. The method of analysis on elastic fracture mechanics understanding have been develop much more compared to elastic-plastic and fully plastic analysis. When it was found that the material with high toughness could not be characterized by LEFM, Wells[4] attempted to analyse elastic-plastic fracture mechanic. The researcher measures the fracture toughness by the crack faces displacement, δ, as illustrates in Figure 1.4[1].

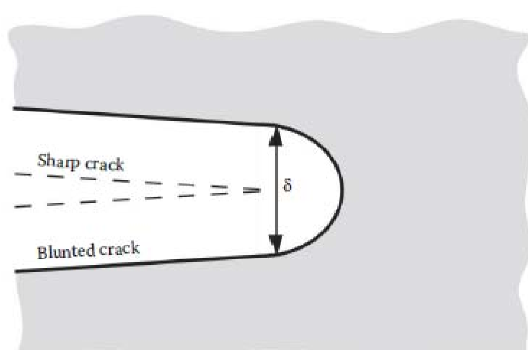

Figure 1.4 Crack tip opening displacement (CTOD).

Rice [5] expand the energy release rate, G, that was previously limited to linear elastic fracture mechanics to J, a fracture characterization parameter for nonlinear materials. CTOD is a measure of fracture toughness that takes plastic deformation that blunted the initial crack tip into consideration. While J is determinable as an energy parameter and predict the stress state as described by Hutchinson [6], Rice and Rosengren [7]. The J contour integral has been widely used to characterise fracture toughness of metal.

When there is sufficient plasticity or significant crack expansion, the single parameter (K, J, CTOD) in fracture mechanics analysis is invalid. Near tip fields vary on configuration in large plasticity, hence the single parameter assumption is unable to characterize plasticity fully plastic crack problems.

In this thesis, a study is made to develop a method to characterize to three-dimensional elastic-plastic crack tip fields using a new three-dimensional method proposed by Yusof (2019)[8].

1.2 Project Background

Rice[5] introduced the path-independent integral, J, surrounding the crack tip in a two-dimensional field of elastic-plastic material. Following this, Hutchinson[6], Rice and Rosengren[7] have shown that the stress and strain crack-tip field for the elastic-plastic material known as the HRR field is governed by J. The J-integral assessment of near-tip behaviours under large-scale and moderate yielding is the restricted condition.

Sham and Hancock[9] conducted research on asymptotic stress field in mode I stress field. The plane stress fields are investigated for the stationary crack in an elastic-plastic solid. The plane stress differ from the Hutchinson field as the complete plasticity assumption is to be relaxed [10]. Two sections of the elastic-plastic crack tip fields were discussed. Rice[11] analysed the plastic sector, while Sham et al[12]. analysed the elastic sector. The SH fields produce a crack tip sector, assuming that the sector consists of a centred fan leading the crack tip and an elastic sector afterward.

Nevertheless, the applicability of the J-integral is further extended by the development of two-parameter fracture mechanics, the K/J-T[13], J-Q[14], and J-A_2[15]. These are then extended to describe the crack tip in three-dimensional to correlate with real-life fracture mechanics at matter to offer a complete solution. The characterization of three-dimension loss of constraint has been proposed such as J-T_Z, J-$\emptyset$, J-A_P, and J-T/Q [16]–[19] but these area also limited to T/Q>0.

Recent development by Yusof (2019)[8] of three-dimensional elastic-plastic finite element (FE) has been able to characterize three-dimensional crack problems for T/Q<0 and T/Q>0. In this present work, an approach was proposed to characterize three-dimensional elastic-plastic crack-tip field through modification of HRR- J-$\Delta\sigma$-SH approach.

1.3 Problem Statement

Analytical mathematical solutions to express three-dimensional elastic plastic crack-tip fields is a current knowledge gap in the study of elastic-plastic crack-tip stresses. Using finite element (FE) is for three-dimensional stress analysis is not efficient and can give inaccurate results depending on the sensitivity of the formulation. Therefore, on analytical solution is essential to overcome the inconsistency issue from finite element solutions.

1.4 Objectives

1.4.1 Objectives 1

To characterizes three-dimensional elastic-plastic stress fields surrounding the crack tip using HRR, SH, Prandtl and J-$\Delta\sigma$ analytical solutions.

1.4.2 Objectives 2

To propose a solution that shows the relationship between two-dimensional characterizing parameter of HRR-J-$\Delta\sigma$-SH approach.

1.5 Scope Of Work

This research is to construct series a analytical solution from the HRR expressions, J-$\Delta\sigma$ expressions and SH expressions to develop an asymptotic elastic-plastic crack-tip analytical solutions to describe three-dimensional elastic-plastic crack tip stress fields under contained yielding using Microsoft excel.

1.6 Thesis Outline

The thesis contains five chapters included Introduction, Literature Review, Methodology, Results and Discussion, and Conclusion and Future Recommendation.

The introduction included the information of the fracture mechanics, the brief explanation on the project, the problem of the investigation, the project's objectives to achieve and briefly explain the method of research.

Literature review briefly describe the previous works of researchers in determining the possibility of developing crack tip constraints. The chapter shows the analytical solution according to the respective researchers.

The methodology describes the steps of developing the three-dimensional intermediate fields and the constraints loss. The analytical method and the calculation are showed.

The results and discussion show the outcome of the study. The comparison of the result from analytical solution was compared with the finite element solution.

The conclusion is to conclude the finding of this research and the future recommendations are made for upcoming research and development projects based on the report's findings.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

Constraints of crack tip is important because it affects fracture toughness and defect tolerance. A review of HRR fields, SH, Prandtl, J-$\Delta\sigma$ approaches is given.

2.2 Hutchinson-Rice-Rosengren fields

Hutchinson, Rice and Rosengren[6], [7] presented crack tip stress fields for both plane stress-strain that emphasizes the crack tip singularity. Stress-strain relationships are combined with the plasticity theory. The topic of both papers is the plastic deformation at the crack tip in a two-dimensional with uniaxial tension from the crack. Elastic fields can continue the solution past the singular terms' elastic-plastic boundary and meet asymptotic boundary requirements. The limiting condition is where the plastic strains are greater than the elastic strains up until the near crack tip zone.

Because elastic strains are insignificant in comparison to plastic strains in the immediate region of the crack tip, only the plastic part of the stress-strains relation enters the asymptotic analysis of near-crack tip fields. Given the Ramberg-Osgood [20] relation, taking the total strain experience by a material under deformation where ε_e= recoverable elastic strain and ε_p is permanent strain:

$$\varepsilon_T = \varepsilon_e + \varepsilon_p \tag{1}$$

$$\varepsilon_T = \frac{\sigma}{E} + \alpha\left(\frac{\sigma}{\sigma_o}\right)^n \tag{2}$$

Where σ is the value of stress, E is Young's Modulus, α is a material constant, yield strength is σ_o and the strain hardening exponent as n.

If ε_e is infinitesimal small, so the equation is,

$$\varepsilon_T = \alpha \left(\frac{\sigma}{\sigma_o}\right)^n \tag{3}$$

Hence power-law material response, for which the uniaxial plastic strain ε is proportional to the uniaxial stress σ by,

$$\frac{\varepsilon}{\varepsilon_o} = \alpha \left(\frac{\sigma}{\sigma_o}\right)^n \tag{4}$$

Where σ_o is the yield stress, $\varepsilon_o = \sigma_o / E$ is the yield strain, α is a material constant and the strain hardening exponent, n.

The type of nonlinear elastic-plastic behaviour provided by power-law model the tensile stress-strain relations of specific metals is the first representation to address. Several values of the strain hardening coefficient are shown in Figure 2.1. Under plane strain conditions, the power-law predictions clearly indicate a trend toward substantially higher tensile stresses within the vicinity of the crack tip.

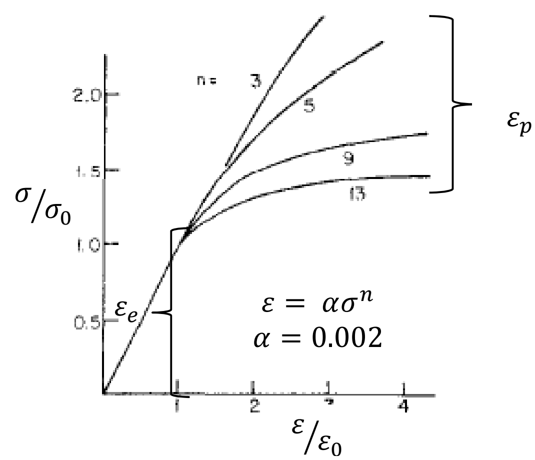

Figure 2.1. Ramberg-Osgood stress-strain relation

To remain path independent, stress-strain must vary as $1/r$ near the crack tip where r is the distance near the crack tip. The plastic zone with distance near to the crack tip, total strain in comparison larger from elastic strain. Ahead from the crack tip, the variation of stress and strain given:

$$\sigma_{ij} = k\left(\frac{J}{r}\right)^{\frac{1}{n+1}} \tag{5}$$

Where k are proportionality constants. For a linear elastic material n=1 and gets $\frac{1}{r}$ singularity while perfectly plastic is n = ∞.

Applying the suitable boundary condition yields the dominating singularity that governs the behaviour at the tip of a line crack. With reference to polar coordinates, r and θ, centered at the crack tip as illustrated in Figure 2.2.

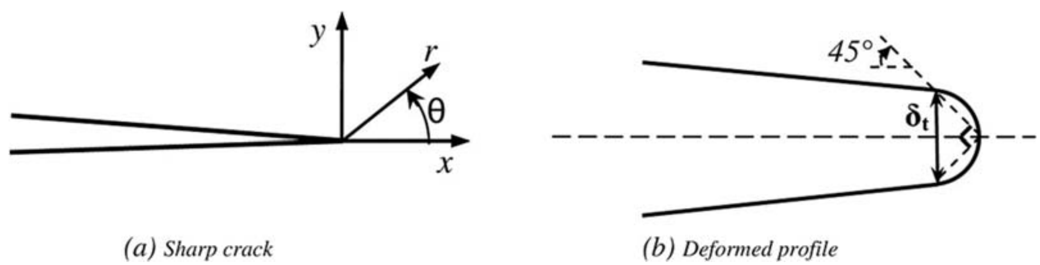

Figure 2.2. Sharp crack and deformed profile for defining crack-tip opening displacement.

$$\sigma_{ij} = \sigma_o\left[\frac{J}{\alpha\sigma_o\varepsilon_o I_n r}\right]^{\frac{1}{n+1}}\tilde{\sigma}_{ij}(\theta,n) \tag{6}$$

$$\varepsilon_{ij} = \alpha\varepsilon_o\left[\frac{J}{\alpha\sigma_o\varepsilon_o I_n r}\right]^{\frac{n}{n+1}}\tilde{\varepsilon}_{ij}(\theta,n) \tag{7}$$

$$u_i = \alpha\varepsilon_o r\left[\frac{J}{\alpha\sigma_o\varepsilon_o I_n r}\right]^{\frac{n}{n+1}}\tilde{u}_i(\theta,n) \tag{8}$$

I_n is an integration constant that depends on n, $\tilde{\sigma}_{IJ}$, $\varepsilon_{ij}$and u_i are dimensionless functions of n and θ, where there are stress, strain, and displacement sequentially. Both values can be obtained from table of HRR singular fields quantities[21] with a strain hardening exponent, n=100. There are dependent of the stress state given plane stress or plane strain.

Figure 2.3(a) shows the asymptotic stresses at the crack tip (r=0) in cartesian co-ordinates. When the radius of the plastic zone is greater five times than the thickness of the plate, the midplane at z/t=0.5 demonstrates that a fully restricted plane-strain field is maintained at the crack tip, the result shows the plane strain in the angular range θ≤45°, the stresses are constant. In the angular range 45° ≤θ≤135°, the $\sigma_{\theta\theta}$ and σ_{rr} decrease linearly with angle while $\sigma_{r\theta}$ remain the same. At 135°≤θ≤180°, the elastic sector appeared.

At the free surface, z/t = 0 the crack tip is as shown in Figure 2.3(b). in angular state $\theta \leq 45°$, the stress variants are independent of angle. At angular range $45° \leq \theta \leq 105°$, the hoop stress, $\sigma_{\theta\theta}$, decrease linearly and the line increase until the $\theta \leq 180°$. Then radial stress, σ_{rr}, decrease from $0° \leq \theta \leq 75°$, and increase when $\theta \geq 75°$. While for shear stress, $\sigma_{r\theta}$, discontinuity developed in the angular range $\theta \geq 150°$, based on the assumption that plasticity surrounds the crack at all angles.

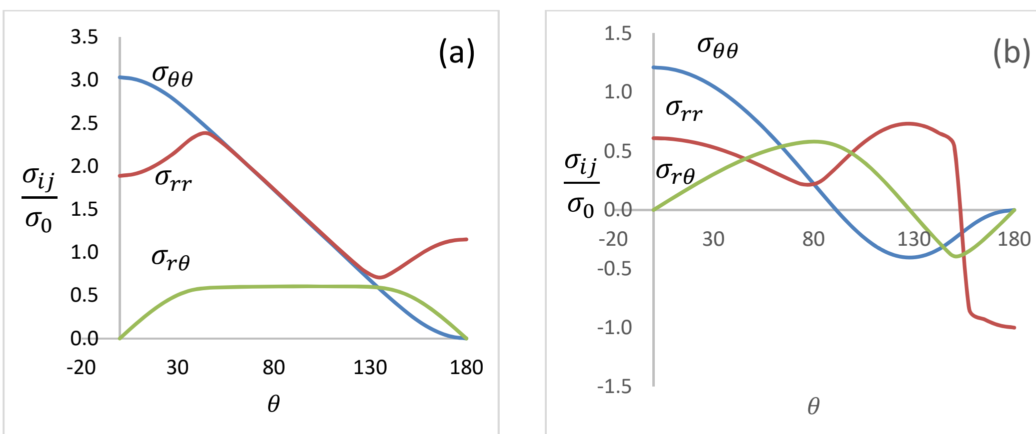

Figure 2.3 The asymptotic cylindrical stresses (r=0) determine from the two-dimensional plane strain (T=0). (a) is the HRR in plane strain and (b) is the HRR in plane stress

2.3 Prandtl Field

Prandtl fields near the tip can be determined and analysed with a theory for plastic fields in completely plastic materials. This foundation allows the stresses to be resolved from the traction-free fracture surface region in Figure 2.4. The yield criterion and the free surface in this location imply a homogeneous tensile field parallel to the crack flanks[22].

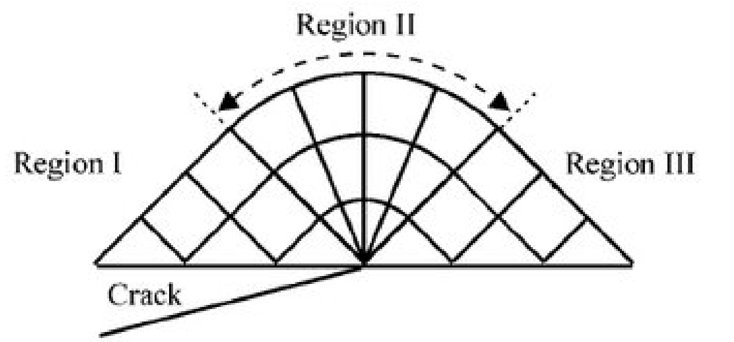

Figure 2.4. Prandtl Field

The stress state region I:

$$\sigma_{\theta\theta} = k(1 - \cos 2\theta) \tag{9}$$

$$\sigma_{rr} = k(1 + \cos 2\theta) \tag{10}$$

$$\sigma_{r\theta} = -k(\sin 2\theta) \tag{11}$$

$$\sigma_m = k \tag{12}$$