
UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2003/2004

April 2004

JIM 312 – Teori Kebarangkalian

Masa : 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab SEMUA soalan.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

...2/-

1. (a) Suatu sistem komunikasi mempunyai 3 antena secaman yang disusun secara linear. Sistem ini dikatakan berfungsi jika tiada kerosakan berlaku pada sebarang 2 antena yang bersebelahan. Andaikan 1 mewakili antena yang elok dan 0 mewakili antena yang rosak, dan memang terdapat 2 antena yang rosak di dalam sistem ini.

(i) Senaraikan kesemua susunan antena di dalam sistem ini.

(ii) Berdasarkan (i) senaraikan susunan-susunan antena yang membolehkan sistem ini berfungsi.

(iii) Berapakah kebarangkalian sistem ini berfungsi?

(50 markah)

(b) Fungsi jisim kebarangkalian bagi pembolehubah rawak X diberikan oleh $p(x) = c(0.8)^x$, $x = 0, 1, 2, \dots$. Cari nilai c .

(20 markah)

(c) Dua biji dadu adil dilemparkan. Andaikan X ialah nilai paling kecil yang dicerap daripada 2 dadu tersebut dan Y ialah hasildarab kedua-dua nilai yang dicerap. Dapatkan fungsi jisim kebarangkalian tercantum $p(x, y)$.

(30 markah)

...3/-

2. (a) Empat buah bas membawa sejumlah 148 orang pelajar. Bas-bas tersebut masing-masing membawa 40, 33, 25 dan 50 orang pelajar. Seorang pelajar dipilih secara rawak. Andaikan X mewakili bilangan pelajar di dalam bas yang mengandungi pelajar terpilih ini. Seorang daripada 4 orang pemandu bas turut dipilih secara rawak. Andaikan Y mewakili bilangan pelajar di dalam bas yang dipandu beliau.
- (i) Bandingkan $p(x)$ dan $p(y)$.
 - (ii) Hitungkan $P(X > 30)$ dan $P(Y > 30)$

(50 markah)

(b) Diberikan $M_X(t) = \frac{1}{1-t^2}, -1 < t < 1$.

Cari $E[X]$ dan $\text{Var}[X]$.

(20 markah)

- (c) X dan Y adalah pembolehubah-pembolehubah rawak tak bersandar dan tertabur secara seragam $(0, 1)$. Hitungkan $P(X \geq Y)$.

(30 markah)

3. (a) Fungsi jisim kebarangkalian tercantum bagi X dan Y diberikan oleh jadual berikut:

		Y	
	$p(xy)$	0	1
X	0	1/8	1/4
	1	1/8	1/2

...4/-

- (i) Dapatkan fungsi jisim kebarangkalian bersyarat Y diberikan $X = 1, 2$.
- (ii) Hitungkan pekali korelasi, $\rho(X, Y)$.

(50 markah)

- (b) X dan Y adalah dua pembolehubah rawak normal piawai yang tak bersandar. Tunjukkan $U = \frac{2X - Y}{\sqrt{5}}$ dan $V = \frac{X + 2Y}{\sqrt{5}}$ adalah tak bersandar secara stokastik.

(50 markah)

4. (a) Dapatkan taburan $\sum_{i=1}^n X_i$ bagi situasi berikut:

- (i) X_1, \dots, X_n adalah sampel rawak daripada populasi bertaburan Binomial (n, p) .
- (ii) X_1, X_2, \dots, X_n adalah pembolehubah rawak tak bersandar yang berkeadaan $X_i \sim \text{Binomial}(n_i, p)$, $i = 1, 2, \dots, n$.

(50 markah)

- (b) X_1 dan X_2 adalah sampel rawak daripada taburan $N(0, 1)$. Tentukan taburan

- (i) X_1^2/X_2^2
- (ii) $\frac{X_1 - X_2}{\sqrt{2}}$

(20 markah)

...5/-

(c) X_1, \dots, X_n adalah sampel rawak daripada taburan χ_1^2 . Bagi $k < n$, takrifkan

$$U = \sum_{i=1}^k X_i \text{ dan } V = \sum_{i=k+1}^n X_i. \text{ Dapatkan min } (n - k)U/kV.$$

(30 markah)

5. (a) $P(A) = 0.4, P(A \cup B) = 0.7$. Andaikan $P(B) = p$. Cari p sekiranya $P(A | B) = 0.6$.

(25 markah)

(b) Diberi $E(X) = 4$ dan $E[X^2] = 25$. Gunakan ketaksamaan Chebyshev untuk mencari batas bawah $P(-2 < X < 10)$.

(25 markah)

(c) $Z \sim$ eksponen (1). Dapatkan $\text{Cov}(Z, Z^2)$.

(25 markah)

(d) (X, Y) tertabur secara trinomial dengan

$$\begin{aligned} p(x, y) &= P(X = x, Y = y) \\ &= \frac{n!}{x!y!(n-x-y)!} p_1^x p_2^y p_3^{n-x-y} \end{aligned}$$

$$x + y \leq n, 0 \leq p_i < 1, i = 1, 2, 3 \text{ dan } \sum_{i=1}^3 p_i = 1.$$

kebarangkalian sut X .

Cari fungsi jisim

(25 markah)

Rumus-Rumus**Modul 1****Pelajaran 1**

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2. $P(A) = P(A \cap \bar{B}) + P(A \cap B)$

3. $P(\bar{A}) = 1 - P(A)$

4. ${}_n P_r = \frac{n!}{(n-r)!}$

5. $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

6. $N = \frac{n!}{n_1! n_2! \dots n_k!}$

Pelajaran 2

1. $P(A | B) = \frac{P(A \cap B)}{P(B)}$

2. $P(A \cap B) = P(A)P(B)$

3. $P(A) = P(A | B) P(B) + P(A | \bar{B}) P(\bar{B})$

4. $P(B_j | A) = \frac{P(A \cap B_j)}{\sum_{j=1}^n P(A | B_j) P(B_j)}$

Pelajaran 3

1. $P(a \leq X \leq b) = \int_a^b f(x) dx$

2. $P(a < X < b) = \sum_{a < x < b} p(x)$

3. $F(t) = P(X \leq t)$

4. $P(a < X \leq b) = F(b) - F(a)$

5. $\frac{d}{dt} F(t) = f(t)$
6. $F_Y(t) = F_X(g^{-1}(t))$
7. $F_Y(t) = 1 - F_X(g^{-1}(t))$
8. $f_Y(t) = f_X(g^{-1}(t)) |J|$
9. $J = \frac{dg^{-1}(t)}{dt}$
10. $f_Y(t) = \sum_{i=1}^k f_X(g_i^{-1}(t)) |J_i|$
11. $J_i = \frac{d}{dt} g_i^{-1}(t)$
12. $P_Y(y) = \sum_{x \in A} P_X(x)$

Modul 2

Pelajaran 1

1. $E(X) = \sum_{x \in \text{Julat } X} xp(x)$
2. $1 + x + x^2 + \dots + x^n + \dots = \frac{1}{1-x}, |x| < 1$
3. $1 + 2x + \dots + nx^{n-1} + \dots = \frac{1}{(1-x)^2}, |x| < 1$
4. $E(X) = \int_{-\infty}^{\infty} x f(x) dx$
5. $E(X) = \int_0^{\infty} [1 - f(x)] dx - \int_{-\infty}^0 F(x) dx$
6. $E[G(X)] = \sum_{x \in \text{Julat } X} G(x) p(x)$

$$7. E[G(X)] = \int_{-\infty}^{\infty} G(x) f(x) dx$$

$$8. E[c] = c$$

$$9. E[cX] = c E[X]$$

$$10. E[X + c] = E[X] + c$$

$$11. \text{Var}(X) = E[X - E[X]]^2$$

$$12. \text{Var}(X) = E[X^2] - \mu_X^2$$

$$13. \text{Var}(X) = \sum_{x \in \text{Julat } X} x^2 p(x) - \mu_X^2$$

$$14. \text{Var}(X) = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu_X^2$$

$$15. \text{Var}(a) = 0$$

$$16. \text{Var}(aX + b) = a^2 \text{Var}(X)$$

$$17. F_X(t_k) = k, 0 < k < 1$$

Pelajaran 2

$$1. m_k = E[X^k]$$

$$2. m_k = \sum_{x \in \text{Julat } X} x^k p(x)$$

$$3. m_k = \int_{-\infty}^{\infty} x^k f(x) dx$$

$$4. \mu_k = E[(X - \mu_X)^k]$$

$$5. \gamma_1 = \mu_3 / \sigma_X^3$$

$$6. \gamma_2 = \frac{\mu_4}{\sigma_X^4} - 3.$$

$$7. \mu_{[k]} = E[X(X-1)(X-2) \dots (X-k+1)]$$

$$8. m(t) = E[e^{tX}]$$

$$9. \quad m(t) = \sum_{x \in \text{Julat } X} e^{tx} p(x)$$

$$10. \quad m(t) = \int_{-\infty}^{\infty} e^{tx} f(x) dx$$

$$11. \quad m_Y(t) = E[e^{tg(X)}]$$

$$12. \quad m_Y(t) = \sum_{x \in \text{Julat } X} e^{tg(x)} p(x)$$

$$13. \quad m_Y(t) = \int_{-\infty}^{\infty} e^{tg(x)} f(x) dx$$

$$14. \quad m_Y(t) = e^{bt} m_X(at)$$

$$15. \quad m^{(i)}(0) = m_i$$

$$16. \quad k(t) = \ln \dot{m}(t)$$

$$17. \quad \psi(t) = E[t^X]$$

$$18. \quad f(t) = \sum_{i=0}^{\infty} \frac{f^{(i)}(a)}{i!} (t-a)^i$$

$$19. \quad \psi^{(i)}(0) = i! p(i)$$

$$20. \quad P(|X| \geq a) < \frac{1}{a^2} E[X^2]$$

$$21. \quad P(|X - \mu| \geq a\sigma) \leq \frac{1}{a^2}$$

$$22. \quad P(|X - \mu| < a\sigma) \geq 1 - \frac{1}{a^2}$$

$$23. \quad P(X \geq a) \leq \frac{E[X]}{a}$$

$$24. \quad E[X^n] = \int_0^{\infty} nx^{n-1} (1 - F(x)) dx$$

Pelajaran 3

1. (i) $p(x) = \begin{cases} q, & x = 0 \\ p, & x = 1 \\ 0, & \text{di tempat lain} \end{cases}$ $X \sim \text{Bernoulli } (p)$
- (ii) $E[X] = p$
- (iii) $\text{Var}(X) = pq$
- (iv) $m(t) = q + pe^t$
2. (i) $p(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & x=0, 1, 2, \dots, n \\ 0, & \text{di tempat lain} \end{cases}$ $X \sim \text{Binomial } (n, p)$
- (ii) $E[X] = np$
- (iii) $\text{Var}(X) = npq$
- (iv) $m(t) = (q + pe^t)^n$
3. (i) $p(x) = \begin{cases} \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}, & x=0, 1, 2, \dots, n \\ 0, & \text{di tempat lain} \end{cases}$ $X \sim \text{hipergeometri } (N, k, n)$
- (ii) $E[X] = \frac{nK}{N}$
- (iii) $\text{Var}(X) = \frac{nK(N-K)(N-n)}{N^2(N-1)}$
4. $(a + b)^n = \sum_{i=0}^n \binom{n}{i} a^i b^{n-i}$

5. (i) $p(x) = \begin{cases} q^{x-1}p, & x = 1, 2, 3, \dots \\ 0, & \text{di tempat lain} \end{cases}$ X ~ geometri (p)
- (ii) $E[X] = 1/p$
- (iii) $\text{Var}(X) = q/p^2$
- (iv) $m(t) = \frac{pe^t}{1 - qe^t}$
6. (i) $p(x) = \begin{cases} \binom{x-1}{r-1} p^r q^{x-r}, & x = r, r+1, r+2 \\ & r = 2, 3, 4, \dots \\ 0, & \text{di tempat lain} \end{cases}$ X ~ negatif binomial (r, p)
- (ii) $E[X] = r/p$
- (iii) $\text{Var}(X) = rq/p^2$
- (iv) $m(t) = \left[\frac{pe^t}{1 - qe^t} \right]^r$
7. (i) $p(x) = \begin{cases} e^{-\lambda} \frac{\lambda^x}{x!}, & x = 0, 1, 2, \dots \\ 0, & \text{di tempat lain} \end{cases}$ X ~ Poisson (λ)
- (ii) $E[X] = \lambda$
- (iii) $\text{Var}(X) = \lambda$
- (iv) $m(t) = e^{\lambda(e^t - 1)}$
8. $\lim_{x \rightarrow 0} (1+x)^{1/x} = e$
9. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$
10. $\lim_{x \rightarrow 0} (1+ax)^{1/x} = e^a$

Pelajaran 4

$$1. \quad (i) \quad f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{di tempat lain} \end{cases} \quad X \sim \text{seragam}(a, b)$$

$$(ii) \quad E[X] = \frac{a+b}{2}$$

$$(iii) \quad \text{Var}(X) = \frac{(b-a)^2}{12}$$

$$(iv) \quad m(t) = \frac{e^{bt} - e^{at}}{t(b-a)}$$

$$2. \quad (i) \quad f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \quad -\infty < x < \infty \quad X \sim N(\mu, \sigma^2)$$

$$(ii) \quad E[X] = \mu$$

$$(iii) \quad \text{Var}(X) = \sigma^2$$

$$(iv) \quad m(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

$$3. \quad \lim_{n \rightarrow \infty} P \left[a \leq \frac{S_n - np}{\sqrt{npq}} \leq b \right] \rightarrow P(Z \geq a) - P(Z > b)$$

$$4. \quad \lim_{\lambda \rightarrow \infty} P \left[a \leq \frac{X - \lambda}{\sqrt{\lambda}} < b \right] \rightarrow P(Z > a) - P(Z \geq b)$$

$$5. \quad (i) \quad f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{di tempat lain} \end{cases} \quad X \sim \text{eksponen}(\lambda)$$

$$(ii) \quad E[X] = 1/\lambda$$

$$(iii) \quad \text{Var}(X) = 1/\lambda^2$$

$$(iv) \quad m(t) = \frac{\lambda}{\lambda - t}$$

$$6. \quad \Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$$

$$7. \quad \Gamma(n) = (n-1) \Gamma(n-1)$$

$$8. \quad \Gamma(n) = (n-1)!$$

$$9. \quad (i) \quad f(x) = \begin{cases} \frac{\lambda^n x^{n-1}}{\Gamma(n)} e^{-\lambda x}, & x > 0 \\ 0, & \text{di tempat lain} \end{cases} \quad X \sim \text{Gamma}(n, \lambda)$$

$$(ii) \quad E[X] = n/\lambda$$

$$(iii) \quad \text{Var}(X) = n/\lambda^2$$

$$(iv) \quad m(t) = \left(\frac{\lambda}{\lambda - t} \right)^n$$

$$10. \quad (i) \quad f(x) = \begin{cases} \frac{1}{2^{v/2} \Gamma\left(\frac{v}{2}\right)} x^{v/2-1} e^{-x/2}, & x > 0 \\ 0, & \text{di tempat lain} \end{cases} \quad X \sim \chi_v^2$$

$$(ii) \quad E[X] = v$$

$$(iii) \quad \text{Var}(X) = 2v$$

$$(iv) \quad m(t) = \left(\frac{1}{1-2t} \right)^{v/2}$$

$$11. \quad B(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt$$

$$12. \quad B(x, y) = \int_0^{\infty} \frac{t^{x-1}}{(1+t)^{x+y}} dt$$

$$13. \quad B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$$14. (i) f(x) = \begin{cases} \frac{1}{B(a,b)} x^{a-1} (1-x)^{b-1}, & 0 < x < 1 \\ 0, & \text{di tempat lain} \end{cases} \quad X \sim \text{Beta}(a, b)$$

$$(ii) F_X(p) = \sum_{x=a}^n \binom{n}{x} p^x (1-p)^{n-x}$$

$$(iii) E[X] = \frac{a}{a+b}$$

$$(iv) \text{Var}(X) = \frac{ab}{(a+b+1)(a+b)^2}$$

Modul 3

Pelajaran 1

$$1. P(X \leq x, Y \leq y) = \sum_{t_1 \leq x} \sum_{t_2 \leq y} p(t_1, t_2)$$

$$2. P(X \leq x, Y \leq y) = \int \int f(t_1, t_2) dt_1 dt_2$$

$$3. F(x, y) = P(X \leq x, Y \leq y)$$

$$4. f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y}$$

Pelajaran 2

$$1. p(x) = \sum_y p(x, y)$$

$$2. p(y) = \sum_x p(x, y)$$

$$3. f(x) = \int f(x, y) dy$$

$$4. f(y) = \int f(x, y) dx$$

$$5. F(x) = F(x, \infty)$$

$$6. \quad F(y) = F(\infty, y)$$

$$7. \quad f(x) = \frac{\partial F(x, \infty)}{\partial x}$$

$$8. \quad f(y) = \frac{\partial F(\infty, y)}{\partial y}$$

$$9. \quad p(x | y) = \frac{p(x, y)}{p(y)}$$

$$10. \quad f(x | y) = \frac{f(x, y)}{f(y)}$$

$$11. \quad p(x, y) = p(x) p(y)$$

$$12. \quad f(x, y) = f(x) f(y)$$

Pelajaran 3

$$1. \quad E[g(X, Y)] = \sum_x \sum_y g(x, y) p(x, y)$$

$$2. \quad E[g(X, Y)] = \iint g(x, y) f(x, y) dx dy$$

$$3. \quad E[g_1(X, Y) + g_2(X, Y)] = E[g_1(X, Y)] + E[g_2(X, Y)]$$

$$4. \quad E[h_1(X) h_2(Y)] = E[h_1(X)] E[h_2(Y)]$$

$$5. \quad (i) \quad \text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$$

$$(ii) \quad \text{Cov}(X, Y) = E[XY] - \mu_X \mu_Y$$

$$6. \quad \text{Cov}(aX, bY) = ab \text{Cov}(X, Y)$$

$$7. \quad \text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2 \text{Cov}(X, Y)$$

$$8. \text{Var} \left(\sum_{i=1}^n X_i \right) = \sum_{i=1}^n \text{Var}(X_i) + 2 \sum_{i < j} \text{Cov}(X, Y)$$

$$9. \rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

$$10. E[g(X, Y) | Y = y] = \sum_x g(x, y) p(x | y)$$

$$11. E[g(X, Y) | Y = y] = \int_{-\infty}^{\infty} g(x, y) f(x | y) dx$$

$$12. E[E[X | Y = y]] = E[X]$$

$$13. E[E[Y | X = x]] = E[Y]$$

$$14. E[E[g(X) | Y = y]] = E[g(X)]$$

$$15. E[E[g(Y) | X = x]] = E[g(Y)]$$

$$16. \text{Var}(X | Y = y) = E[X^2 | Y = y] - (E[X | Y = y])^2$$

$$17. m(t_1, t_2) = E[e^{t_1 X_1 + t_2 X_2}]$$

$$18. m(t_1, t_2, \dots, t_n) = E \left[e^{\sum_{i=1}^n t_i X_i} \right]$$

$$19. m(t_1) = \lim_{t_2 \rightarrow 0} m(t_1, t_2)$$

$$20. m(t_1, t_2, \dots, t_n) = m(t_1) m(t_2) \dots m(t_n)$$

Pelajaran 4

$$1. (i) p(x_1, x_2, \dots, x_k) = \frac{n!}{x_1! x_2! \dots x_k!} p_1^{x_1} p_2^{x_2} \dots p_k^{x_k}$$

$$(ii) p(x_i) = \binom{n}{x_i} p_i^{x_i} (1 - p_i)^{n - x_i}$$

$$(iii) p(x_i, x_j) = \frac{n!}{x_i! x_j! (n - x_i - x_j)!} p_i^{x_i} p_j^{x_j} (1 - p_i - p_j)^{n - x_i - x_j}$$

$$(iv) E[X_i X_j] = n(n - 1) p_i p_j$$

$$(v) \text{Cov}(X_i, X_j) = -n p_i p_j$$

$$2. (i) f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[\left(\frac{x-\mu_x}{\sigma_x} \right)^2 - 2\rho \left(\frac{x-\mu_x}{\sigma_x} \right) \left(\frac{y-\mu_y}{\sigma_y} \right) + \left(\frac{y-\mu_y}{\sigma_y} \right)^2 \right] \right\},$$

$$-\infty < x < \infty, -\infty < y < \infty$$

$$(ii) f(x|y) = \frac{1}{\sigma_x\sqrt{2\pi(1-\rho^2)}} \exp \left\{ -\frac{1}{2(1-\rho^2)\sigma_x^2} \left[x - \mu_x - \rho \frac{\sigma_x}{\sigma_y} (y - \mu_y) \right]^2 \right\}$$

$$-\infty < x < \infty$$

$$(iii) m(t_1, t_2) = \exp \left[t_1\mu_x + t_2\mu_y + \frac{1}{2} (t_1^2\sigma_x^2 + 2\rho t_1 t_2\sigma_x\sigma_y + t_2^2\sigma_y^2) \right]$$

$$(iv) E[XY] = \mu_x\mu_y + \rho\sigma_x\sigma_y$$

$$(v) \text{Cov}(X, Y) = \rho\sigma_x\sigma_y$$

Modul 4

Pelajaran 1

$$1. M_k = \frac{1}{n} \sum_{i=1}^n X_i^k$$

$$2. E[M_k] = m_k$$

$$3. \text{Var}(M_k) = \frac{1}{n} [m_{2k} - m_k^2]$$

$$4. E[\bar{X}] = \mu$$

$$5. \text{Var}(\bar{X}) = \frac{1}{n} \sigma^2$$

$$6. S^2 = \frac{1}{(n-1)} \sum_{i=1}^n (X_i - \bar{X})^2$$

7. $E[S^2] = \sigma^2$
8. $\text{Var}(S^2) = \frac{1}{n} \left(\mu_4 - \frac{(n-3)}{(n-1)} \sigma^4 \right)$
9. $\sum_{i=1}^n (X_i - \mu)^2 = \sum_{i=1}^n (X_i - \bar{X})^2 + n(\bar{X} - \mu)^2$
10. $\bar{X} - \mu = \frac{1}{n} \sum_{i=1}^n (X_i - \mu)$

Pelajaran 2

1. $p(u, v) = p_{X,Y} (g_1^{-1}(u, v), g_2^{-1}(u, v))$
2. $f(u, v) = f_{X,Y} (g_1^{-1}(u, v), g_2^{-1}(u, v)) |J|$

$$3. J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$4. f(u, v) = \sum_{i=1}^m |J_i| f_{X,Y} (g_i^{-1}(u, v), h_i^{-1}(u, v))$$

$$5. J_i = \begin{vmatrix} \frac{\partial g_i^{-1}(u, v)}{\partial u} & \frac{\partial g_i^{-1}(u, v)}{\partial v} \\ \frac{\partial h_i^{-1}(u, v)}{\partial u} & \frac{\partial h_i^{-1}(u, v)}{\partial v} \end{vmatrix}$$

$$6. m_{u,v}(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{t_1 g(x,y) + t_2 h(x,y)} f(x,y) dx dy$$

$$7. m_v(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{tg(x,y)} f(x,y) dx dy$$

$$8. \quad (i) \quad f_{u=x+y}(u) = \int_{-\infty}^{\infty} f_{x,y}(x, u-x) dx$$

$$(ii) \quad f_{u=x+y}(u) = \int_{-\infty}^{\infty} f_{x,y}(u-y, y) dy$$

$$9. \quad (i) \quad f_{u=x-y}(u) = \int_{-\infty}^{\infty} f_{x,y}(x, x-u) dx$$

$$(ii) \quad f_{u=x-y}(u) = \int_{-\infty}^{\infty} f_{x,y}(u+y, y) dy$$

$$10. \quad (i) \quad f_{u=xy}(u) = \int_{-\infty}^{\infty} \frac{1}{|x|} f_{x,y}(x, u/x) dx$$

$$(ii) \quad f_{u=xy}(u) = \int_{-\infty}^{\infty} \frac{1}{|y|} f_{x,y}(u/y, y) dy$$

$$11. \quad f_{u=xy}(u) = \int_{-\infty}^{\infty} |y| f_{x,y}(uy, y) dy$$

Pelajaran 3

$$1. \quad (i) \quad f(x) = \frac{\Gamma[(n+1)/2]}{\Gamma(n/2)\sqrt{\pi n}} \left(1 + \frac{x^2}{n}\right)^{-(n+1)/2}, \quad -\infty < x < \infty \quad X \sim t_n$$

$$(ii) \quad T = \frac{Z}{\sqrt{V/n}}$$

$$(iii) \quad E[X] = 0$$

$$(iv) \quad \text{Var}[X] = \frac{n}{n-2}$$

$$2. \quad (i) \quad f(x) = \begin{cases} \frac{\Gamma[(m+n)/2]}{\Gamma(m/2)\Gamma(n/2)} \left(\frac{m}{n}\right)^{m/2} \frac{x^{(m-2)/2}}{[1+(m/n)x]^{(m+n)/2}}, & x > 0 \\ 0 & , \text{ di tempat lain} \end{cases} \quad X \sim F_{m,n}$$

$$(ii) \quad F = \frac{U/m}{V/m}$$

$$(iii) \quad E[X] = \frac{n}{n-2}$$

$$(iv) \quad \text{Var}(X) = \frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$$

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