UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2002/2003 Academic Session

February - March 2003

ZCT 304E/3 - Electricity and Magnetism II

Time : 3 hours

Please check that the examination paper consists of **ELEVEN** printed pages before you commence this examination.

Answer all FIVE questions. Students are allowed to answer all questions in English OR bahasa Malaysia OR combination of both.

1. The surface of a sphere of radius a (centered at the origin) is charged with a uniform surface charge density σ.

   (a) Calculate the total charge Q' on the sphere. (5/20)

   (b) Find the force exerted by this charge distribution on a point charge q located on the z axis if:

      (i) $z > a$ (10/20)

      (ii) $z < a$ (5/20)

2. A square of side a is located in the xy plane with its center at the origin.

   (a) Calculate the magnetic induction ($\vec{B}$) at a point located on the z-axis if a current I' flows around the square. (15/20)

   (b) Show that your answer gives the result $\dfrac{2\sqrt{2}\mu_0 I'}{\pi a}$ for the magnetic induction at the center of the square. (5/20)

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3. A certain $\vec{B}$ field is given in cylindrical coordinates by:

$$\vec{B} = 0 \qquad 0 < \rho < a$$

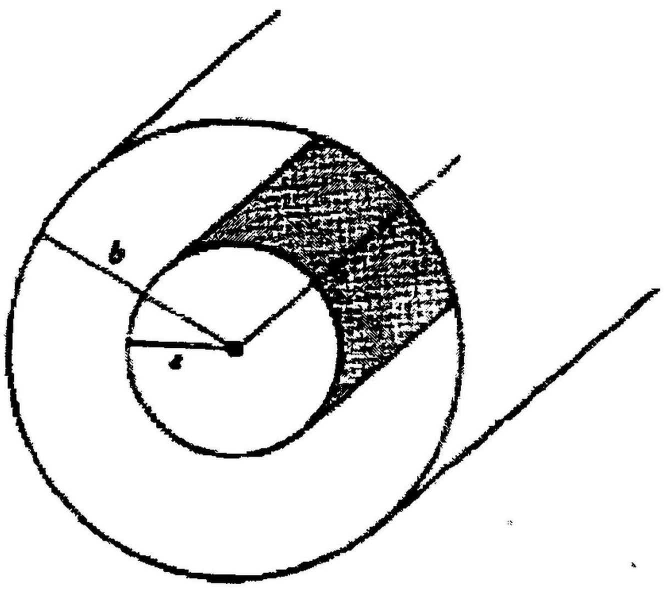
$$\vec{B} = \frac{\mu_0 I}{2\pi\rho}\left(\frac{\rho^2 - a^2}{b^2 - a^2}\right)\hat{\phi} \qquad a < \rho < b$$

$$\vec{B} = \frac{\mu_0 I}{2\pi\rho}\hat{\phi} \qquad b < \rho$$

(a) Find the current density $\vec{J}$ everywhere. (15/20)

(b) How might one produce such a field? (5/20)

4. The region between two cylinders (coaxial and infinite, as shown in the following figure) is filled with charge of density $\rho_{ch} = Ae^{-\alpha\rho}$. Calculate $\vec{E}$ everywhere.



(20/20)

5. An infinitely long, thin wire carrying current I is surrounded coaxially by a cylindrical shell (radii a and b) of l.i.h magnetic material of susceptibility χ.

(a) Find $\vec{B}$ and $\vec{H}$ everywhere. (10/20)

(b) Find the distribution of magnetisation current densities. (10/20)

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Mathematical Guidance

Possibly Useful Integrals:

$$\int_{-1}^{1}\frac{(z-r\mu)d\mu}{(r^2+z^2-2zr\mu)^{3/2}}=\frac{1}{z^2}\left(\frac{z-r}{|z-r|}+\frac{z+r}{|z+r|}\right)$$

$$\int_{-1}^{1}\frac{d\mu}{(r^2+z^2-2zr\mu)^{1/2}}=\frac{1}{zr}\left(|z+r|-\lfloor z-r\rfloor\right)$$

$$\int\frac{dx}{(x^2+a^2)^{3/2}}=\frac{1}{a^2}\cdot\frac{x}{(x^2+a^2)^{1/2}}$$

$$\int xe^{ax}dx=e^{ax}\left[\frac{x}{a}-\frac{1}{a^2}\right]$$

<u>Useful Constants</u>

$$k=\frac{1}{4\pi\varepsilon_0}=8.99\times10^9\,\frac{N\cdot m^2}{C^2}$$

$$e=1.60\times10^{-19}C$$

$$\varepsilon_0=8.85\times10^{-12}\frac{C^2}{N\cdot m^2}$$

$$\mu_0=4\pi\times10^{-7}\frac{T\cdot m}{A}$$

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Vector Calculus

Cartesian Coordinates

$$\vec{\nabla} u = \hat{x}\frac{\partial u}{\partial x} + \hat{y}\frac{\partial u}{\partial y} + \hat{z}\frac{\partial u}{\partial z}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = \hat{x}(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}) + \hat{y}(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}) + \hat{z}(\frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x})$$

$d\tau = dxdydz$ $\quad da_x = \pm dydz \quad da_y = \pm dxdz \quad da_z = \pm dxdy$

Cylindrical Coordinates

$$\vec{\nabla} u = \hat{\rho}\frac{\partial u}{\partial \rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial u}{\partial \phi} + \hat{z}\frac{\partial u}{\partial z}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = \hat{\rho}(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}) + \hat{\phi}(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}) + \hat{z}[\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{1}{\rho}\frac{\partial A_\rho}{\partial \phi}]$$

$d\tau = \rho d\rho d\phi dz$ $\quad da_\rho = \pm\rho d\phi dz \quad da_\phi = \pm d\rho dz \quad da_z = \pm\rho d\rho d\phi$

$\hat{\rho} = \cos\phi\hat{x} + \sin\phi\hat{y}$ $\quad \hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y}$

Spherical Coordinates

$$\vec{\nabla} u = \hat{r}\frac{\partial u}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial u}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial u}{\partial \phi}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\vec{\nabla}\times\vec{A} = \frac{\hat{r}}{r\sin\theta}[\frac{\partial}{\partial \theta}(\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial \phi}] + \frac{\hat{\theta}}{r}[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)] + \frac{\hat{\phi}}{r}[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}]$$

$d\tau = r^2 \sin\theta dr d\theta d\phi$ $\quad da_r = \pm r^2 \sin\theta d\theta d\phi \quad da_\theta = \pm r\sin\theta dr d\phi \quad da_\phi = \pm r dr d\theta$

$\hat{r} = \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z}$ $\quad \hat{\theta} = \cos\theta\cos\phi\hat{x} + \cos\theta\sin\phi\hat{y} - \sin\theta\hat{z}$

$\hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y}$

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Important Equations

Maxwell's Equations:

$\vec{\nabla}\cdot\vec{D} = \rho_f$ $\vec{\nabla}\cdot\vec{B} = 0$ $\vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{dt}$ $\vec{\nabla}\times\vec{H} = J_f + \frac{\partial\vec{D}}{dt}$

Lorentz Force: $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

Equation of Continuity: $\vec{\nabla}\cdot\vec{J}_f + \frac{\partial\rho_f}{dt} = 0$

Coulomb's Law: $\vec{F}_q = \sum_i \frac{qq_i\vec{R}_i}{4\pi\varepsilon_0 R_i^3}$ (for a collection of point charges)

$\vec{F}_q = \frac{q}{4\pi\varepsilon_0}\int_{L'}\frac{\lambda(\vec{r}\,')\vec{R}ds'}{R^3}$ (for a line charge distribution)

$\vec{F}_q = \frac{q}{4\pi\varepsilon_0}\int_{S'}\frac{\sigma(\vec{r}\,')\vec{R}da'}{R^3}$ (for a surface charge distribution)

$\vec{F}_q = \frac{q}{4\pi\varepsilon_0}\int_{V'}\frac{\rho(\vec{r}\,')\vec{R}d\tau'}{R^3}$ (for a volume charge distribution)

Electric Field: $\vec{E} = \frac{\vec{F}_q}{q}$

Electric Flux: $\Phi_e = \int \vec{E}\cdot d\vec{a}$

Gauss' Law: $\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_t}{\varepsilon_0}$ (integral form)

$\vec{\nabla}\cdot\vec{E} = \frac{\rho(\vec{r})}{\varepsilon_0}$ (differential form)

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Scalar Potential: $\phi(\vec{r}) = \sum_i \frac{q_i}{4\pi\varepsilon_0 R_i}$ (for a collection of point charges)

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{L'} \frac{\lambda(\vec{r}\,')ds'}{R}$$ (for a line charge distribution)

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{S'} \frac{\sigma(\vec{r}\,')da'}{R}$$ (for a surface charge distribution)

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho(\vec{r}\,')d\tau'}{R}$$ (for a volume charge distribution)

Potential Energy: $U_e(\vec{r}) = q\phi(\vec{r})$ (for an isolated point charge)

$$U_e = \frac{1}{2}\sum_i q_i\phi_i(\vec{r}_i)$$ (for a collection of point charges)

$$U_e = \frac{1}{2}\int_L \lambda(\vec{r})\phi(\vec{r})ds$$ (for a line charge distribution)

$$U_e = \frac{1}{2}\int_S \sigma(\vec{r})\phi(\vec{r})da$$ (for a surface charge distribution)

$$U_e = \frac{1}{2}\int_V \rho(\vec{r})\phi(\vec{r})d\tau$$ (for a volume charge distribution)

$u_e = \frac{1}{2}\varepsilon_0 E^2$ (energy density in an electric field)

$U_e = \int u_e d\tau$ (total energy)

Multipole Moments: $Q = \sum_i q_i$ or $Q = \int_L \lambda ds$ or $Q = \int_S \sigma da$ or $Q = \int_V \rho d\tau$ (monopole)

$\vec{p} = \sum_i q_i\vec{r}_i$ or $\vec{p} = \int_L \lambda\vec{r}ds$ or $\vec{p} = \int_S \sigma\vec{r}da$ or $\vec{p} = \int_V \rho\vec{r}d\tau$ (dipole)

Boundary Conditions: $E_{t2} - E_{t1} = 0$ and $E_{n2} - E_{n1} = \frac{\sigma}{\varepsilon_0}$ (electric field)

$\phi_2 = \phi_1$ (scalar potential)

$B_{n2} - B_{n1} = 0$ and $\vec{B}_{t2} - \vec{B}_{t1} = \mu_0\vec{K}\times\hat{n}$ (magnetic induction)

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Electricity in Matter:

$\rho = \rho_f + \rho_b$ (free charge and bound charge)

$\rho_b = -\vec{\nabla} \cdot \vec{P}$ and $\sigma_b = \vec{P} \cdot \hat{n}$ (bound charge densities)

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ (definition of electric displacement)

$\vec{D} = \kappa_e \varepsilon_0 \vec{E} = \varepsilon \vec{E}$ (for an l.i.h. dielectric)

$u_e = \frac{1}{2} \vec{D} \cdot \vec{E}$ (energy density in matter)

$\oint \vec{D} \cdot d\vec{a} = Q_{f,in}$ and $\vec{\nabla} \cdot \vec{D} = \rho_f$ (Gauss' Laws for $\vec{D}$)

Electric Current: $I = \frac{dq}{dt} = \int \vec{J} \cdot d\vec{a} = \int \vec{K} \cdot d\vec{s}$

$\vec{J} = \rho \vec{v}$ $\vec{K} = \sigma \vec{v}$ (current density)

$I d\vec{s} = \vec{K} da = \vec{J} d\tau$ (current elements)

$\vec{J}_f = \sigma \vec{E}$ (Ohm's Law)

Magnetostatic Force: $\vec{F}_{C' \to C} = \frac{\mu_0}{4\pi} \oint_C \oint_{C'} \frac{I d\vec{s} \times \left(I' d\vec{s}' \times \hat{R} \right)}{R^2}$

Magnetic Induction: $\vec{B} = \frac{\mu_0}{4\pi} \oint_{C'} \frac{I' d\vec{s}' \times \hat{R}}{R^2}$ (for a filamentary current)

$\vec{B} = \frac{\mu_0}{4\pi} \int_{S'} \frac{\vec{K}' \times \hat{R} da'}{R^2}$ (for a surface current)

$\vec{B} = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{J}' \times \hat{R} d\tau'}{R^2}$ (for a volume current)

Ampere's Law: $\oint_C \vec{B} \cdot d\vec{s} = \mu_0 I_t$ (integral form)

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ (differential form)

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Vector Potential: $\vec{B} = \vec{\nabla} \times \vec{A}$

$$\vec{A} = \frac{\mu_0}{4\pi}\oint_{C'} \frac{I'd\vec{s}\,'}{R}$$ (for a filamentary current)

$$\vec{A} = \frac{\mu_0}{4\pi}\int_{S'} \frac{\vec{K}'da'}{R}$$ (for a surface current)

$$\vec{A} = \frac{\mu_0}{4\pi}\int_{V'} \frac{\vec{J}'d\tau'}{R}$$ (for a volume current)

Magnetic Flux: $\Phi_b = \int \vec{B} \cdot d\vec{a}$

Faraday's Law: $\varepsilon_t = \oint \vec{E}_t \cdot d\vec{s} = \frac{-d\Phi_b}{dt}$ (integral form)

$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{dt}$$ (differential form)

Magnetism in Matter:

$\vec{J} = \vec{J}_f + \vec{J}_m$ (free current plus magnetisation current)

$\vec{J}_m = \vec{\nabla} \times \vec{M}$ (magnetisation volume current density)

$\vec{K}_m = \vec{M} \times \hat{n}$ (magnetisation surfacet density)

$\vec{B} = \mu_0(\vec{H} + \vec{M})$ (definition of magnetic field)

$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H}$ (for l.i.h. material)

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