UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2003/2004

April 2004

JIM 414 – Pentaabiran Statistik

Masa : 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH DUA** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab SEMUA soalan.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

...2/-

1. (a) $X_1, X_2, \ldots, X_n$ adalah suatu sampel rawak daripada taburan Bernoulli (θ).

Takrifkan $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Buktikan

(i) $\bar{X}_n \xrightarrow{P} p$

(ii) $1 - \bar{X}_n \xrightarrow{P} 1-p$

(50 markah)

(b) (i) $X \sim \chi^2_p$. Tunjukkan X juga tertabur secara $F_{1,p}$.

(ii) $X^2 \sim F_{1,p}$. Tunjukkan X tertabur secara t_p.

(20 markah)

(c) Daripada sampel rawak bersaiz 20 yang diambil daripada taburan normal kita perolehi

$$\sum_{i=1}^{20} x_i = 257.6 \text{ dan } \sum_{i=1}^{20} x_i^2 = 5234.17.$$

Dapatkan selang keyakinan 90% bagi σ^2.

(30 markah)

2. (a) $X_1, \ldots, X_n$ adalah sampel rawak daripada taburan Poisson (θ)

$f(x, \theta) = \frac{e^{-\theta}\theta^x}{x!} I_{\{0,1,2,\ldots\}}(x)$, $\theta > 0$. Katakan $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$.

(i) Tentukan sama ada $S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2$ penganggar saksama ataupun pincang bagi θ.

(ii) Tentukan hal yang sama bagi $\bar{X}_n$.

(iii) Selain daripada memerhatikan kesaksamaan, bagaimanakah kita boleh menilai S_n^2 dan $\bar{X}_n$ sebagai penganggar θ? Huraikan cara penilaian ini.

(50 markah)

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(b) $X_1, \ldots, X_n$ adalah sampel rawak daripada taburan Poisson (θ). Adakah $\sum_{i=1}^{n} X_i$ lengkap? Huraikan.

(20 markah)

(c) $X_1, \ldots, X_n$ adalah sampel rawak daripada taburan $N(\mu, \sigma^2)$. Pertimbangkan statistik $Z = \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}}$.

(i) Sahkan yang statistik ini ialah kuantiti pangsian.

(ii) Apakah yang boleh dibina dengan kuantiti pangsian ini? Huraikan

(30 markah)

3. (a) $X_1, \ldots, X_n$ adalah sampel rawak daripada tabuaran gamma (4, θ). Takrifkan $\bar{X} = \dfrac{1}{n} \sum_{i=1}^{n} X_i$ Terbitkan

(i) selang keyakinan hampiran 95% bagi θ apabila $n = 100$.

(ii) Selang keyakinan 95% bagi θ apabila $n = 5$.

(50 markah)

(b) $H_0 : p = 0.1$ lawan $H_1 : p \neq 0.1$.

$X_1, X_2, \ldots, X_{10}$ adalah sampel rawak daripada taburan Bernoulli (p). Sekiranya $Y = \sum_{i=1}^{10} X_i$ digunakan untuk menentukan rantau genting berbentuk $Y \geq 6$,

(i) nilaikan kebarangkalian ralat Jenis I.

(ii) nilaikan kebarangkalian-kebarangkalian ralat Jenis II jika alternatif-alternatif kepada H_0 sebenarnya adalah $p = 0.2$ dan $p = 0.4$.

(50 markah)

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4. (a) $X_1, \ldots, X_n$ adalah sampel rawak daripada taburan yang berfungsi ketumpatan

$$f(x; \theta) = (2\theta x + 1 - \theta) I_{(0,1)}(x), \quad -1 \le \theta \le 1.$$

Katakan kita ingin menguji $H_0 : \theta \le \theta_0$ dan $H_1 : \theta > \theta_0$.

(i) Dapatkan rantau genting paling berkuasa secara seragam saiz-α jika ia wujud.

(ii) Cari statistik $T = u(X_1, \ldots, X_n)$ yang mana nisbah $L(\theta_1; x_1, \ldots, x_n)/L(\theta_2; x_1, \ldots, x_n)$ merupakan fungsi berekanada bagi $t = u(x_1, \ldots, x_n)$, $\theta_1 \in \Theta_0$ dan $\theta_2 \in \Theta_1$, jika ianya wujud.

(iii) Bolehkah kita menggunakan $f(x; \theta) = a(\theta)\, b(x) \exp[c(\theta) d(x)]$ untuk mendapatkan rantau genting paling berkuasa secara seragam saiz-α? Jelaskan.

(iv) Dapatkan rantau genting saiz-α ujian nisbah kebolehjadian hipotesis ini. Adakah (i) sama dengan (iv)?

(80 markah)

(b) Sekeping syiling dilempar 500 kali. Dua ratus lapan puluh kepala dan 220 bunga muncul. Adakah syiling ini adil? Jelaskan.

(20 markah)

5. (a) $X_1, \ldots, X_n$ adalah sampel rawak daripada taburan seragam (a, b). Dapatkan taburan statistik tertib ke-j sampel ini.

(25 markah)

(b) $X_1, \ldots, X_n$ adalah sampel rawak daripada taburan Gamma $\left(\frac{1}{2}, \frac{\sigma^2}{2}\right)$.

Dapatkan batas bawah Cramer-Rao bagi σ^2.

(25 markah)

(c) X_1, X_2 adalah sampel rawak daripada taburan seragam $(0, \theta + 1)$. Untuk menguji $H_0: \theta = 0$ lawan $H_1 : \theta > 0$, dua ujian dipertimbangkan:

I: Tolak H_o jika $X_1 > 0.95$

II: Tolak H_o jika $X_1 + X_2 > C$.

Cari C supaya saiz Ujian II sama dengan saiz Ujian I.

(25 markah)

(d) L(x) dan U(x) memenuhi $P(L(X) \le \theta) = \alpha_1$ dan $P(U(X) \ge \theta) = \alpha_2$. Nilaikan $P(L(X) \le \theta \le U(X))$.

(25 markah)

Lampiran

1. $\underset{n \to \infty}{had} F_n(z) = \Phi(z)$

2. $E[cX] = c\,E[X]$

3. $Var(aX + b) = a^2\ Var(X)$

4. $E[\overline{X}] = \mu$

5. $Var(\overline{X}) = \dfrac{\sigma^2}{n}$

6. $P(|X - \mu| \geq \varepsilon) \leq \dfrac{\sigma^2}{\varepsilon^2}$

7. $Z_n = \dfrac{\sqrt{n}(\overline{X}_n - \mu)}{\sigma}$

8. $M_{\sum X_i}(t) = [M_X(t)]^n$

9. $M_{\overline{X}}(t) = [M(t/n)]^n$

10. $g_1(y) = n[1 - F(y)]^{n-1} f(y)$

11. $g_\alpha(y) = \dfrac{n!}{(\alpha - 1)!(n - \alpha)!}[F(y)]^{\alpha - 1} f(y)[1 - F(y)]^{n-\alpha}$

12. $g_{\alpha,\beta}(x,y) = \dfrac{n!}{(\alpha-1)!(\beta-\alpha-1)!(n-\beta)!}[F(x)]^{\alpha-1} f(x)[F(y)-F(x)]^{\beta-\alpha-1} f(y)[1-F(y)]^{n-\beta}$,
$\alpha < \beta$

13. $g_n(y) = n[F(y)]^{n-1} f(y)$

14. $f_Y(t) = f_X[g^{-1}(t)] \, |J|$

15. $J = \dfrac{dg^{-1}(t)}{dt}$

16. $L(\theta; x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} f(x_i, \theta)$

17. $f(x; \theta) = a(\theta)\, b(x) \exp[c(\theta)\, d(x)]$

18. $\mathrm{Var}(T) \geq \dfrac{[\tau'(\theta)]^2}{nE[\{\frac{\partial}{\partial\theta}\log f(x;\theta)\}^2]}$

19. $E[\{\frac{\partial}{\partial\theta}\log f(x;\theta)\}^2] = -E[\frac{\partial^2}{\partial\theta^2}\log f(x;\theta)]$

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Rumus-Rumus

Modul 1

Pelajaran 1

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2. $P(A) = P(A \cap \bar{B}) + P(A \cap B)$

3. $P(\bar{A}) = 1 - P(A)$

4. ${}^{n}P_{r} = \dfrac{n!}{(n-r)!}$

5. $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

6. $N = \dfrac{n!}{n_1!\, n_2! \dots n_k!}$

Pelajaran 2

1. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

2. $P(A \cap B) = P(A)P(B)$

3. $P(A) = P(A \mid B)\, P(B) + P(A \mid \bar{B})\, P(\bar{B})$

4. $P(B_i \mid A) = \dfrac{P(A \cap B_i)}{\sum_{j=1}^{n} P(A \mid B_j)\, P(B_j)}$

Pelajaran 3

1. $P(a \leq X \leq b) = \int_a^b f(x)\, dx$

2. $P(a < X < b) = \sum_{a<x<b} p(x)$

3. $F(t) = P(X \leq t)$

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4. $P(a < X \leq b) = F(b) - F(a)$

5. $\frac{d}{dt} F(t) = f(t)$

6. $F_Y(t) = F_X(g^{-1}(t))$

7. $F_Y(t) = 1 - F_X(g^{-1}(t))$

8. $f_Y(t) = f_X(g^{-1}(t)) \, | J |$

9. $J = \frac{dg^{-1}(t)}{dt}$

10. $f_Y(t) = \sum_{i=1}^{k} f_X \, (g_i^{-1}(t)) \, | J_i |)$

11. $J_i = \frac{d}{dt} g_i^{-1}(t)$

12. $P_Y(y) = \sum_{x \in A} P_X(x)$

Modul 2

Pelajaran 1

1. $E(X) = \sum_{x \in \text{Julat } X} x p(x)$

2. $1 + x + x^2 + \ldots + x^n + \ldots = \frac{1}{1-x}, \; |x| < 1$

3. $1 + 2x + \ldots + n x^{n-1} + \ldots = \frac{1}{(1-x)^2}, \; |x| < 1$

4. $E(X) = \int_{-\infty}^{\infty} x f(x) \, dx$

5. $E(X) = \int_{0}^{\infty} [1 - f(x)] \, dx - \int_{-\infty}^{0} F(x) \, dx$

6. $E[G(X)] = \sum_{x \in \text{Julat } X} G(x) \, p(x)$

7. $E[G(X)] = \int_{-\infty}^{\infty} G(x)\, f(x)\, dx$

8. $E[c] = c$

9. $E[cX] = c\,E[X]$

10. $E[X + c] = E[X] + c$

11. $Var(X) = E[X - E[X]]^2$

12. $Var(X) = E[X^2] - \mu_X^2$

13. $Var(X) = \sum_{x \in \text{Julat } X} x^2 p(x) - \mu_X^2$

14. $Var(X) = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu_X^2$

15. $Var(a) = 0$

16. $Var(aX + b) = a^2\, Var(X)$

17. $F_X(t_k) = k,\ 0 < k < 1$

Pelajaran 2

1. $m_k = E[X^k]$

2. $m_k = \sum_{x \in \text{Julat } X} x^k\, p(x)$

3. $m_k = \int_{-\infty}^{\infty} x^k f(x)\, dx$

4. $\mu_k = E[(X - \mu_X)^k$

5. $\gamma_1 = \mu_3 / \sigma_X^3$

6. $\gamma_2 = \dfrac{\mu_4}{\sigma_X^4} - 3.$

7. $\mu_{[k]} = E[X(X-1)(X-2) \ldots (X-k+1)]$

8. $m(t) = E[e^{tX}]$

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9. $m(t) = \sum_{x \in \text{Julat } X} e^{tx}\, p(x)$

10. $m(t) = \int_{-\infty}^{\infty} e^{tx}\, f(x)\, dx$

11. $m_Y(t) = E[e^{tg(X)}]$

12. $m_Y(t) = \sum_{x \in \text{Julat } X} e^{tg(x)}\, p(x)$

13. $m_Y(t) = \int_{-\infty}^{\infty} e^{tg(x)}\, f(x)\, dx$

14. $m_Y(t) = e^{bt}\, m_X(at)$

15. $m^{(i)}(0) = m_i$

16. $k(t) = \ell n\, m(t)$

17. $\psi(t) = E[t^X]$

18. $f(t) = \sum_{i=0}^{\infty} \frac{f^{(i)}(a)}{i!} (t-a)^i$

19. $\psi^{(i)}(0) = i!\, p(i)$

20. $P(|X| \geq a) < \frac{1}{a^2} E[X^2]$

21. $P(|X-\mu| \geq a\sigma) \leq \frac{1}{a^2}$

22. $P(|X-\mu| < a\sigma) \geq 1 - \frac{1}{a^2}$

23. $P(X \geq a) \leq \frac{E[X]}{a}$

24. $E[X^n] = \int_0^{\infty} nx^{n-1}(1-F(x))\, dx$

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Pelajaran 3

1. (i) $$p(x) = \begin{cases} q, & x = 0 \\ p, & x=1 \\ 0, & \text{ditempat lain} \end{cases}$$ X ~ Bernoulli (p)

   (ii) $E[X] = p$

   (iii) $Var(X) = pq$

   (iv) $m(t) = q + pe^t$

2. (i) $$p(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & x=0, 1, 2, \ldots, n \\ 0, & \text{di tempat lain} \end{cases}$$ X ~ Binomial (n, p)

   (ii) $E[X] = np$

   (iii) $Var(X) = npq$

   (iv) $m(t) = (q + pe^t)^n$

3. (i) $$p(x) = \begin{cases} \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}, & x=0, 1, 2, \ldots, n \\ 0, & \text{di tempat lain} \end{cases}$$ X ~ hipergeometri (N, k, n)

   (ii) $E[X] = \frac{nK}{N}$

   (iii) $Var(X) = \frac{nK(N-K)(N-n)}{N^2(N-1)}$

4. $(a + b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$

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5. (i) $p(x) = \begin{cases} q^{x-1}p \text{ , } x = 1, 2, 3, \ldots \\ 0 \text{ , di tempat lain} \end{cases}$ $X \sim$ geometri (p)

(ii) $E[X] = 1/p$

(iii) $Var(X) = q/p^2$

(iv) $m(t) = \dfrac{pe^t}{1 - qe^t}$

6. (i) $p(x) = \begin{cases} \binom{x-1}{r-1} p^r q^{x-r} \text{ , } x = r, r+1, r+2 \\ \qquad\qquad r = 2, 3, 4, \ldots \\ 0 \text{ , di tempat lain} \end{cases}$ $X \sim$ negatif binomial (r, p)

(ii) $E[X] = r/p$

(iii) $Var(X) = rq/p^2$

(iv) $m(t) = \left[\dfrac{pe^t}{1 - qe^t}\right]^r$

7. (i) $p(x) = \begin{cases} e^{-\lambda} \dfrac{\lambda^x}{x!} \text{ , } x = 0, 1, 2, \ldots \\ 0 \text{ , di tempat lain} \end{cases}$ $X \sim$ Poisson (λ)

(ii) $E[X] = \lambda$

(iii) $Var(X) = \lambda$

(iv) $m(t) = e^{\lambda(e^t - 1)}$

8. $\underset{x \to 0}{\text{had}} (1 + x)^{1/x} = e$

9. $\underset{x \to \infty}{\text{had}} \left(1 + \dfrac{1}{x}\right)^x = e$

10. $\underset{x \to 0}{\text{had}} (1 + ax)^{1/x} = e^a$

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Pelajaran 4

1. (i) $f(x) = \begin{cases} \frac{1}{b-a}, & a<x<b \\ 0, & \text{di tempat lain} \end{cases}$ X ~ seragam (a, b)

(ii) $E[X] = \frac{a+b}{2}$

(iii) $Var(X) = \frac{(b-a)^2}{12}$

(iv) $m(t) = \frac{e^{bt} - e^{at}}{t(b-a)}$

2. (i) $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, -\infty < x < \infty$ X ~ $N(\mu, \sigma^2)$

(ii) $E[X] = \mu$

(iii) $Var(X) = \sigma^2$

(iv) $m(t) = e^{\mu t - \frac{1}{2}\sigma^2 t^2}$

3. $\underset{n\to\infty}{\text{had}}\ P\left[a \le \frac{S_n - np}{\sqrt{npq}} \le b\right] \to P(Z \ge a) - P(Z > b)$

4. $\underset{\lambda\to\infty}{\text{had}}\ P\left[a \le \frac{X-\lambda}{\sqrt{\lambda}} < b\right] \to P(Z > a) - P(Z \ge b)$

5. (i) $f(x) = \begin{cases} \lambda e^{-\lambda}, & x \ge 0 \\ 0, & \text{di tempat lain} \end{cases}$ X ~ eksponen (λ)

(ii) $E[X] = 1/\lambda$

(iii) $Var(X) = 1/\lambda^2$

(iv) $m(t) = \frac{\lambda}{\lambda - t}$

...15/-

6. $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$

7. $\Gamma(n) = (n-1)\,\Gamma(n-1)$

8. $\Gamma(n) = (n-1)!$

9. (i) $f(x) = \begin{cases} \dfrac{\lambda^n x^{n-1}}{\Gamma(n)} e^{-\lambda x}, & x > 0 \\ 0, & \text{di tempat lain} \end{cases}$ $X \sim \text{Gamma}(n, \lambda)$

(ii) $E[X] = n/\lambda$

(iii) $\text{Var}(X) = n/\lambda^2$

(iv) $m(t) = \left(\dfrac{\lambda}{\lambda - t}\right)^n$

10. (i) $f(x) = \begin{cases} \dfrac{1}{2^{\upsilon/2}\Gamma\left(\dfrac{\upsilon}{2}\right)} x^{\upsilon/2-1} e^{-x/2}, & x > 0 \\ 0, & \text{di tempat lain} \end{cases}$ $X \sim \chi^2_\upsilon$

(ii) $E[X] = \upsilon$

(iii) $\text{Var}(X) = 2\upsilon$

(iv) $m(t) = \left(\dfrac{1}{1-2t}\right)^{\upsilon/2}$

11. $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, dt$

12. $B(x, y) = \int_0^\infty \dfrac{t^{x-1}}{(1+t)^{x+y}}\, dt$

13. $B(x, y) = \dfrac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$

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14. (i) $f(x) = \begin{cases} \dfrac{1}{B(a,b)} x^{a-1}(1-x)^{b-1} & , 0 < x < 1 \\ 0 \;, & \text{di tempat lain} \end{cases}$ $\quad X \sim \text{Beta}(a, b)$

(ii) $F_X(p) = \sum_{x=a}^{n} \binom{n}{x} p^x (1-p)^{n-x}$

(iii) $E[X] = \dfrac{a}{a+b}$

(iv) $\text{Var}(X) = \dfrac{ab}{(a+b+1)(a+b)^2}$

Modul 3

Pelajaran 1

1. $P(X \le x,\ Y \le y) = \sum_{t_1 \le x} \sum_{t_2 \le y} p(t_1, t_2)$

2. $P(X \le x,\ Y \le y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(t_1, t_2)\, dt_1\, dt_2$

3. $F(x, y) = P(X \le x,\ Y \le y)$

4. $f(x, y) = \dfrac{\partial^2 F(x, y)}{dxdy}$

Pelajaran 2

1. $p(x) = \sum_{y} p(x, y)$

2. $p(y) = \sum_{x} p(x, y)$

3. $f(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$

4. $f(y) = \int_{-\infty}^{\infty} f(x, y)\, dx$

5. $F(x) = F(x, \infty)$

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6. $F(y) = F(\infty, y)$

7. $f(x) = \dfrac{\partial F(x, \infty)}{\partial x}$

8. $f(y) = \dfrac{\partial F(\infty, y)}{\partial y}$

9. $p(x \mid y) = \dfrac{p(x, y)}{p(y)}$

10. $f(x \mid y) = \dfrac{f(x, y)}{f(y)}$

11. $p(x, y) = p(x)\, p(y)$

12. $f(x, y) = f(x)\, f(y)$

Pelajaran 3

1. $E[g(X, Y)] = \sum_x \sum_y g(x, y)\, p(x, y)$

2. $E[g(X, Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f(x,y) dx\, dy$

3. $E[g_1(X, Y) + g_2(X, Y)] = E[g_1(X, Y)] + E[g_2(X, Y)]$

4. $E[h_1(X)\, h_2(Y)] = E[h_1(X)]\, E[h_2(Y)]$

5. (i) $\text{Cov}(X, Y) = E[X - \mu_X)(Y - \mu_Y)]$

   (ii) $\text{Cov}(X, Y) = E[XY] - \mu_X \mu_Y$

6. $\text{Cov}(aX, bY) = ab\, \text{Cov}(X, Y)$ - 9 -

7. $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\, \text{Cov}(X, Y)$

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8. $\text{Var}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \text{Var}(X_i) + 2\sum_{i<j}\sum \text{Cov}(X, Y)$

9. $\rho(X, Y) = \dfrac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$

10. $E[g(X, Y) \mid Y = y] = \sum_{x} g(x, y)\, p(x \mid y)$

11. $E[g(X, Y) \mid Y = y] = \int_{-\infty}^{\infty} g(x,y) f(x|y)\, dx$

12. $E[E[X \mid Y = y]] = E[X]$

13. $E[E[Y \mid X = x]] = E[Y]$

14. $E[E[g(X) \mid Y = y]] = E[g(X)]$

15. $E[E[g(Y) \mid X = x]] = E[g(Y)]$

16. $\text{Var}(X \mid Y = y) = E[X^2 \mid Y = y] - (E[X \mid Y = y)^2$

17. $m(t_1, t_2) = E\left[e^{t_1X_1 + t_2X_2}\right]$

18. $m(t_1, t_2, ..., t_n) = E\left[e^{\sum_{i=1}^{n} t_iX_i}\right]$

19. $m(t_1) = \lim_{t_2 \to 0} m(t_1, t_2)$

20. $m(t_1, t_2, ..., t_n) = m(t_1)\, m(t_2) \ldots m(t_n)$

Pelajaran 4

1. (i) $p(x_1, x_2, ..., x_k) = \dfrac{n!}{x_1!\, x_2! \ldots x_k!} p_1^{x_1} p_2^{x_2} \ldots p_k^{x_k}$

(ii) $p(x_i) = \binom{n}{x_i} p_i^{x_i}(1 - p_i)^{n-x_i}$

(iii) $p(x_i, x_j) = \dfrac{n!}{x_i!x_j!(n - x_i - x_j)!} p_i^{x_i} p_j^{x_j}(1 - p_i - p_j)^{n-x_i-x_j}$

(iv) $E[X_iX_j] = n(n - 1)\, p_ip_j$

(v) $\text{Cov}(X_i, X_j) = -np_ip_j$

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2. (i) $$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_X}{\sigma_X}\right)^2 - 2\rho\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right) + \left(\frac{y-\mu_Y}{\sigma_Y}\right)^2\right]\right\},$$

$$-\infty < x < \infty, -\infty < y < \infty$$

(ii) $$f(x|y) = \frac{1}{\sigma_x\sqrt{2\pi(1-\rho^2)}} \exp\left\{-\frac{1}{2(1-\rho^2)\sigma_X^2}\left[x - \mu_X - \rho\frac{\sigma_X}{\sigma_Y}(y-\mu_Y)\right]^2\right\}$$

$$-\infty < x < \infty$$

(iii) $$m(t_1,t_2) = \exp\left[t_1\mu_X + t_2\mu_Y + \frac{1}{2}\left(t_1^2\sigma_X^2 + 2\rho t_1 t_2\sigma_X\sigma_Y + t_2^2\sigma_Y^2\right)\right]$$

(iv) $E[XY] = \mu_X\mu_Y + \rho\,\sigma_X\sigma_Y$

(v) $\mathrm{Cov}(X, Y) = \rho\,\sigma_X\sigma_Y$

Modul 4

Pelajaran 1

1. $M_k = \frac{1}{n}\sum_{i=1}^{n} X_i^k$

2. $E[M_k] = m_k$

3. $\mathrm{Var}(M_k) = \frac{1}{n}[m_{2k} - m_k)^2]$

4. $E[\bar{X}] = \mu$

5. $\mathrm{Var}(\bar{X}) = \frac{1}{n}\sigma^2$

6. $S^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(X_i - \bar{X})^2$

...20/-

7. $E[S^2] = \sigma^2$

8. $\mathrm{Var}(S^2) = \frac{1}{n}\left(\mu_4 - \frac{(n-3)}{(n-1)}\sigma^4\right)$

9. $\sum_{i=1}^{n}(X_i - \mu)^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2 + n(\bar{X} - \mu)^2$

10. $\bar{X} - \mu = \frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)$

Pelajaran 2

1. $p(u, v) = p_{X,Y}\left(g_1^{-1}(u, v),\ g_2^{-1}(u, v)\right)$

2. $f(u, v) = f_{X,Y}\left(g_1^{-1}(u, v), g_2^{-1}(u, v)\right) \mid J \mid$

3. $J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$

4. $f(u,v) = \sum_{i=1}^{m} |J_i| f_{X,Y}\left(g_i^{-1}(u,v), h_i^{-1}(u,v)\right)$

5. $J_i = \begin{vmatrix} \frac{\partial g_i^{-1}(u,v)}{\partial u} & \frac{\partial g_i^{-1}(u,v)}{\partial v} \\ \frac{\partial h_i^{-1}(u,v)}{\partial u} & \frac{\partial h_i^{-1}(u,v)}{\partial v} \end{vmatrix}$

6. $m_{u,v}(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{t_1 g(x,y) - t_2 h(x,y)} f(x,y)\,dx\,dy$

7. $m_u(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{t g(x,y)} f(x,y)\,dx\,dy$

...21/-

8. (i) $f_{u=X-Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(x, u-x)\, dx$

(ii) $f_{u=X+Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(u-y, y)\, dy$

9. (i) $f_{u=X-Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(x, x-u)\, dx$

(ii) $f_{u=X-Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(u+y, y)\, dy$

10. (i) $f_{u=XY}(u) = \int_{-\infty}^{\infty} \frac{1}{|x|} f_{X,Y}(x, u/x)\, dx$

(ii) $f_{u=XY}(u) = \int_{-\infty}^{\infty} \frac{1}{|y|} f_{X,Y}(u/y, y)\, dy$

11. $f_{u=X/Y}(u) = \int_{-\infty}^{\infty} |y|\, f_{X,Y}(uy, y)\, dy$

Pelajaran 3

1. (i) $f(x) = \frac{\Gamma[(n+1)/2]}{\Gamma(n/2)\sqrt{\pi n}}\left(1+\frac{x^2}{n}\right)^{-(n+1)/2}, -\infty < x < \infty$ $X \sim t_n$

(ii) $T = \frac{Z}{\sqrt{V/n}}$

(iii) $E[X] = 0$

(iv) $Var\,[X] = \frac{n}{n-2}$

2. (i) $$f(x) = \begin{cases} \dfrac{\Gamma[m+n)/2}{\Gamma(m/2)\Gamma(n/2)} \left(\dfrac{m}{n}\right)^{m/2} \dfrac{x^{(m-2)/2}}{[1+(m/n)x]^{(m-n)/2}}, x>0 \\ 0 \quad \text{, di tempat lain} \end{cases} \qquad X \sim F_{m,n}$$

(ii) $F = \dfrac{U/m}{V/m}$

(iii) $E[X] = \dfrac{n}{n-2}$

(iv) $Var(X) = \dfrac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$

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