CURVE RECONSTRUCTION BY METAHEURISTICS ALGORITHMS ON CUBIC RATIONAL BÉZIER FUNCTION

NAJIHAH BINTI MOHAMED

UNIVERSITI SAINS MALAYSIA

2019

CURVE RECONSTRUCTION BY METAHEURISTICS ALGORITHMS ON CUBIC RATIONAL BÉZIER FUNCTION

by

NAJIHAH BINTI MOHAMED

Thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

November 2019

DEDICATIONS

To my loving and caring mother and late father for their assistance, words and endless prayers for my success without which I would never have been able to stand today.

To my highly motivated husband, who confident and put his trust in me to end this long journey. To dearest daughters Alya Maisarah, Anis Sakinah and son Muhammad Adam Yusoff for their support, patient and prayers, also for colouring my life.

ACKNOWLEDGEMENT

Thanks to Allah the Almighty, with the blessing overflow and award, this thesis was able to be completed.

First and foremost, I would like to offer immeasurable gratitude to my supervisor Dr. Ahmad Lutfi Amri Ramli, my ex-supervisor Associate Professor Dr. Ahmad Abd Majid, and my co-supervisors Professor Dr. Abd Rahni Mt Piah and Professor Dr. Zainul Ahmad Rajion whose knowledge and support over the last seven years have been invaluable. Thank you for always being a constant and calming presence, without which this thesis would not have been possible. My deep gratitude also goes to my mentor, Dr. Zainor Ridzuan Yahya, who opened my eyes on how this journey should be started.

To all staffs of the school, students and friends of the department, my times spent here have been pleasant and I thank everyone for their help and warmth during that time. Particular thanks also go to *Jasmida, Dani, Izzati, Fatihah, Nurul, Sufiah* and *Syaza*.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

CAD	Computer-Aided Design
CAGD	Computer-Aided Geometric Design
CT	Computed Tomography
DICOM	Digital Imaging and Communication in Medicine
DC	Decompressive Craniectomy
GA	Genetic Algorithm
GHS	Global-best Harmony Search
HA	Hydroxyapatite
HM	Harmony Memory
HMCR	Harmony Memory Considering Rate
HMS	Harmony Memory Size
HS	Harmony Search
ICHT	Intracranial Hypertension
ICP	Intracranial Pressure
L-BFGS	Limited-memory Broyden-Fletcher-Goldfarb-Shanno
MHS	Modified Harmony Search
MRI	Magnetic Resonance Imaging
NI	Number of Improvisation
NP-hard	Non-Deterministic Polynomial-time
NURBS	Non-Uniform Rational B-Spline
PAR	Pitch Adjusting Rate
PE	Polyethylene
PEEK	Polyetheretherketone

PMMA	Polymethylmethacrylate
PSO	Particle Swarm Optimisation
2D	Two Dimensional
3D	Three Dimensional

LIST OF SYMBOLS

$\mathbf{R}_i$	Data points
$N+1$	Total number of data points
$\mathbf{P}_i$	Data points built from parametric equations of fitting function
E	Total error of the least-squared error
$\mathbf{C}_i$	Control points for curve, $\mathbf{C}_0, \mathbf{C}_1, \mathbf{C}_2, \cdots$ and $\mathbf{C}_n$
$n+1$	Total number of control points of Bézier curve
$\mathbf{P}(t)$	Bézier curve function
$B_i^n(t)$	Basis functions of Bézier or Bernstein polynomials *n* degrees
$m+1$	Total number of control points of rational Bézier curve
$\mathbf{Q}(t)$	Rational Bézier curve function
h_i and w_i	Weight for rational Bézier curve function
x_i	Decision variables
N_{DV}	Total number of decision variable
X	Set of the possible range of values for each decision variable
$L\,x_i$	Lower bound for each decision variable
$U\,x_i$	Upper bound for each decision variable
x'	A new harmony vector, $\left(x_1', x_2', \cdots, x_{N_{DV}}'\right)$ in HS
$u(0,1)$	Random number in the interval [0,1]
r	Random number in the interval [0,1]
bw	Arbitrary distance bandwidth
$\vec{x}$	Position vectors in PSO

$\vec{v}$	Velocity vectors in PSO
N_{PSO}	Neighbourhood relation in PSO's swarm
$pBest$	Personal best position in PSO
$gBest$	Global best position in the entire PSO's swarm
ω	Inertial weight factor in PSO
φ_1	Random number in the interval $[0,1]$ to determine $pBest$
φ_2	Random number in the interval $[0,1]$ to determine $gBest$
C_{PSO1}	Constant multiplier terms or "*self-confidence*"
C_{PSO2}	Constant multiplier terms or "*swarm confidence*"
D_l	Temporary harmony vectors in MHS
α	Random number in the interval $[0,1]$
s_i	Parameter of data position after reparameterisation
$\mathbf{A}_1'$	Initial guesses of control point $\mathbf{C}_1$
$\mathbf{A}_2'$	Initial guesses of control point $\mathbf{C}_2$
α_{size}	Value to control the size of search spaces
$\mathbf{R}_{mid}$	Middle points in the set of points $\mathbf{R}_i$
$\mathbf{R}_{Q1}$	Last data in the first quarter in the set of points $\mathbf{R}_i$
$\mathbf{R}_{Q3}$	Last data in the third quarter in the set of points $\mathbf{R}_i$
pt	Value in the interval of $[0,1]$, to project the position of $\mathbf{A}_1'$ and $\mathbf{A}_2'$
β	Parameter for Equation (5.13)
$\mathbf{C}_i'$	Control points for second segment with C^1 continuity

$\mathbf{CP}_i$	Control points for second segment with G^1 continuity
T	Parameter to locate new position of $\mathbf{CP}_1$
A and **B**	Points selected by user clicked on MATLAB figure
C and **D**	Reflected points by mirror technique
$\mathbf{M}_x$	Middle vertical line
$PO(u)$	Offset curve
$C(u)$	Design curve
$Dis(u)$	Offset distance
M	Middle point
$\mathbf{D}'$ and $\mathbf{A}'$	Offset points
t_D	Distance involving **D** and $\mathbf{D}'$
t_A	Distance involving **A** and $\mathbf{A}'$

PEMBINAAN SEMULA LENGKUNG OLEH ALGORITMA METAHEURISTIK DENGAN MENGGUNAKAN FUNGSI NISBAH KUBIK BÉZIER

ABSTRAK

Pembinaan semula lengkung kerap digunakan dalam kejuruteraan berbalik. Sementara itu, penyesuaian lengkung merupakan salah satu komposisi utama bagi pembinaan semula lengkung yang biasanya diwakili oleh fungsi matematik, sesuai untuk mewakili satu set titik data, dan perlu memenuhi beberapa kekangan. Pelbagai kajian penyesuaian lengkung telah dilakukan oleh ramai penyelidik secara khusus menggunakan teknik pengoptimuman. Pengoptimuman terdiri daripada algoritma tepat, dan algoritma anggaran. Algoritma anggaran adalah teknik yang baik untuk diketengahkan kerana kaedah penyesuaian lengkung akan lebih ringkas dan mudah, menjimatkan pengiraan yang berat, boleh menyelesaikan masalah berskala besar dan menghasilkan hasil akhir dengan kualiti yang lebih baik.. Metaheuristik memiliki mekanisme yang kuat dan cerdas untuk menghindari terjebak dalam minimum tempatan. Konsep metaheuristik adalah cukup sederhana; tidak memerlukan maklumat kecerunan dan boleh digunakan dalam berbagai masalah merentasi pelbagai disiplin ilmu. Metaheuristik yang dibincangkan dalam tesis ini adalah Carian Harmoni, Algoritma Genetik dan Pengoptimuman Sekawan Partikel. Bagi memeriahkan bidang metaheuristik, kaedah metaheuristik baru dicadangkan iaitu Carian Harmoni Diubahsuai yang meminjam mekanisme dari Algoritma Genetik dan Pengoptimuman Sekawan Partikel lalu dimasukkan ke dalam Carian Harmoni klasik dalam menghasilkan vektor penyelesaian baharu yang meningkatkan kadar konvergensi dan ketepatan algoritma. Fungsi Bézier yang wajar dan tidak wajar digunakan sebagai fungsi penyesuaian, dan skema penyesuaian lengkung baru ini juga mengusulkan teknik penentuan ruang carian pemboleh ubah keputusan tertentu. Bagi menjaga kelancaran keluk gabungan fungsi Bézier, syarat kesinambungan C^1 dan G^1 diterapkan. Gabungan kesinambungan ini dikekalkan dalam skema melalui penerapan teknik geometri. Skema ini diuji pada beberapa imej dan prosedur perbandingan dilakukan antara algoritma dan kekangan tertentu. Akhirnya, skema ini digunakan dalam pembinaan implan tengkorak di bahagian yang cacat dalam kepingan tengkorak imej kontur, langkah pengimbangan juga diusulkan bagi menyelesaikan proses.

CURVE RECONSTRUCTION BY METAHEURISTICS ALGORITHMS ON CUBIC RATIONAL BÉZIER FUNCTION

ABSTRACT

Curve reconstruction regularly used in reverse engineering. Meanwhile, curve fitting is one of the main compositions of curve reconstruction that is usually represented by mathematical functions, most suitable for representing a set of data points, and may need to meet some constraints. Various of curve fitting studies had been done by many researchers specifically using optimisation technique. The optimisation technique consists of exact algorithm, and approximate algorithm. The approximate algorithm is a good technique to be highlighted since it is a feasible way to develop an easier, more convenient curve fitting method, that will save great computation, solve a large scale problem and produce a better quality end result. Metaheuristics has strong and intelligent mechanisms to avoid being trapped in the local minimum. Metaheuristics depends on a fairly simple concept; does not require gradient information and can be used in a variety of problems covering different disciplines. The metaheuristics discussed in this thesis are Harmony Search, Genetic Algorithm and Particle Swarm Optimisation. To enliven the metaheuristics field, a new metaheuristics method is proposed namely the Modified Harmony Search which borrows its mechanism from the Genetic Algorithm and the Particle Swarm Optimisation incorporated into the classical Harmony Search in generating new solution vectors that enhance the convergence rate and the accuracy of Harmony Search algorithm. Rational and non-rational Bézier functions were used as the fitting functions, and this new curve fitting scheme also proposes the technique of setting up the search spaces of certain decision variables. In order to take care of the smoothness

of the joining curves of Bézier functions, C^1 and G^1 continuity conditions were applied. These continuities joint were maintained in the scheme through the application of the geometrical technique. This scheme was tested on several images and comparison procedures were performed between the selected algorithms and constraints. Finally, this scheme was applied in the construction of cranial implant for defected part in contour image skull slices, with proposed offsetting step to complete the process.

CHAPTER 1

INTRODUCTION

In this thesis, curve fitting is dealt in the sense of curve reconstruction, regularly used in reverse engineering which is also a branch of geometric modeling. In order to reconstruct curves, rational and non-rational Bézier functions through optimisation method were used. Optimisation method is an alternative method that can be used to solve a problem with difficulties in getting the exact solution. Even though optimisation produces an optimal solution, utilisation of a good mechanism will enhance the quality of the end result, reducing the computational time and can solve large scale problems. Optimisation methods especially metaheuristics are increasingly accepted as an option to solve complex calculation including curve or surface reconstruction in geometric modeling. Meanwhile, the fitting function is chosen among quadratic Bézier, cubic Bézier, rational quadratic Bézier and rational cubic Bézier functions which will be compared based on the performance of the minimum value of the total least-squares error.

Metaheuristics algorithm is an approximate algorithm which efficiently explore the search space in order to find near-optimal solutions. This algorithm has strategies or mechanisms that guide the search process. Metaheuristics do not only provide a good solution but is also computationally efficient. The goal of metaheuristics is to get the most suitable solution to the problem involving the fitness function or objective function that represents a particular problem, and the objective function used in this thesis is the total least-squares error. The metaheuristics discussed in this thesis are Harmony Search (HS), Genetic Algorithm (GA) and Particle Swarm Optimisation (PSO). A metaheuristics method developed namely the Modified Harmony Search (MHS) borrows some elements from GA and PSO and transferred them into HS.

1.1 Data Fitting in Geometric Modeling

Geometric modeling is also a process of constructing a complete mathematical description to model a physical entity or system. Data fitting is one of the main composition of curve reconstruction. Data fitting is a fundamental tool in scientific research and engineering applications, where it contains the element of geometric modeling and reverse engineering. As the amount of data to be operated at this time is increasingly large, efficient data fitting methods are needed in order for the current computer hardware to be able to handle large scale of data. Therefore, the best way to handle this problem is to build simple and convenient data fitting methods, which also deal with data points with fitting accuracy. In this way, the appropriate data fitting method can save heavy and complex calculations, as well as fitting more data points.

1.1.1 Curve Fitting

Curve fitting is a process of constructing a curve by interpolating or approximating a set of data points to an appropriate and suitable mathematical function, which may need to meet certain constraints. Interpolation should accurately fit all of the data, meanwhile approximation is where a function is constructed to approximately fit the data. Approximation and interpolation are considered as entirely different problems, as approximation pass near the given points while interpolation pass through the given points. Approximation of curves by parametric rational curves needs a linkage between the curve points and the parameter values. The approximation quality is highly dependent on the reparameterisation phase. This quality criterion can be measured as the curve is given by dense or continuous sets of points, an optimal parameterisation can thus be defined as a minimisation of the distance between the given curve and the approximation function. The quality of an approximation by a parametric curve, exhibits a very high dependency on the parameterisation law (Bouras, et al., 1918). Curve approximation and evaluation of fit is a common task in many CAD environment (Wolters and Farin, 1997).

The idea of curve fitting is the formulation of a least-squares problem as the data points on the curve are fixed. However, the result of curve fitting is not always encouraging. According to Kang, et al. (2015), this situation happened due to two reasons. Firstly, it is not easy to derive both the locations and distributions of analytic expressions and the general characteristics of optimal data points. Second, it is difficult to compute the unknown number and position of data points resulting in a large and nonlinear optimisation problems. To accommodate this weakness, metaheuristics can be a technique to be featured.

The rational and non-rational Bézier were considered in this work as the fitting function as it requires less computation compared to the B-spline function. Bézier curve can be straightforward and encouraging to implement. Then, the quadratic and cubic of rational and non-rational Bézier curve were chosen as the higher degree is difficult to handle as it tends not to meet the process of approximating the shape of the curve and it has more parameters. The shape of curve using non-rational Bézier curves are determined by only the control points, while rational Bézier curves can be more flexible by adding more parameters called the weight factor or shape parameter besides their control points. In order to make it easier for implementation, continuity conditions C^1 and G^1 are also considered. Both rational and non-rational Bézier curves are widely used in the fields of CAD and computer aided geometric design (CAGD), as the function's presentation is meaningful in geometry yet simple, and easily can change shape just by moving control points or changing weighting values.

1.2 Optimisation Algorithms

Curve fitting technique consists of mathematical functions with some number of parameters, and these model parameters values are optimised in such a way that it approximates the data points on the measured curve as much as possible. Normally, the mathematical function is an error function to be minimised. Another name for this error function is objective function and cost function. Each curve has visuals such as peaks and valleys. Each of these peaks or valleys, are usually represented by several parameters that specify their position and shape. Therefore, the problem becomes a multi-dimensional optimisation problem.

The optimisation algorithm can roughly be broken down into two categories, the exact algorithm and the approximate algorithm. Exact algorithm guarantees that the optimum solutions will be obtained within a limited time. However, for highly complex and difficult optimisation problems for instance Non-Deterministic Polynomial-time (NP-hard) or global optimisation, the exact algorithm is less suitable.

1.2.1 Exact Algorithm

Exact algorithm also known as the deterministic method, normally, requires derivative information of the function, which are first order and/or second order derivatives. A few examples of exact algorithms are Levenberg-Marquardt algorithm, Berndt-Hall-Hall-Hausman algorithm, Nonlinear conjugate gradient method, Broyden-Fletcher-Goldfarb-Shanno algorithm and Newton's method.

1.2.2 Approximate Algorithm

Approximate algorithm is also known as the non-deterministic method, where heuristic algorithm is the example for this category. However, heuristic algorithm does not have the assurance that optimal solutions can be provided, with probabilities of returning solutions that are worse than the optimal, as they may get stuck in a local minimum. Examples of heuristic algorithm are greedy algorithm, Newell and Simon and alpha-beta pruning. Meanwhile, metaheuristics is a problem solving algorithm with the high level framework that provides a set of guidelines or strategies in developing a heuristic optimisation algorithm. In other words, metaheuristics is a great and powerful heuristic as it possesses the elements and the mechanisms to avoid getting stuck in the local minimum. This is because, metaheuristics has the guiding powers over the search space in exploiting the best ability to achieve a better solution. Metaheuristics optimisation algorithms are becoming increasingly popular in engineering applications due to the dependency on a fairly simple and easy-to-understand concept; do not require gradient information; can overcome local optimum problems; and can be used in a variety of problems covering different disciplines. (Mirjalili and Lewis, 2016).

1.3 Background of Study

Suppose a set of data points, $\mathbf{R}_i$, $i = 0,1,\ldots,N$, were given where $\mathbf{R}_i$ are data points on the curve and $\mathbf{R}_i \in \mathbb{R}^2$, while N is the total number of the data points. The objective function involved here is based on the total error of the least-squared error between $\mathbf{R}_i$ and $\mathbf{P}_i$ denoted as E that has to be minimum

$$E = \sum_{i=0}^{N} \left| \mathbf{R}_i - \mathbf{P}_i \right|^2 . \tag{1.1}$$

$\mathbf{P}_i$ are data points built from the parametric equations of a fitting function. The approach is to assume that E depends on some parameters that need to be calculated. The procedure is to find the parameters values minimising error function, E, where E underlying mathematical function represents total distance between the points generated by the function, $\mathbf{P}_i$ and corresponding points, $\mathbf{R}_i$.

1.4 Problem Statement

The statements of problem are :

1. Metaheuristics is a problem solving technique with high level framework that provides a set of strategies with rooms for improvement.
2. The amount of data to be operated in data fitting can be large and is hard to handle. Furthermore, the process becomes harder if smoothness of the model

is also considered as the main issue. The result of curve fitting is not always accurate due to the unknown general characteristics of the optimal data points.

3. Cranial implant construction is currently being commercialized by some companies for their business and profits.
4. The use of expensive and exclusive software by commercial companies in the redesign of skull contour images.

1.5 Objectives of Study

This thesis will explore the performance of optimisation method specifically metaheuristics algorithms and its implementation in the curve fitting problem. The objectives of this thesis are :

1. To develop a better metaheuristics algorithm with a good performance in terms of speed, successive behaviour and scalability study.
2. To develop a better scheme in curve fitting problem by applying metaheuristics algorithms
3. To apply the curve fitting scheme in the construction of cranial implant required part in three dimensional form.
4. To develop suitable concept of an offsetting curve in order to complete the process of redesigning the contour image of skull slice.

1.6 Research Methodology

The methodologies of this thesis are :

1. In order to develop better metaheuristics, an algorithm called the Modified Harmony Search (MHS) will be proposed. MHS is an improvement to the classical HS by employing a concept from GA and PSO.
2. In order to prove the good performance in terms of speed, successive behaviour and scalability study, several algorithms are chosen to test on ten benchmark functions. A complete observation and statistical analysis will be conducted.
3. In order to develop a better scheme in curve fitting problem, metaheuristics algorithms will be applied in reconstructing curves or approximating curves. Suitable steps and techniques will be provided to the scheme. Several algorithms are chosen to test on four outline boundary images. A complete observation and statistical analysis will be conducted.
4. In order to apply the curve fitting scheme in the construction of cranial implant, suitable steps and techniques will be provided to the scheme.
5. In order to complete the process of redesigning the contour image of skull slice, concept of an offsetting curve will be improvised and proposed.

1.7 Outline of the Thesis

This thesis is organised as follows: Literature review in Chapter 2. The background of Bézier and rational Bézier curve functions together with the properties and the continuity conditions are explained in details in Chapter 3. Chapter 4 contains all details of the selected metaheuristics algorithms such as GA, PSO and HS. A better metaheuristics algorithm namely the MHS is discussed in details in this chapter together with its characteristics, consecutive behaviour, scalability study and empirical study on the best parameters setting toward benchmark functions.

Development of better curve fitting scheme using selected metaheuristics algorithms which are GA, PSO, HS and MHS is explained in details in Chapter 5. All the basic elements such as parameterisation, least-squares error and objective function are also explained in this chapter. The performance and comparative studies of all algorithms that have been done based on different type of continuity conditions are also included. Meanwhile Chapter 6 explains the anatomy of cranioplasty and decompressive craniectomy. The application of curve fitting scheme in the design of the cranial implant and the validation study carried out, where a schematic review is made on the perfect data are also included. This thesis is concluded in Chapter 7 which describes the contribution, limitation and future works of this research.

CHAPTER 2

LITERATURE REVIEW

Generally, geometric modeling is one of the frequently used and popular in the field of engineering research and problem solving. It is one of the core solution in many problems in the area of graphics design, computer-aided design (CAD), scientific analysis and engineering designs. Looking into the view of computational geometry and topology, reconstruction will become a routine matter and will be greatly needed. Based on the available data, either curve or point and structure or surface, even the formation of structure, the reconstruction of mathematical model is a geometric modeling problem. In addition to that, many approaches and methods were developed for the purpose of multiple formations reconstructions over time as the subjects are viewed from different perspectives (C. J. Li, et al., 2018; Wu, et al., 2018; Hong, et al., 2018; Atmosudiro, et al., 2016; Pérez-Arribas, 2014).

2.1 Curve Fitting and The Applications

Curve fitting has many applications in geometric modeling, such as image processing, computer graphics, shape modeling, pattern recognition, data mining, statistical data analysis and many other industrial applications (X. Yang, 2004) as well as in manufacturing of ship hulls, car bodies, air fuselage and other free-form object (Gálvez, et al., 2007).

Recently, the curve fitting technique can be seen implemented in various fields and areas. Y. Zheng, et al. (2018) used curve fitting technique to measure peak areas by estimate the accurate parameters in mathematical models of spectral peaks in the problem of flight time of the secondary ion mass spectrometry spectra. Liu, et al. (2018) formally analysed the parameter selection process in the problem of cyberspace faces. C. Wang, et al. (2018) used curve fitting technique in longitudinal-submerged arc-welded pipes problem. J. Wang and Feng (2016) used curve fitting technique in the projections of fossil fuels. Niu, et al. (2016) used curve fitting technique for lane detection. Chen, et al. (2015) used curve fitting technique in data of oil storage tanks. Majeed, et al. (2017) used curve fitting technique in the reconstruction of multiple bones fracture. Ferreiro, Rabaçal, and Costa (2016) estimated the kinetic parameters of agricultural residues pyrolysis using the fitting procedure. Zhuang, et al. (2018) used the curve reconstruction technique in soft robotics. Goshtasby and Shyu (1995) used curve fitting technique in the problem of edge detection. Z. Yang, et al. (2015) used curve fitting technique in computer numerical control machining problem for data compression and smooth tool path generation, where quadratic B-spline as the fitting curve. Kim, et al. (2016) used curve fitting technique in vortex filament motion problem in dynamic modeling and analysis, by making the B-spline and Bézier curves, as well as Lagrange interpolating polynomials as the tools. Biswas, Ghoshal and Hazra (2016) used curve fitting technique for image segmentation in image analysis. Meanwhile in medical area, Andronikou, Arthur, and Rees (2018) used curve reconstruction technique in demonstrating of multifocal metastatic lung nodules image in medical imaging. And Galaz and Acevedo (2017) used curve reconstruction technique in poroelastography imaging in medical imaging. Hamidi, Ghassemian and Imani (2018) used curve fitting to get the information available in the sequence of heart sound signal in the problem of cardiovascular disease. Meanwhile, Jiang, Zhang and Lu (2018) used curve fitting technique on radial artery pulse diagnosis. The element of waveform based on discrete Fourier series were proposed, where the coefficients of discrete Fourier series were being optimised.

2.2 Bézier Function as Fitting Function

Many researchers used Bézier curves in their works, such as Gálvez, et al. (2007), Hasegawa, Rosso and Tsuzuki (2013), Słapek and Paszkiel (2017) and Brakhage (2018). Meanwhile, the idea of rational Bézier curves usage as the fitting curve is coming from Mineur, et al. (1998) where Bézier curves was chosen in the work due to the control of polygon that is constrained to give the desired curve shape. A curve approximation method in building feature-based CAD models from three dimensional (3D) point clouds presented by Stamati and Fudos (2008), where a set of piecewise cubic rational Bézier curves was used with G^1 joint to approximate the boundaries of the features. Lu (2011) presented the polynomial approximation of rational Bézier curves by an iteration method. Other than that, to obtain a smooth curve fitting, two types of continuity of the parametric curve, which are parametric continuity and geometric continuity are highlighted.

2.3 Curve Fitting using Optimisation Algorithms

As optimisation can be divided into two categories, past researches works on curve reconstructing in the sense of geometric modeling using both method categories were identified.

2.3.1 Curve Fitting by Exact Algorithm

Borges and Pastva (2002) implemented curve fitting through the application of the Gauss-Newton method to evaluate the Jacobian on implicit differentiation of a pseudo-inverse. A new curve and surface fitting method was proposed by C. J. Li, et al. (2018), where they adopted the diagonalisable differential systems with variable coefficients based on the homogeneous linear differential systems. X. Yang (2018) proposed fitting and fairing techniques by matrix weighted Non-Uniform Rational B-Spline (NURBS) curves and using the normal or tangents on Hermite type data. M. M. Li and Verma (2016) used Radial Basis Function to fit stopping power data by an additional linear neuron in neural network. Kang, et al. (2015) adapted sparse optimisation model of splines to do curve fitting. In their work, they computed knots numbers and position either to be removed or adjusted, so that the final knot vector was obtained.

Casciola and Romani (2009) represented a fitting procedure by a fast Newton-type algorithm on a piecewise rational Hermite interpolants. W. Zheng, et al. (2012) proposed a method using B-spline curve on the Limited-memory Broyden-Fletcher-Goldfarb-Shanno optimisation technique. This technique is actually coming from the family of quasi-Newton methods used in solving unconstrained nonlinear optimisation problems. An iterative method proposed by Chambelland, Daniel, and Brun (2006), where the researchers used rational pole curve in fitting a set of data points. The objective function is the sum of squared Euclidean norms to has to be minimum in order to obtain the values of weights, control points and the data points.

L. Zhang, Ge and Zheng (2018) investigated a generalised B-splines' geometric fitting method and proposed a set of different weights using the progressive iterative approximation method, while Lu (2011) also used rational Bézier curves in the fitting method. Progressive iterative approximation was also used by Lin (2012), where they developed the adaptive data fitting algorithms. Meanwhile Lee (2000) improved the Moving Least Squares method and also provides detailed literature on this problem domain. However, Gu, et al. (2016) used Moving Total Least Squares instead of Moving Least Squares and claimed that the method was more reasonable in dealing errors in variables model. Saini, Kumar and Gulati (2015) used NURBS on a Levenberg–Marquardt optimisation algorithm. The algorithm is an iterative optimisation technique, which combines the steepest descent and Gauss-Newton methods. Galaz and Acevedo (2017) also used Levenberg–Marquardt optimisation algorithm.

2.3.2 Curve Fitting by Metaheuristics Algorithms

B-spline, NURBS, Bézier and rational Bézier curves and surfaces fitting are among the commonly used methods in the field of computer modelling. Specifically, these methods are conducted using the approximation of data points by optimising the control points, weights, and/or data points or even in phases of reparameterisation. Past researches on curve fitting by metaheuristics algorithms with the fitting curve based on B-spline, NURBS, Bézier and rational Bézier are discussed in this section.

Sevaux and Mineur (2007) developed a curve model using GA and B-spline with specific curvature variations in designing of car bodies. Gálvez, et al. (2007) used two artificial intelligent techniques which were GA and functional networks in Bézier curve and surface fitting. Gálvez and Iglesias (2012) developed B-spline surface reconstruction through an iterative step based on GA. Gálvez and Iglésiais (2013) invented a new iterative hybrid GA and PSO approach for curve fitting on B-spline curves.

Gálvez and Iglésiais (2011) applied PSO algorithm to data fitting problem using B-spline curves, based on the idea of considering the internal knots as free variables of the problem. Gálvez and Iglésiais (2012) improved the same technique in (Gálvez and Iglésiais, 2011) for usage in the clouds data points but this time the researchers used NURBS surface function. Wu, et al. (2018) proposed a fitting algorithm based on the PSO algorithm using ball B-spline curves. While Yahya, Mt Piah and Abd Majid (2012) also used the same algorithm on conic curve.

Uyar and Ülker (2017) optimised the data points by invasive weed optimisation to curve fits by B-spline, where it is a stochastic optimisation algorithm and it was inspired by the behaviour of the weed colonies. Ueda, et al. (2016) curve fitted Bézier curve by minimising the distance between the curve and the given sequence of points using multi objective simulated annealing. Simulated annealing is also a technique used by Sarfraz (2010) over cubic splines on vectorising outlines of generics shapes. Hasegawa, Rosso and Tsuzuki (2013) used Bézier function in curve fitting problem and the technique used is the parallel differential evolution algorithm. Khanna and Rajpal (2015) used fuzzy logic and ant colony optimisation at two different steps in their work.

2.4 Complexity of Algorithms

Minimisation of Equation (1.1) is a nonlinear problem requiring iterative methods. Chernov, Lesort and Simányi (2004) stated that the complexity of this algorithm is $\mathrm{O}(m)+\mathrm{O}(k)$, where $\mathrm{O}(m)$ is the cost of evaluation of number of unknown parameters and $\mathrm{O}(k)$ is the cost of some k iterations spent on the subsequent minimisation of E. Meanwhile, Xie, et al. (2012) stated that a heuristic search was used in their study in a time complexity $\mathrm{O}(p)$ where p is the number of points and the number of data points were the parameters to be optimised.

2.5 Metaheuristics for Optimisation

The metaheuristics algorithms to be discussed in this thesis are HS, PSO and GA. There were several researchers who used these algorithms to solve various problems involving optimisation.

Harmony Search

HS has been used by researchers to solve various optimisation problems, such as load frequency control (Khooban, et al., 2017), wireless sensor networks (Alia, 2017), power system stabilisers (Naresh, Ramalinga Raju and Narasimham, 2016), scheduling algorithm (Dong, Jiao and Wu, 2015) and vehicle routing problem (Taha Yassen, et al., 2015). Meanwhile, some researchers have taken the initiative to extend the HS algorithm to improve the optimisation methods in their fields such as hybridising HS and PSO (Shankar, Shanmugavel and Rajesh, 2016; Ouyang, et al., 2016), hybridising genetic programming and HS (Elola, et al., 2017), proposed global-best harmony (Omran and Mahdavi, 2008), as well as make modifications to the HS by adapting new parameters (Das, et al., 2011).

Genetic Algorithm

GA is seen today as a powerful tool in solving complex optimisation problems in various application areas. Examples include adaptive community detection (Guerrero, et al., 2017), economic load dispatching (Shang, et al., 2017), forensic identification (Bermejo, et al., 2017) and bankruptcy forecasting models (Zelenkov, Fedorova and Chekrizov, 2017). Many researchers were also interested in improving the performances and to overcome certain weaknesses, therefore the GA was either modified (San José-Revuelta, 2007) or combined with other approaches, such as evolutionary (Gong, et al., 1997) and fuzzy-logic (Berlanga, et al., 2010) techniques.

Particle Swarm Optimisation

Some applications of PSO can be seen to even daily distribution of industrial electric loads (Lopes, et al., 2018), to meet railway design criteria and reduce comprehensive cost in mountain railway alignment design problem (Pu, et al., 2019), to apply in the missing value imputation for breast cancer diagnosis (Nekouie and Moattar, 2019), to apply in the capacitated location-routing problem (Peng, Manier, and Manier, 2017) and in university course scheduling problem with selective search (Imran Hossain, et al., 2019). PSO has also been upgraded for quality improvement by many researchers such as, bio-inspired PSO based on prey-predator relationship, which employs the three strategies of catch, escape and breeding (H. Zhang, et al., 2018), update the rule for PSO which is based on normal distribution (Kiran, 2017) and combines strong points of firefly and PSO, a local search strategy proposed by controlling previous global best fitness value (Aydilek, 2018).

2.6 Past Research on Cranial Reconstruction

Xiao, et al. (2012) developed a simple formula for clinicians to estimate the volume of skull defect, based on the post-operative computed tomography (CT) studies. In their study, they collected sample sets of post-operative CT images from patients undergoing craniectomy and they compared the skull defect volume by computer-assisted volumetric analysis and their method, called the **ABC** method, where **A** represents length, **B** as thickness while **C** as the height. To obtain the volume, the product $\mathbf{A}\times\mathbf{B}\times\mathbf{C}$ was calculated. The result from both shows no statistically significant difference between the two.

Hieu, et al. (2003) studied two types of anatomical data, which were solid bone models (STereoLithography files – STL) and bone slice contours (Initial Graphics Exchange Specification – IGES and StrataSys Layer files – SSL). The bone solids and contours were constructed based on CT scanning data. NURBS based method was used to cater bone slice contours data images, where each slice is divided to the outer wall and inner wall. The NURBS used to design the outer wall surface is based on a technique mirrored of intact part.

Gopakumar (2004) proposed the usage of medical modeller software for 3D reconstruction. In order to design the implants, they use the CAD based on mirror imaging techniques, in which a non-defected part is taken from contra-lateral side of the skull. A suitable mirroring axis for the skull was obtained by trial and error.

Meanwhile, Jardini, et al. (2014) designed a customised implant by converting the images in DICOM format into a 3D Virtual model using InVesalius software (CT1-ProMED, Brazil). The software export them in an STL file and made it possible to isolate the bone structure through segmentation for the threshold. The STL file was edited using Magics 15.0 software (Materiise, Belgium) in order to minimise surface imperfections. This treatment enables model softening in the upper skull region that contained more widely spaced slices. Other than that, a few researchers also proposed the reconstruction of cranial implant using various of curve functions such as cubic Bézier, rational cubic Ball, GC^1 cubic Ball, B-spline and NURBS (Rusdi, et al., 2018; Majeed, et al., 2015; Majeed, Yahya, et al., 2015; Majeed, et al., 2018; Majeed, et al., 2017).

CHAPTER 3

RATIONAL AND NON-RATIONAL BÉZIER CURVES, WITH THE C^1 AND G^1 CONTINUITIES

3.1 Bézier Technique

The Bézier technique is an important element in CAGD and 3D modeling as the technique can represent a highly geometric and intuitive form through the sophisticated mathematical concepts. The fundamentals of the Bézier technique is introduced in this chapter through the utilisation of the Bézier curve and the building block of the Bézier technique called the Bernstein polynomials.

Parametric Curves

Parametric curves by G. E. Farin, Hoschek, and Kim (2002) are the primary element in curve modeling and CAGD. An example of simple quadratic function as shown in Figure 3.1 represented in a parametric curve is written as a two dimensional (2D) parametric curve, taking the form of

$$\mathbf{x}(t) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} t \\ 2 - 2t + t^2 \end{bmatrix}, \quad t \in \mathbb{R}. \tag{3.1}$$

G. Farin (2002) describes each coordinate is a function of parameter *t*, and the real line is the domain of the curve. The notation for points and vectors is as follows

$$\mathbf{x}(t) = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{a}_2 t^2, \tag{3.2}$$

where $\mathbf{a}_0 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $\mathbf{a}_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$.

The $\mathbf{a}_i$ refers to the coefficients of the curve and $1, t, t^2$ are the quadratic monomial basis functions.

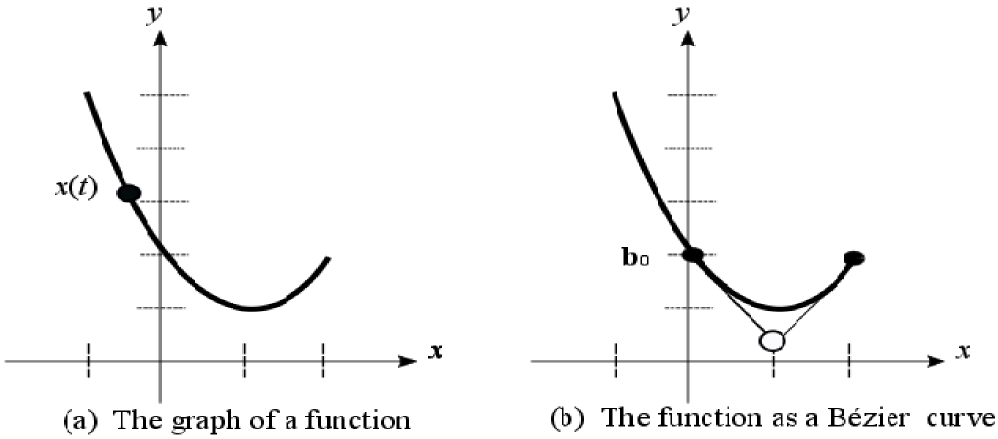

Figure 3.1: Two Different Forms of the Same Graph

However, monomial form does not provide the most geometric intuitive interpretation. Better formulation for the building block of Bézier curve can be achieved using the Bernstein basis functions. Figure 3.1 (b) illustrates the quadratic Bézier form.

3.1.1 Bézier Curves

G. E. Farin, Hoschek, and Kim (2002) defined Bezier curves as :

Given $n+1$, $\mathbf{C}_0, \mathbf{C}_1, \mathbf{C}_2, \cdots$ and $\mathbf{C}_n$ with $\mathbf{C}_i \in \mathbb{R}^2$ or $\mathbb{R}^3$ are called the control points for the curve. Bézier curve can be defined as

$$\mathbf{P}(t) = \sum_{i=0}^{n} B_i^n(t)\mathbf{C}_i, \quad 0 \le t \le 1, \tag{3.3}$$

where functions $B_i^n(t)$ is determined as

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}, \quad 0 \le t \le 1. \tag{3.4}$$

The functions $B_i^n(t)$ for $i = 0,1,\ldots,n$ are referred to as the basis functions of Bézier or Bernstein polynomials n degrees. Binomial coefficients are known as $\binom{n}{i}$ or ${}^nC_i$ and are described as

$$\binom{n}{i} = \begin{cases} \dfrac{n!}{i!(n-i)!}, & 0 \le i \le n \\ 0, & \text{others.} \end{cases} \tag{3.5}$$

The control points $\mathbf{C}_0, \mathbf{C}_1, \mathbf{C}_2, \cdots$ and $\mathbf{C}_n$ will form a control polygon on its curve. The connection of all control points in the order of their numbering with straight lines will form the Bézier control polygon. All points on the Bézier curve lie within the convex hull of the control polygon. Meanwhile, the shape of the curve resembles the shape of the control polygon. The Bézier curve interpolates the first and final data points. The curvature starts at $\mathbf{C}_0$ and ends at $\mathbf{C}_n$. The illustration of control polygon, convex hull and example shape of the curve are shown Figure 3.2 and Figure 3.3.

Properties of Bézier Curves

The essential properties of Bézier curve described by Agoston (2005) are as follows.

1. The Bézier curve passes through a polygon endpoint, indicating the segment curve should start at $\mathbf{C}_0$ and end at $\mathbf{C}_n$, or vice versa.

2. The Bézier curve associated with the control polygon are known as convex hull properties. The important characteristic that ensures the Bézier curve lies within the hull of the curve is

$$B_i^n(t) \geq 0 \text{ and } \sum_{i=0}^{n} B_i^n(t) = 1. \tag{3.6}$$

Figure 3.2 shows the control polygon of the culmination at 10 control points while Figure 3.3 shows the convex hull. Control point $\mathbf{C}_3, \mathbf{C}_4, \mathbf{C}_5$ and $\mathbf{C}_8$ are the interior of the convex hull.

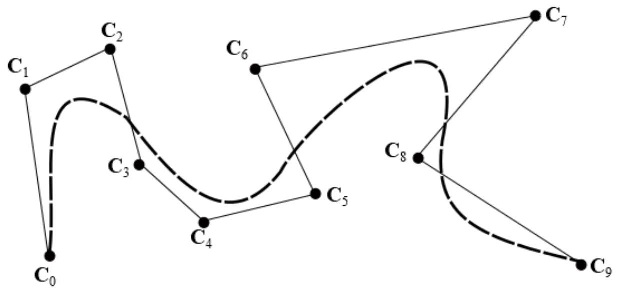

Figure 3.2: Control Polygon for Bézier Curve

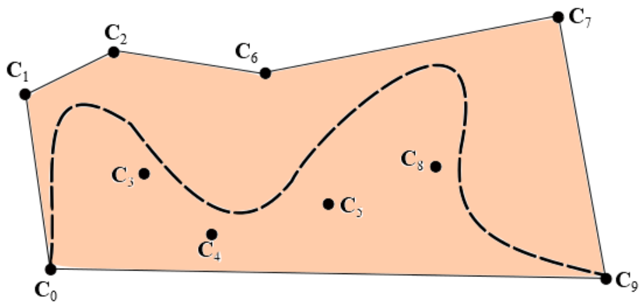

Figure 3.3: Convex Hull for Bézier Curve

3. The Bézier curve also has a property of variation diminishing. The number of intersections of a straight line with a planar Bézier curve is no greater than the number of intersections of the line with the control polygon. Example as in Figure 3.4.

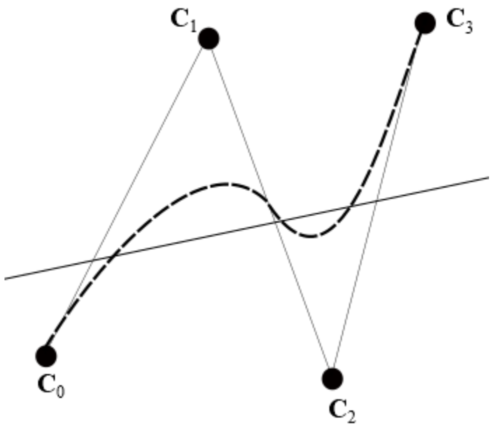

Figure 3.4: Variation of Diminishing

Properties of Bernstein polynomials

The essential properties of Bernstein polynomial by Agoston (2005) are as follows.

1. Bézier curve of degree *n* is determined by $n+1$ control points. In each of the basis functions, exponent *t* is $i+(n-i)=n$. Thus, the curve has the degree of *n*. To generate a Bézier curve, the end points of a curve are the two most important points. According to Equation (3.3) of the Bézier curve, $\mathbf{P}(t)$ interpolates the two end control points, $\mathbf{C}_0$ and $\mathbf{C}_n$. This is determined as $t=0$.

$$B_0^n(0)=\binom{n}{0}0^0(1-0)^{n-0}=1,\quad 0^0=1. \tag{3.7}$$

Then, the $\mathbf{P}(t)$ becomes

$$\mathbf{P}(0)=\sum_{i=0}^{n}B_i^n(0)\mathbf{C}_i=B_0^n(0)\mathbf{C}_0=\mathbf{C}_0, \tag{3.8}$$

and as $t=1$,