Second Semester Examination
2018/2019 Academic Session

June 2019

ESA380 – Orbital Mechanics
[Mekanik Orbit]

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **ELEVEN (11)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEBELAS (11)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **FOUR (4)** questions. **All questions are COMPULSORY**.
Please refer to the **APPENDIXES** to assist you in answering the questions.

*[**Arahan :** Jawab **EMPAT (4)** soalan. **Semua soalan WAJIB dijawab.**]*
*Sila rujuk **LAMPIRAN** untuk membantu anda menjawab soalan-soalan.*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

1. (a). **Figure 1** shows a system of four body problem. Assume that m_4 is an earth satellite and that m_1 is the Earth. While the remaining masses m_2 and m_3 are the moon and other planet, respectively. By considering only gravitational forces of these objects,

(i). Develop the equation of motion of the satellite with respect to the inertial frame.

(ii). Develop the equation of motion of the satellite relative to the earth.

(**50 marks**)

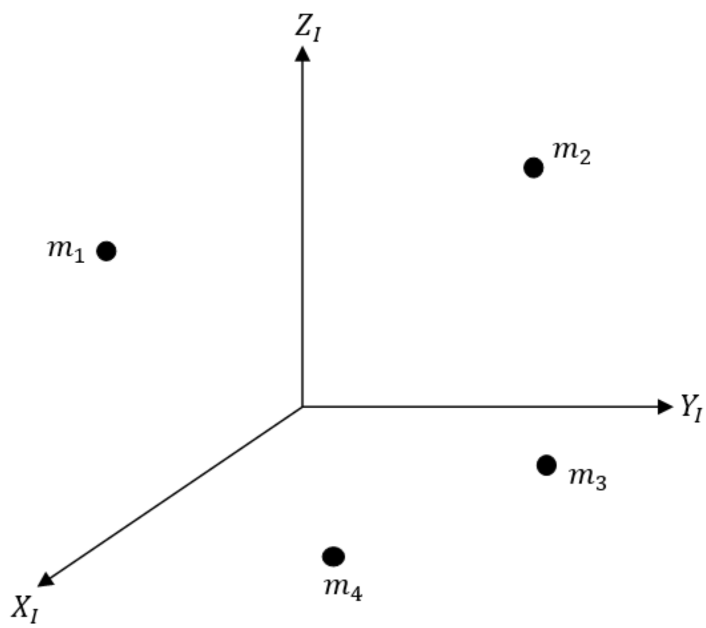


Figure 1

(b). The following position data for a satellite are given:

Point A = 500 km altitude at a true anomaly of 50°
Point B = 1700 km altitude at a true anomaly of 130°

(i). Calculate the eccentricity of the orbit.

(20 marks)

(ii). Calculate, in terms of the period T, the time required to fly from A to B. Given the mean anomaly and eccentric anomaly of this type of orbit are as follow

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}$$

(**30 marks**)

2. (a). At a given instant the position $\mathbf{r}$ and velocity $\mathbf{v}$ of a satellite in the geocentric equatorial frame are $\mathbf{r} = 12670\hat{\mathbf{K}}\,(km)$ and $\mathbf{v} = -3.874\hat{\mathbf{J}} - 0.7905\hat{\mathbf{K}}\,(km/s)$. Find the orbital element.

(**70 marks**)

(b). The unit vector in a uvw cartesian coordinate frame have the following components in the xyz frame

$$\hat{\mathbf{u}} = 0.26726\hat{\mathbf{i}} + 0.53452\hat{\mathbf{j}} + 0.80178\hat{\mathbf{k}}$$
$$\hat{\mathbf{v}} = -0.44376\hat{\mathbf{i}} + 0.80684\hat{\mathbf{j}} - 0.38997\hat{\mathbf{k}}$$
$$\hat{\mathbf{w}} = -0.85536\hat{\mathbf{i}} - 0.25158\hat{\mathbf{j}} + 0.45284\hat{\mathbf{k}}$$

If, in the xyz frame, $\mathbf{V} = -50\hat{\mathbf{i}} + 100\hat{\mathbf{j}} + 75\hat{\mathbf{k}}$, find the components of the vector $\mathbf{V}$ in the uvw frame.

(**30 marks**)

3. Consider a transfer from a circular parking orbit at 185 km and i = 5° to the geostationary orbit (T_{GEO} = 23h 56m 4s, e = 0, i = 0°)
μ_{Earth} = 398600 km^3/s^2, R_{Earth} = 6317 km)

(a). Compute the circular velocities in the parking orbit and target orbit. Then, find the velocities in the pericentre and apocentre of a Hohnmann transfer (assuming that the initial and target orbit are coplanar)

(**40 marks**)

(b). Compute the total ΔV

(i) if the sequence of the maneuvers was full inclination change in initial orbit

(ii) Hohnmann transfer to geostationary orbit (GEO)

(**44 marks**)

(c). Compute the total ΔV if the full inclination change is combine with the second Hohnmann transfer (the Hohnmann transfer inclination is still 5°)

(**16 marks**)

4. European Space Agency (ESA) is planning to send spacecraft to Saturn with a mission to orbit the Saturn ring and take image of the polar of Saturn.

(a). Sketch the patched-conic approximation for interplanetary trajectory and list the forces that influence the interplanetary spacecraft by considering the coordinate system is Sun-centred.

(20 marks)

(b). Calculate the Sphere of Influence (SOI) for Saturn and synodic period for Saturn relative to the Earth.

(10 marks)

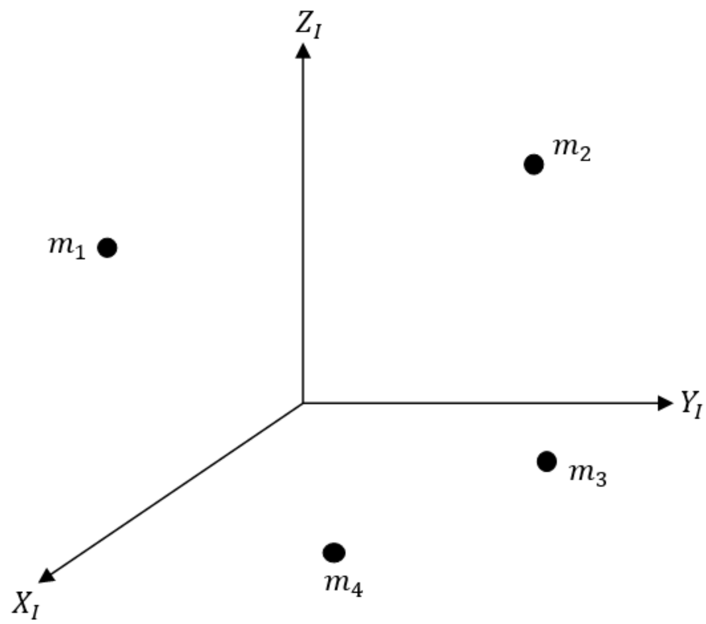
(c). Calculate the propellant mass (m_p) required to launch a 2000 kg spacecraft from a 180 km circular Earth orbit on a Hohnmann transfer trajectory to the orbit of Saturn. Calculate the time required for the mission. Assume the propulsion system has a specific impulse of 300 s.

(70 marks)

1. *(a). **Rajah 1** menunjukkan sebuah sistem masalah empat jasad. Anggap m_4 sebagai sebuah satelit Bumi dan m_1 sebagai Bumi. Sementara jasad m_2 dan m_3 adalah bulan dan planet lain. Dengan hanya mempertimbangkan tarikan graviti jasad-jasad tersebut,*

   (i). Hasilkan persamaan gerakan satelit relatif kepada bingkai tegar.

   (ii). Hasilkan persamaan gerakan satelit relatif kepada Bumi.



Rajah 1

(50 markah)

(b). *Data kedudukan sebuah satelit adalah seperti berikut:*

Kedudukan A = 500 km altitude pada 'true anomaly' of 50°

Kedudukan B = 1700 km altitude pada 'true anomaly' of 130°

(i). *Kirakan keesentrikan orbit.*

(*20 markah*)

(ii). *Kirakan, dalam bentuk tempoh T, masa yang diperlukan untuk bergerak dari titik A ke B. Diberi 'mean anomaly' dan 'eccentric anomaly' untuk jenis orbit ini seperti berikut*

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}$$

(*30 markah*)

2. (a). *Pada satu ketika, kedudukan* $\mathbf{r}$ *dan halaju* $\mathbf{v}$ *sebuah satelit dalam bingkai geosentrik khatulistiwa adalah* $\mathbf{r} = 12670\hat{\mathbf{K}}\ (km)$ *dan* $\mathbf{v} = -3.874\hat{\mathbf{J}} - 0.7905\hat{\mathbf{K}}\ (km/s)$. *Cari unsur orbit.*

(70 markah)

(b). *Vektor-vektor dalam bingkai Cartesian koordinat* uvw *mempunyai komponen berikut dalam bingkai* xyz

$$\hat{\mathbf{u}} = 0.26726\hat{\mathbf{i}} + 0.53452\hat{\mathbf{j}} + 0.80178\hat{\mathbf{k}}$$
$$\hat{\mathbf{v}} = -0.44376\hat{\mathbf{i}} + 0.80684\hat{\mathbf{j}} - 0.38997\hat{\mathbf{k}}$$
$$\hat{\mathbf{w}} = -0.85536\hat{\mathbf{i}} - 0.25158\hat{\mathbf{j}} + 0.45284\hat{\mathbf{k}}$$

Jika di dalam bingkai xyz, $\mathbf{V} = -50\hat{\mathbf{i}} + 100\hat{\mathbf{j}} + 75\hat{\mathbf{k}}$, *cari komponen-komponen vektor* $\mathbf{V}$ *dalam bingkai* uvw.

(30 markah)

3. *Anggap perpindahan daripada orbit singgah bulat di 185 km dan i = 5° ke orbit geopegun (T_{GEO} = 23 jam 56 minit 4 saat, e = 0, i = 0°)*
μ_{Earth} = 398600 km^3/s^2, R_{Earth} = 6317 km)

   (a). *Kira kelajuan bulat di orbit singgah dan orbit yang dituju. Kemudian, cari kelajuan di peripusat dan apopusat untuk perpindahan Hohnmann. (anggap yang orbit mula dan orbit tuju adalah sesatah)*

   (***40 markah***)

   (b). *Kira jumlah ΔV*

   (i). *jika urutan pergerakan adalah kecondongan penuh berubah di orbit mula.*

   (ii). *Hohnmann transfer ke orbit geopegun (GEO)*

   (***44 markah***)

   (c) *Kira jumlah ΔV jika perubahan kecondongan penuh disatukan dengan perpindangan Hohmann kedua (kecondongan perpindahan Hohnmann masih 5°)*

   (***16 markah***)

4. Agensi Angkasa Eropah (ESA) merancang untuk menghantar kapal angkasa ke Zuhal dengan misi untuk mengorbit lingkaran Zuhal dan mengambil gambar kutub di Zuhal.

(a). Lakarkan anggaran 'patched-conic' untuk trajektori antara planet and senaraikan daya yang mempengaruhi kapal angkasa antara planet dengan menganggap sistem koordinat adalah berpusat pada matahari.

(20 markah)

(b). Kira 'Sphere of Influence' (SOI) untuk Zuhal dan masa sinodik untuk Zuhal bernisbahkan kepada Bumi.

(10 markah)

(c). Kira jisim bahan dorong (m_p) yang diperlukan untuk melancarkan kapal angkasa seberat 2000 kg dari orbit bulat Bumi setinggi 180 km di perpindahan Hohnmann ke orbit Zuhal. Kira masa yang diperlukan untuk misi tersebut. Anggap sistem dorongan mempunyai 300 s untuk impuls tertentu.

(70 markah)

APPENDIX 1/ *LAMPIRAN 1*

Table 1: Astronomical data for the Sun, the planets and the moon

Object	Radius (r) (km)	Mass (kg)	Sidereal Rotation period	Inclination of equator to orbit plane	Semimajor axis of orbit (R) (km)	Orbit eccentricity	Inclination of orbit to the ecliptic plane	Orbit sidereal period (T)
Sun	696000	$1.989 \cdot 10^{30}$	25.38 d	7.25 °	-	-	-	-
Mercury	2440	$330.2 \cdot 10^{21}$	58.65 d	0.01 °	$57.91 \cdot 10^{6}$	0.2056	7.0°	87.97 d
Venus	6052	$4.869 \cdot 10^{24}$	24.3 d	177.4 °	$108.2 \cdot 10^{6}$	0.0067	3.39°	224.7 d
Earth	6378	$5.974 \cdot 10^{24}$	23.93 h	23.45 °	$149.6 \cdot 10^{6}$	0.0167	0.0°	365.25 d
Moon	1737	$73.48 \cdot 10^{21}$	27.32 d	6.68 °	$384.4 \cdot 10^{3}$	0.0549	5.145°	27.32 d
Mars	3396	$641.9 \cdot 10^{21}$	24.6 h	25.19 °	$227.9 \cdot 10^{6}$	0.0935	1.85°	1.881 y
Jupiter	71490	$1.899 \cdot 10^{27}$	9.92 h	3.13 °	$778.6 \cdot 10^{6}$	0.0489	1.304°	11.86 y
Saturn	60270	$568.5 \cdot 10^{24}$	10.66 h	26.73 °	$1.433 \cdot 10^{9}$	0.0565	2.485°	29.46 y
Uranus	25560	$86.83 \cdot 10^{24}$	17.24 h	97.77 °	$2.872 \cdot 10^{9}$	0.0457	0.772°	84.01 y
Neptune	24760	$102.4 \cdot 10^{24}$	16.11 h	28.32 °	$4.495 \cdot 10^{9}$	0.0113	1.796°	164.8 y
Pluto	1195	$12.5 \cdot 10^{21}$	6.387 d	122.5 °	$5.870 \cdot 10^{9}$	0.2444	17.16°	247.7 y

Table 2: Gravitational parameter (*μ*) for the Sun, the planets and the moon

Object	**μ (km^3/s^2)**	**Object**	**μ (km^3/s^2)**
Sun	132 712 000 000	Jupiter	126 686 000
Mercury	22 030	Saturn	37 931 000
Venus	324 900	Uranus	5 794 000
Earth	398 600	Neptune	6 835 100
Moon	4903	Pluto	830
Mars	42 828		

APPENDIX 2/ *LAMPIRAN 2*

EQUATIONS

Mean anomaly

$$M_e = E - e\sin E$$

$$E = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}\right)$$

Coordinate transformation

$$[\mathbf{R}_1(\phi)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

$$[\mathbf{R}_2(\phi)] = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$[\mathbf{R}_3(\phi)] = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\mathbf{Q}]_{\bar{x}X} = \begin{bmatrix} \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i & -\cos\Omega\sin\omega - \sin\Omega\cos i\cos\omega & \sin\Omega\sin i \\ \sin\Omega\cos\omega + \cos\Omega\cos i\sin\omega & -\sin\Omega\sin\omega + \cos\Omega\cos i\cos\omega & -\cos\Omega\sin i \\ \sin i\sin\omega & \sin i\cos\omega & \cos i \end{bmatrix}$$

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