Second Semester Examination
2021/2022 Academic Session

July/August 2022

EMT212 – Computational Engineering
(Kejuruteraan Pengkomputeran)

Duration: 2 hours
(Masa : 2 jam)

Please check that this examination paper consists of FIVE (5) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA (5) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **SIX (6)** questions.

*[**Arahan**: Jawab **ENAM (6)** soalan.*

1. In the solution process of the simplex method, explain why it is necessary to convert the objective function and the constraint equations from inequalities to equalities.

(6 marks)

2. Consider the following continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

where ρ is the density function of the fluid and $\mathbf{v}$ is the velocity vector of the fluid.

(a) Explain the quantity that is being conserved so that the equation can be derived using the divergence theorem.

(9 marks)

(b) Rewrite the continuity equation if the density does not change with time.

(5 marks)

3. Consider a solid cone that floats in water where its flat base is below the surface of the water and only ONE THIRD (1/3) of its height is immersed.

(a) Sketch the vector field acting on the sphere that is relevant in modeling the buoyancy on the cone.

(5 marks)

(b) Express the magnitude of the total force $\mathbf{F}_T$ on the cone due to the water pressure in terms of the surface integral over the affected area. DO NOT evaluate the integral.

(5 marks)

(c) Use the divergence theorem to express the magnitude of the buoyancy in terms of the diameter of base D, the height of the sphere h, and the density of water ρ. Evaluations of the integrals are not necessary.

(20 marks)

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4. A 10-mm-thick high-density polyethylene sheet is cooled from 150°C to the ambient temperature of 25°C. Cooling fans are used to provide sufficient heat transfer to allow the surface temperature of the plate to quickly reach ambient temperature. This problem can be simplified to a one-dimensional problem to obtain the temperature profile along the thickness of the sheet.

   (a) State the boundary conditions for the above problem.

   (3 marks)

   (b) Describe the strategy or steps you would use to determine the temperature profile along the thickness of the polyethylene sheet using the finite difference approximation. You may use an equation or figure to support your explanations. DO NOT solve the differential equation.

   (7 marks)

5. Use the central finite difference approach to approximate the solution of the following boundary value problem (BVP)

$$-\frac{d^2u}{dx^2} = \pi^2 \sin(\pi x) \qquad 0 \le x \le 1$$
$$u(0) = 0, \quad u(1) = 0$$

   Using the step size $\Delta x = 1/4$ and FIVE (5) grid points, determine the approximation values of $u(x)$ at each grid point.

   (20 marks)

6. Consider the following Poison's equation

$$-\frac{\partial^2 u}{\partial x^2} = 1 \qquad 0 \le x \le 1$$

   with boundary conditions $u(0) = u(1) = 0$. The problem is discretised using FDM with FIVE (5) grid points.

   Using For loop, write the code to construct system matrix $\mathbf{A}$ and the load vector $\mathbf{b}$ as MATLAB arrays containing the boundary condition data. DO NOT write the code to solve the linear system.

   (20 marks)

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APPENDIX 1

1. Newton's Method

$$x_{i+1} = x_i - \frac{f'(x_i)}{f''(x_i)}$$

2. Formulas for first finite differences

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h} + O(h)$$
$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + O(h)$$
$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + O(h^2)$$

3. Formulas for second finite differences

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + O(h)$$
$$f''(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2} + O(h)$$
$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2} + O(h^2)$$

4. Heat equation

$$-\alpha \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial t} = Q(x,t)$$

5. Convective boundary condition

$$hu + ku' = hu_\infty$$

6. Discrete form of 1D Poisson's equation

$$-k\frac{u_{i-1} - 2u_i + u_{i+1}}{h^2} = f_i$$

7. Explicit and implicit methods for heat equation

$$-\lambda(u_{i+1}^l - 2u_i^l + u_{i-1}^l) = u_i^{l+1} - u_i^l - sf_i^{l+1}$$

$$-\lambda u_{i+1}^{l+1} + (1+2\lambda)u_i^{l+1} - \lambda u_{i-1}^{l+1} = u_i^l + sf_i^{l+1}$$

$$\lambda = \frac{\alpha s}{h^2}$$

8. Integrals of sine and cosine

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$
$$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$$

9. Spherical coordinates

$$x = \rho \sin\varphi \cos\theta\,;\;\; y = \rho \sin\varphi \sin\theta\,;\; x = \rho\cos\varphi$$
$$\rho \geq 0; 0 \leq \;\varphi \leq \;\pi$$
$$dV = \rho^2 \sin\varphi\;\; d\rho d\theta d\varphi$$

10. Cylindrical coordinates

$$x = r\cos\theta\,;\;\; y = r\sin\theta\,;\; z = z$$
$$dV = rdzdrd\theta$$

11. Taylor series at point a

$$u(x) = u(a) + u'(a)(x-a) + u''(a)\frac{(x-a)^2}{2!} + u'''(a)\frac{(x-a)^3}{3!} + \cdots$$
$$\ldots + u^{(n)}(a)\frac{(x-a)^n}{n!} + \cdots$$

12. Miscellaneous

$$\nabla \times \mathbf{u} = \left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)\mathbf{k}$$

$$\oint_C \; M(x,y)dx + N(x,y)dy = \iint_D \; \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dA$$

$$\oint_{\partial S} \mathbf{F}(x,y,z) \cdot d\mathbf{r} = \iint_S \; (\nabla \times \mathbf{F})\, dS$$

$$\oint_S \; (\mathbf{F} \cdot \mathbf{n})\, dS = \iiint_V \; \nabla \cdot \mathbf{F}\, dV$$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume of a cone $= \frac{1}{3}Ah$