Second Semester Examination
2018/2019 Academic Session

June 2019

ESA251 – Control System Theory
[Teori Sistem Kawalan]

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **ELEVEN (11)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEBELAS (11)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **FOUR (4)** questions. **All questions are COMPULSORY**.

*[**Arahan :** Jawab **EMPAT (4)** soalan. **Semua soalan WAJIB dijawab**.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

1. **Figure 1** shows a mass-spring-damper model for automotive suspension systems. K_1 and K_2 are the spring constant, M is the system's mass and fv is the damping effect.

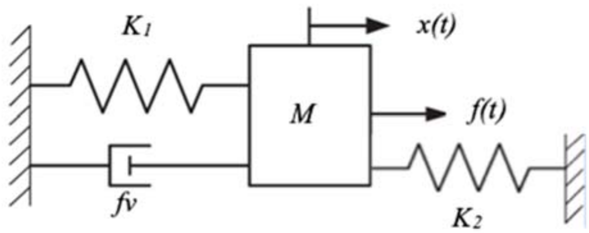


Figure 1

(a). Using a free body diagram, model the system using **differential equations**, **state-space representation** and the **transfer function** that relates the output and the input of the system.

(20 marks)

(b). Based on your transfer function in [a], for each of the case below, determine the **system poles** and define the system damping e.g. un-damped**.** Also find the **output response function in time domain *c(t)*** and **sketch the step response** if the step input is 1 $u(t)$. For all cases, assume M = 1 kg and K_2 = 1 N/m.

(i). K_1 = 99 N/m fv = 0 N-s/m
(ii). K_1 = 9 N/m fv = 10 N-s/m
(iii). K_1 = 24 N/m fv = 10 N-s/m
(iv). K_1 = 49 N/m fv = 10 N-s/m

(30 marks)

(c). Based on your transfer function in [b](iv), determine the damping ratio (ξ), natural frequency,(ω_n), overshoot (OS%), peak time (T_p) and settling time (T_s). Also, determine the steady state error (E_{ss}) for the step same input given in [b].

(50 marks)

2. **Figure 2** shows a closed loop control system with a unity feedback. The performance and the stability of the output *Y(s)* is determined by the value of *K* in the controller.

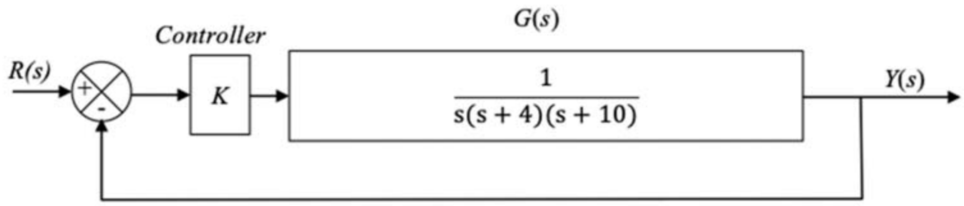


Figure 2

(a). **State** the stability for Routh-Hurwitz criterion. **Obtain** the closed-loop transfer function *T(s)* and **determine** the gain margin (range of *K*) to keep the system within the stability limit.

(20 marks)

(b). **Find** a value of *K* that causes the system to operate at marginally stable condition and determine the un-damped natural frequency.

(30 marks)

(c). **State** the stability criterion in root locus. Using the root-locus method, **determine** the parameters –whichever appropriate e.g. break-away/break-in point, gain at break-away/break-in point, imaginary crossing, gain at imaginary crossing, departure/arrival angle. Then **place** the open-loop poles and zeros and **plot** the root locus on the graph paper provided.

(50 marks)

3. Refer to **Figure 2**.

(a). **Construct** a Bode-plot of the system on the semi-log graph paper provided (starting frequency = 0.01 rad/s).

(20 marks)

(b). From your Bode-plot, **obtain** the gain margin GM(dB) and the frequency (ωGM), the phase margin ΦM(degrees) and the frequency ($\omega\Phi$M). Also, **determine** the range of K to keep the system stable.

(30 marks)

(c). From your Bode-plot,

(i). Determine the value of K and the phase margin ΦM if the requirement for Gain Margin = 15 dB.

(ii). Determine the value of K and the gain margin GM if the requirement for Phase Margin = 30°.

(50 marks)

4. Refer to **Figure 2**, (please treat every sub-question below as case-by-case basis).

   (a). Give your comments about the system's damping ratio, settling time *Ts* and the stability between K=500 and K=560. (you may refer to your Routh Hurwitz table in Q2(a)).

   (20 marks)

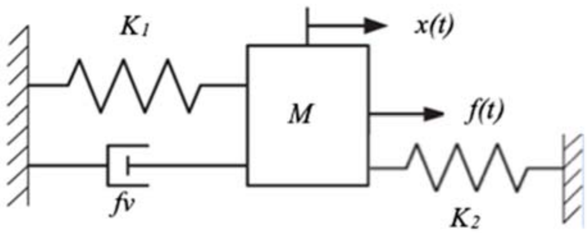
   (b). Assume K=1, find a steady state error (E_{ss}) if the input is a ramp input of 10 tu(t). If such error exists, please propose a controller that is able to completely eliminate the error and prove your answer with your proposed controller.

   (30 marks)

   (c). Assume controller = K(s+5)(s+6), based on the poles and zeros, please sketch the root locus (without detail calculation) and based on your sketch, comment on the system stability for any value of K.

   (50 marks)

1. ***Gambarajah 1** menunjukkan satu sistem mass-spring-damper untuk suspensi sebuah kenderaan. K_1 dan K_2 merupakan pemalar spring untuk kedua-dua spring, M adalah berat dan fv adalah kesan redaman.*



Gambarajah 1

*(a). Dengan menggunakan gambarajah blok bebas, dapatkan model sistem tersebut menggunakan **persamaan perbezaan**, **'state space'** dan **rangkap pindah.***

(20 markah)

*(b). Berdasarkan rangkap pindah anda dalam [a], untuk setiap kes dibawah, tentukan 'system poles' dan tentukan jenis redaman- contoh, un-damped. Dapatkan juga **fungsi sambutan keluaran dalam time domain c(t)** dan **lakarkan sambutan langkah** jika satu masukan langkah dikenakan ialah 1 u(t). Untuk semua kes dibawah, anggapkan nilai M = 1 kg dan K_2 = 1 N/m.*

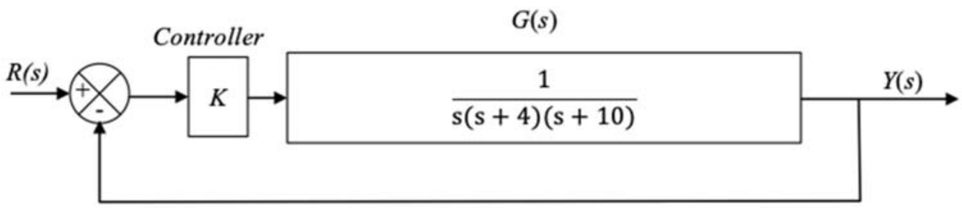
(i). K_1 = 99 N/m fv =0 N-s/m
(ii). K_1 = 9 N/m fv =10 N-s/m
(iii). K_1 = 24 N/m fv =10 N-s/m
(iv). K_1 = 49 N/m fv =10 N-s/m

(30 markah)

(c). Berdasarkan rangkap pindah anda di S1[b](iv), dapatkan nisbah redaman (ξ), Frekuensi semulajadi (ωn), 'overshoot' (OS%), masa puncak (T_p) *dan 'settling time' (T_s). Tentukan juga nilai ralat keadaan mantap (Ess) untuk masukan langkah sebanyak 1 u(t).*

(50 markah)

2. ***Gambarajah 2*** *menunjukkan satu sistem dengan suapbalik uniti. Nilai prestasi dan kestabilan keluaran Y(s) ditentukan oleh nilai pengawal K.*



Gambarajah 2

(a). *Terangkan secara ringkas kriteria kestabilan Routh-Hurwitz. Dapatkan rangkap pindang gelung tutup T(s) dan tentukan julat gandaan (Julat K) untuk memastikan sistem didalam keadaan stabil.*

(20 markah)

(b). *Hitungkan nilai gandaan K yang menyebabkan sistem ini beroperasi pada kestabilan marginal dan tertukan nilai frekunsi tanpa redaman.*

(30 markah)

(c). *Terangkan secara ringkas kriteria stabiliti dalam teknik londar punca. Dengan menggunakan kaedah londar punca, dapatkan parameter yang berkaitan [titik keluar/masuk , gandaat pada titik masuk/keluar, titik lintasan paksi khayalan, gandaan di titik paksi hayala, sudut berlepas/ mendarat] Berdasarkan parameter tersebut, lakarkan londar punca diatas kertas geraf yang disediakan.*

(50 markah)

3. Berdasarkan ***Gambarajah 2****.*

(a). Bina sebuah Bode-plot untuk system tersebut diatas kertas geraf semi-log yang disediakan.(frekuensi permulaan = 0.01 rad/s).

(20 markah)

(b). Daripada lakaran Bode-plot tersebut, hitungkan jidar gandaan, GM(dB) dan frekuensi jidar gandaan (ωGM), jidar fasa ΦM dan frekuensi jidar fasa ($\omega\Phi$M). Kirakan juga julat K untuk kestabilan sistem.

(30 markah)

(c). Daripada Bode-plot anda,

(i). Tentukan nilai K dan jidar fasa ΦM untuk memenuhi keperluan Jidar gandaan GM =15dB.

(ii). Tentukan nilai K dan jidar gandaan GM untuk memenuhi keperluan jidar fasa =30°.

(50 markah)

4. Sila rujuk ***Gambarajah 2****. (Sila anggap setiap soalan dibawah secara berasingan)*

(a). Berikan komen anda tentang nisbah redaman sistem, masa penetapan T_s dan kestabilan antara nilai K=500 dan K=560.(anda boleh rujuk kepada jawapan anda dalam jadual Routh Hurwitz dalam S2(a))

(20 markah)

(b). Anggapkan K=1, kira ralat keadaan mantap (Ess) jika masukan ramp sebanyak 10 tu(t). Sekiranya wujud ralat tersebut, sila cadangkan sebuah pengawal yang boleh menghilangkan ralat tersebut ubtuk masukan ramp dan buktikan jawapan anda dengan menggunakan pengawal tersebut.

(30 markah)

(c). Anggapkan pengawal = K(s+5)(s+6), berdasarkan poles dan zeros, sila lakar londar punca (tanpa pengiraan yang lengkap) dan daripada lakaran tersebut, berikan komen anda tentang kestabilan sistem untuk semua nilai K.

(50 markah)

APPENDIX A/ *LAMPIRAN A*

LAPLACE TRANSFORM TABLE

Laplace transform	*Time function*	*Description of time function*
1		A unit impulse
$\frac{1}{s}$		A unit step function
$\frac{e^{-st}}{s}$		A delayed unit step function
$\frac{1-e^{-st}}{s}$		A rectangular pulse of duration T
$\frac{1}{s^2}$	t	A unit slope ramp function
$\frac{1}{s^3}$	$\frac{t^2}{2}$	
$\frac{1}{s+a}$	e^{-at}	Exponential decay
$\frac{1}{(s+a)^2}$	$t e^{-at}$	
$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	
$\frac{a}{s(s+a)}$	$1 - e^{-at}$	Exponential growth
$\frac{a}{s^2(s+a)}$	$t - \frac{(1-e^{-at})}{a}$	
$\frac{a^2}{s(s+a)^2}$	$1 - e^{-at} - ate^{-at}$	
$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}$	
$\frac{ab}{s(s+a)(s+b)}$	$1 - \frac{b}{b-a}e^{-at} + \frac{a}{b-a}e^{-bt}$	
$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)} + \frac{e^{-bt}}{(c-a)(a-b)} + \frac{e^{-ct}}{(a-c)(b-c)}$	
$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	Sine wave
$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	Cosine wave
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	Damped sine wave
$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	Damped cosine wave
$\frac{\omega^2}{s(s^2+\omega^2)}$	$1 - \cos \omega t$	
$\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}$	$\frac{\omega}{\sqrt{(1-\zeta^2)}}e^{-\zeta\omega t}\sin[\omega\sqrt{(1-\zeta^2)}t]$	
$\frac{\omega^2}{s(s^2+2\zeta\omega s+\omega^2)}$ with $\zeta < 1$	$1 - \frac{1}{\sqrt{(1-\zeta^2)}}e^{-\zeta\omega t}\sin[\omega\sqrt{(1-\zeta^2)}t+\phi]$ with $\zeta = \cos\phi$	

APPENDIX B/ *LAMPIRAN B*

SECOND ORDER TIME DOMAIN SPECIFICATION
(*SPESIFIKASI DOMAIN MASA SISTEM TERTIB KEDUA*)

% Overshoot, $\%C_P = 100e^{-\left[\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right]}$

Peak Time, *Masa puncak*, $t_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

Settling time, *Masa pengenapan*, $t_s = \dfrac{4}{\zeta\omega_n}$

Error Steady State, *Ralat keadaan mantap*, $e_{ss} = \lim_{s\to 0}\dfrac{sR(s)}{1+G(s)H(s)}$

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