TWO-DIMENSIONAL FATIGUE CRACK GROWTH ANALYSIS USING EXTENDED FINITE ELEMENT METHOD (XFEM)

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Declaration

This work has not previously been accepted in substance for any degree and is not being concurrently submitted in candidature for any degree.

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Abstrak

Tesis ini mencadangkan keupayaan teknik FEM yang bertambah baik yang dikenali sebagai kaedah elemen terhingga lanjutan (XFEM) untuk menganalisis pertumbuhan retakan lesu di bawah beban amplitud yang berterusan. Tesis ini membandingkan penyebaran retakan berangka model dalam spesimen keluli lentur dengan uji kaji dan simulasi sebelum ini menggunakan teknik FEM tradisional. Tujuan kajian ini adalah untuk melakukan analisis pereputan retakan retakan untuk menentukan bentuk sambungan retak. Model spesimen ujian disediakan berdasarkan parametres geometrik dan keadaan beban kitaran. Analisis berangka dilakukan menggunakan Abaqus 6.12-2. Penyebaran retak disimulasikan dengan menggunakan kaedah elemen terhingga berangka numerik (XFEM) di bawah kriteria lesu kitaran rendah menggunakan pendekatan kitaran langsung. Tesis ini, mengesahkan kesahihan kaedah berangka ini untuk penyebaran retak. Keputusan berangka arah perambatan retakan dibandingkan dengan penemuan eksperimen dalam spesimen sebenar dan sebelum ini disimulasikan menggunakan kaedah teknik penyesuaian dalam kesusasteraan [1]. Hasil daripada kaedah yang dicadangkan ini tidak seperti yang dijangkakan kerana pembiakan retak itu tidak seperti digambarkan dalam kesusasteraan. Telah ditunjukkan bahawa laluan retak yang dikira tidak membenarkan pertimbangan sebelumnya.

Abstract

This thesis proposes the ability an improved FEM technique known as extended finite element method (XFEM) to analyse fatigue crack growth under constant amplitude loading. This thesis compares numerically modelled crack propagation in a bending steel specimen with experimentally conducted and simulated previously using traditional FEM technique. The aim of the study was to perform numerical analysis of crack propagation in order to determine the shape of the crack extension. Model of the test specimen was prepared based on geometric parametres and cyclic load condition. The numerical analysis was performed using Abaqus 6.12-2. Crack propagation was simulated by using the numerical extended finite element method (XFEM) under low-cycle fatigue criterion using direct cyclic approach. This thesis, verifies the validity of this numerical method for crack propagation. The numerical results of the direction of the crack propagation were compared with the experimental findings in a real specimen and previously simulated using adapative technique method in the literature [1]. The outcome of the proposed method is not as what was expected due to the crack propagation was not like illustrated in the literature. It is shown that the computed crack path does not justify the previous findings.

Chapter 1 Introduction

1.1 Research Background

Modeling crack growth in a traditional finite element framework is inefficient due to the need for the mesh to match the geometry of the discontinuity. This becomes a major difficulty when treating problems with evolving discontinuities where the mesh must be regenerated at each step. Moreover, the crack tip singularity needs to be accurately represented by the approximation [1]. Due to the fact that standard finite element methods are based on piecewise differentiable polynomial approximations, they are not well suited to problems with discontinuous and/or singular solutions. Typically, finite element methods require significant mesh refinement or meshes which conform with these features to get accurate results. In response to this deficiency of standard finite element methods, extended finite elements have been developed.

Extended finite element method (XFEM) which is also known as partition of unity method (PUM) is a numerical technique that extends the classical finite element method (FEM) approach by extending the solution space for solutions to differential equations with discontinuous functions. The extended finite element method was developed to ease difficulties in solving problems with localized features that are not efficiently resolved by mesh refinement

In isotropic linear elastic fracture analysis, two sets of functions are used to handle the presence of a crack: a discontinuous function for the crack line and a set of asymptotic functions for the crack tip [5]. For the purpose of fracture analysis, the enrichment functions typically consist of the near-tip asymptotic functions that capture the singularity around the crack tip and a discontinuous function that represents the jump in displacement across the crack surfaces. The approximation for a displacement vector function $\mathbf{u}$ with the partition of unity enrichment is

$$u = \sum_{I=1}^{N} N_I(x)\,[u_I + H(x)a_I + \sum_{\alpha=1}^{4} F_\alpha(x)b_I^\alpha], \tag{1.1}$$

where $N_I(x)$ are the usual nodal shape functions; the first term on the right-hand side of the above equation, u_I, is the usual nodal displacement vector associated with the continuous part of the finite element solution; the second term is the product of the nodal enriched degree of freedom vector, a_I, and the associated discontinuous jump function *H(x)* across the crack surfaces; and the third term is the product of the nodal enriched degree of freedom vector, b_I^α, and the associated elastic asymptotic crack-tip functions, $F_\alpha(x)$. The first term on the right-hand side is applicable to all the nodes in the model; the second term is valid for nodes whose shape function support is cut by the crack interior; and the third term is used only for nodes whose shape function support is cut by the crack tip.

The modified Heaviside function associated with the crack line is

$$H(x) = f(x) = \begin{cases} +1, & if\ (x - x^*) \cdot n \geq 0 \\ -1, & otherwise \end{cases} \tag{1.2}$$

where x is a sample (integration) point, x is the projection of x onto the crack surface, and n is the unit outward normal to the crack at x. Figure 1.1 shows the type of enrichment active on each node in an isotropic fracture model.

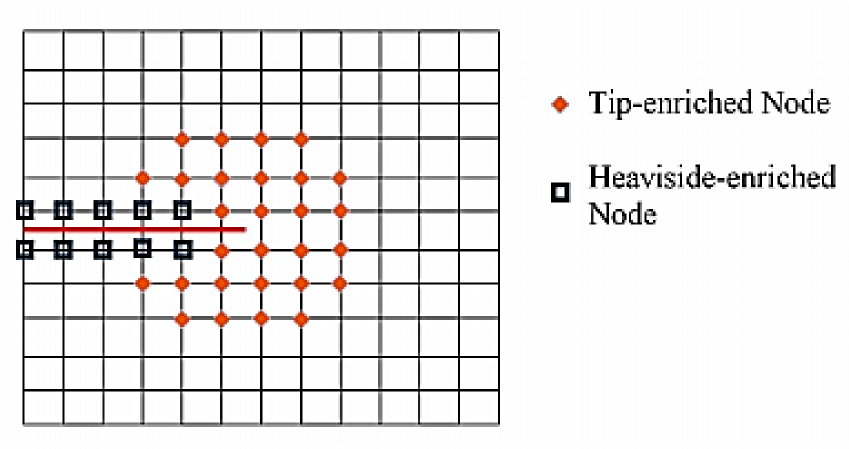

Figure 1.1 : Nodal enrichment scheme for bulk fracture [3].

Components are subjected to a fluctuating load of a certain magnitude for a sufficient amount of time, small cracks will nucleate in the material. Over time, the cracks will propagate, up to the point where the remaining cross-section of the component is not able to carry the load, at which the component will be subjected to sudden fracture [1]. This process is called fatigue, and is one of the main causes of failures in structural and mechanical components [2]. In order to assess the safety of the component, engineers need to estimate its expected lifetime. The fatigue life is the sum of the number of loading cycles required for a fatigue crack to nucleate/initiate, and the number of cycles required for the crack to propagate until its critical size has been reached. Estimations of the fatigue crack propagation rate, da/dN, are normally based on a relation with the range of the stress intensity factor, ΔK, which is a linear elastic fracture mechanics (LEFM) parameter for quantifying the load and geometry of the crack.

The loading and displacement of a crack can be described by the three modes of fracture, each with its own stress intensity factor. The different modes require different values for the constants in the crack propagation law. In the case of mixed-mode fatigue, it may be necessary to use an effective mixed-mode stress intensity factor. The crack propagation life can be estimated by integrating equation (2.4). However, the stress intensity factors Kmax and Kmin are normally functions of the crack length a, and depend on the geometry of the structure. Analytical integration of equations (2.3) and (2.4) is rarely viable for complicated geometries. Instead, crack propagation problems are normally solved using some computational method, the Finite Element Method (FEM) [12]. The crack propagation process is then solved in a step-wise manner. For each step, the crack is advanced a small length, and the number of cycles required for the next crack increment is estimated using one of the crack propagation laws. In order to overcome remeshing techniques which costs time XFEM is suitable for its capability for crack propagation problems.

1.2 Problem Statement

Inspections are used to look for cracks, and if a crack is found the component can be fixed or replaced as necessary. Inspection intervals could then be set at some fraction of the fatigue fracture life of the structure. Fatigue fracture life is calculated as the number of cycles required to grow a crack from a minimum detectable size to a critical size when the structure fails. Analytical fatigue crack growth models are available in (ABAQUS, C., 2012. Analysis user’s manual) for many generalized geometries including an edge crack, center crack, and etc. As geometry, loading and boundary conditions become more complex, the analytical equations quickly become more complicated and it may be difficult to account for all effects on crack propagation. The Finite Element Method (FEM) has been used for decades to assist engineers in analysing complex, cracked structures. Until recently, cracks had to be modeled as part of the structure's geometry. As the crack grew, the model would be rebuilt and remeshed, requiring significant user interaction or specialized programs. The eXtended Finite Element Method (XFEM) was developed in 1999 by [7], where cracks could be defined arbitrarily, independent of the mesh. This method fit to fatigue crack growth, where a crack propagates along a solution dependent path, independent of the mesh. The main aimof this thesis is to take a look at the ability of XFEM to evaluate crack growth.

1.3 Scope of Research

Before the XFEM can be used to model fatigue crack growth and estimate fatigue fracture life in everyday structures with complex geometry, complex loading and boundary conditions, material variability, and numerous other external factors, we must first focus on a simplified model with a known analytical solution. This thesis focuses on a specially designed specimen under cyclic Lateral Force Bending with Holes (LFBH). The LFBH offers simplifieded geometry, simplified boundary and loading conditions, and has been studied since the beginning of fracture mechanics. The scope is further limited to Linear Elastic Fracture Mechanics (LEFM) in two dimensions. LFBH crack propagation data and illustration of the simulation of fatigue crack growth was previously collected and illustrated by [1] for Structural Steel E335, thus the analysis will be limited to the same homogeneous, isotropic material and specimen geometry. All finite element analyses were performed in Abaqus 6-12.2.

1.4 Objectives

The primary goals of this thesis are to investigate the capabilities of the eXtended Finite Element Method for modeling crack propagation and estimating fatigue fracture life. The following objectives were met to realize these goals:

1. To develop the FCG life and to illustrate graphically FCG life and crack propagation of the specimen used in [1].

2. To verify the result of simulation of [1] using XFEM by simulating the specimen in two-dimensional under low-cycle fatigue analysis using direct cyclic approach.

Chapter 2 Literature Review

The following literature review has been provided to give a brief background into fracture mechanics and fatigue crack growth laws that will be used in this thesis. XFEM technique in fatigue fracture analysis are considered, and procedure of their use in Abaqus are discussed.

Bulk fracture mechanics is concerned with systems where only a single material factors in to the fracture analysis. The term crack will be used to denote any physical discontinuity in the material such as voids or fractures. When a given crack is subjected to loading, these loads can be decomposed into three modes, as shown in Figure 2.1. Plane strain conditions are assumed for the present work and therefore mode III loading may be neglected. A single type of loading tends to be the exception, rather than the rule, and so the term mode-mixity is used when there is more than one loading mode operating on a given crack [8].

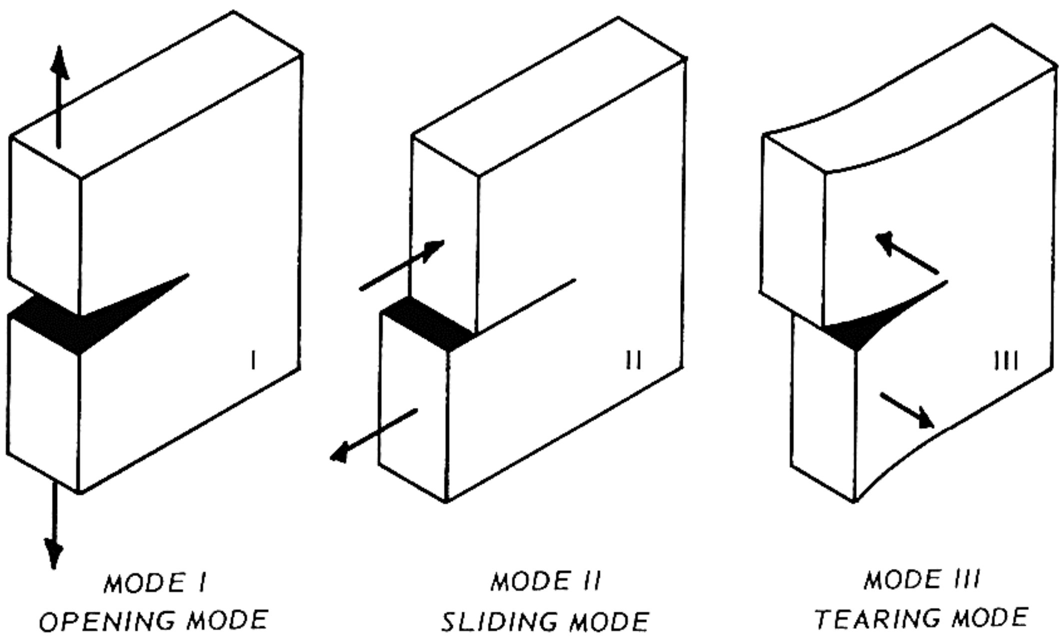

Figure 2.1 : Three modes of crack displacement at the crack tip.

One of the central assumptions of fracture mechanics is that, if one get sufficiently close to the crack tip, the stress, strain, and displacement fields become independent of the specimen geometry and the manner in which the sample is loaded. The fields near the crack tip may then be characterized by the three stress intensity factors(SIF) - KI, KII, and KIII corresponding to the three types of loading shown in Figure 2.1.

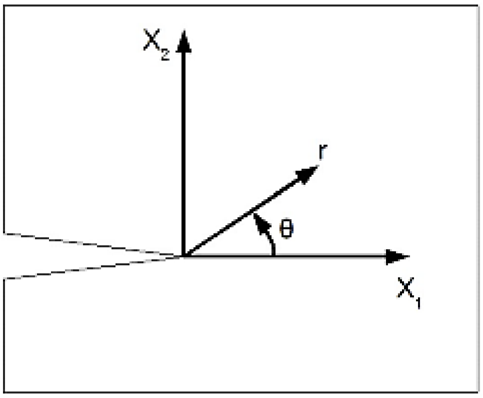

Figure 2.2 : Crack Tip.

It is evident that the above displacement fields (and the corresponding stress fields) are asymptotic in nature. This would imply that the stresses go to infinity as one approaches the crack tip. This, of course, does not occur, but it does go to the heart of the validity of the specimen-independence assumption listed above.

For a crack in a perfectly elastic material, the asymptotic solution are expected to become increasingly accurate as one approaches the crack tip [5]. In regions far away from the tip, the fields are influenced by boundary conditions, sample geometry, etc. and therefore do not conform to the asymptotic solution. At the other extreme, when very close to the crack tip, the solution also breaks down - the infinite stresses do not occur at least in part because of material plasticity, a clear violation of the elastic assumption inherent in LEFM [5].

In between these two cases there exists a region termed the region of K dominance, where one is close enough to the crack tip to ignore the specifics of the test specimen but far enough away for the assumptions of linearity and elasticity to still hold. It is within this area that the fields are governed solely by the stress intensity factors [5].

Another way of characterizing a crack is to use so-called energy methods. The energy release rate is one such method. It is based on the assumption that, whatever the specifics involved in a particular crack propagation, energy is dissipated[9]. It takes a small amount of energy to create the new free surfaces produced by crack growth. Even if the assumptions of LEFM are violated very near the crack tip (plasticity, etc.), as long as such processes remain constant during propagation, the energy dissipated will also remain constant and may be quantified [10]. The energy release rate is shown below in the Griffith energy balance where W is the external work, U is the elastic energy, *V* is the energy required for crack growth, and a is the length of the crack.

$$\frac{dW}{da} - \frac{dU}{da} = \frac{dV}{da} \tag{2.1}$$

When the left hand side is divided by the sample thickness, the energy release rate, G, is obtained. If G ≥ Gc then the crack will propagate, with Gc being the fracture toughness of the material [10]. With E and *v* denoting the Young's modulus and Poisson's ratio, respectively

$$G = \frac{(K_I^2 + K_{II}^2)}{\bar{E}}, \quad \bar{E} = \frac{E}{(1-V^2)} \text{ (plane strain)} \tag{2.2}$$

2.1 Fracture Mechanics

LEFM assumes small deformations and minimal yielding at the crack tip, while EPFM can account for large deformations and plastic effects [1]. In LEFM, the Stress Intensity Factor (SIF), K, is a measure of the stress field at a crack tip as shown in Figure 2.2 and is calculated with Eq. (2.3) where F is a dimensionless geometry factor, σ is a remote nominal stress, and a is the crack length. This equation was developed from a theory of elasticity solution of the stress field around a sharp notch [7]. From the elastic solution it was concluded that the stress field is proportional to $\frac{1}{\sqrt{r}}$ where r is the radial distance from the crack tip. This stress proportionality results in a stress singularity at the crack tip where r → 0 [8].

$$K = F\sigma\sqrt{\pi a} \tag{2.3}$$

SIFs are divided into three modes based on the displacement at the crack tip, as shown in Fig. 2.1. Mode I, the opening mode, is caused by a displacement perpendicular to the crack plane, which is typically a result of tensile stresses [10]. Mode I is primarily responsible for crack growth. Displacements perpendicular to the crack tip edge from in-plane shear stresses cause the sliding mode, Mode II. Out-of-plane shear stresses result in displacements parallel to the crack tip edge and a tearing mode, Mode III [8].

2.2 Fatigue Crack Growth Models

Besides static loading, crack growth can occur when a subcritical load is repetitively applied. Paris conclude that the fatigue crack growth rate, *da/dN*, was related to the stress intensity range, $\Delta K = K_{max} - K_{min}$ [17]. On a log-log plot, it is now known that *da/dN* has a sigmoidal relation with ΔK as seen Figure 2.3. The crack growth curve can be divided into three regions. In the threshold region, crack growth is slow as ΔK asymptotically approaches the threshold value, ΔK_{th}, where crack growth may not occur. The slope of the crack growth curve is approximately linear in the intermediate, or Paris region. The unstable region is characterized by rapid, unstable crack growth where K_{max} asymptotically approaches K_c and fracture is about to happen.

Paris and Erdogan approximated the intermediate crack growth region with a power law relationship known today as the Paris equation, where C and m are determined empirically material constants. One disadvantage of this relationship is that it applies to a single stress ratio, $R = \sigma_{max}/\sigma_{min}$, where stresses are defined far from the crack tip.[11]

$$\frac{da}{dN} = C(\Delta K)^m \tag{2.4}$$

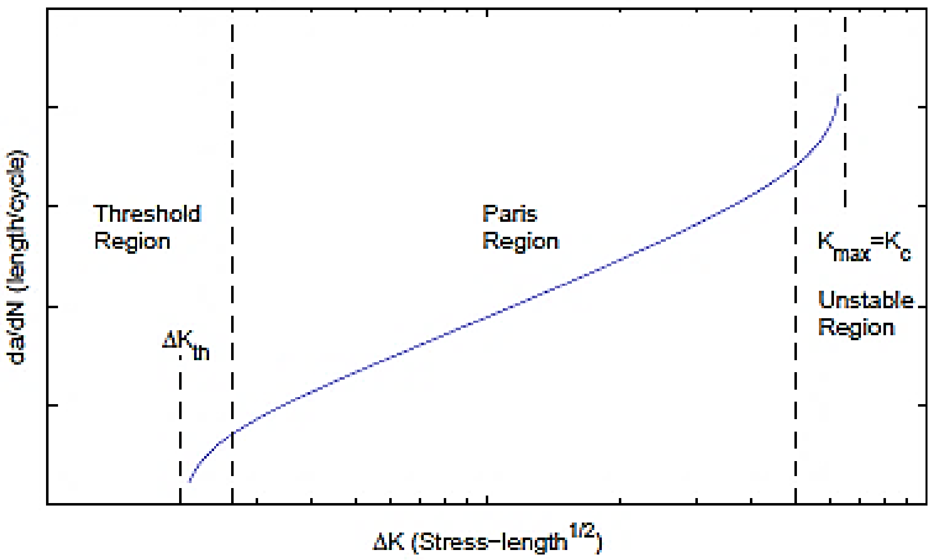

Figure 2.3 : Crack growth curve on log-log axes [12].

Experiments have shown that the crack length a is an exponential function of the number of cycles N. This means that crack growth is very slow until the final stage in the fatigue life, where a relative short number of cycles will result in fast crack growth leading to failure. The initial fatigue crack length a_i seems to be a very important parameter for the fatigue life Nf [13].

For an initially undamaged material, it takes Ni cycles to initiate a crack by dislocation movement. At this fatigue crack initiation life the initial crack has been formed, but in most cases it is so small that it cannot be detected. In this stage I, the crack propagation rate is very low, typically < 0.25 nm/cycle. After Ni cycles, in stage II of crack growth, crack propagation is faster, typically μm's per cycle. The crack growth is triggered by tensile stresses and involves plastic slip on multiple slip planes at the crack tip, resulting in striations [13].

After a large number of cycles the crack reaches a length a1, which can be detected by non-destructive techniques. The crack growth is now much faster and after the fatigue life Nf its length is af and after a few cycles ac the critical crack length is reached and failure occurs. For higher loading amplitudes, the crack growth will be faster. After N cycles, the cycles to go until failure at Nf , is indicated as Nr. The rest-life is the ratio of Nr and Nf.[13].

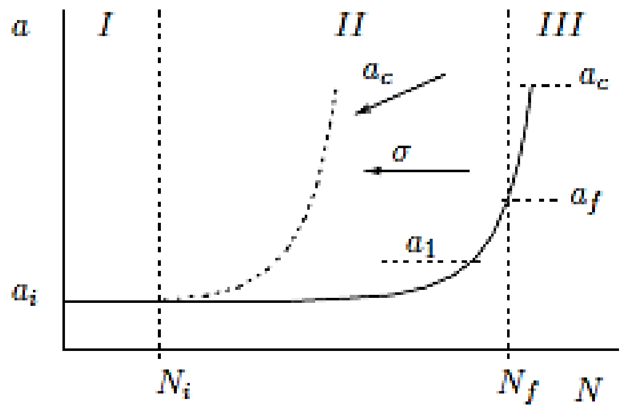

Figure 2.4 : Crack length increase with number of cycles [8].

$$\frac{N_r}{N_f} = 1 - \frac{N}{N_f} \tag{2.5}$$

To predict the fatigue life of structures, crack growth models have been proposed, which relate grow rate da dN to load amplitude or maximum load, which can be expressed in the stress intensity factor K, because we assume to be in the high cycle fatigue regime, where stresses are low [13].

Sequencing of the loads can play a large role in crack growth rates. An example is when overloading occurs prior to normal service loads. Overloading the crack can lead to large plastic deformation at the crack tip. The plasticly deformed region is surrounded by undeformed material that, upon removal of the applied overload, elasticily returns to its original configuration. The undeformed region places a compressive stress on the deformed region around the crack tip [13].

2.3 Onset of fatigue crack growth

The onset of fatigue crack growth refers to the beginning of fatigue crack growth at the crack tip in the enriched elements. In a low-cycle fatigue analysis the onset of the fatigue crack growth criterion is characterized by , which is the relative fracture energy release rate when the structure is loaded between its maximum and minimum values [6]. The fatigue crack growth initiation criterion is defined as

$$f = \frac{N}{c_1 \Delta G^{c_2}} \geq 1.0, \tag{2.6}$$

Where c_1 and c_2 are material constants and N is the cycle number. The enriched elements ahead of the crack tips will not be fractured unless the above equation is satisfied and the maximum fracture energy release rate, ΔG, which corresponds to the cyclic energy release rate when the structure is loaded up to its maximum value, is greater than G_{thresh} [6].

2.4 Fatigue crack growth using the Paris law

Once the onset of the fatigue crack growth criterion is satisfied at the enriched element, the crack growth rate, *da/dN*, can be calculated based on the relative fracture energy release rate, ΔG [6]. The rate of the crack growth per cycle is given by the Paris law if $G_{thresh} < G_{max} < G_{Pl}$,

$$\frac{da}{dN} = c_3 \Delta G^{C_4} \tag{2.7}$$

where $c_3$and c_4 are material constants.

At the end of cycle , Abaqus/Standard extends the crack length, , from the current cycle forward over an incremental number of cycles, to by fracturing at least one enriched element ahead of the crack tips. Given the material constants and , combined with the known element length and the likely crack propagation direction at the enriched elements ahead of the crack tips, the number of cycles necessary to fail each enriched element ahead of the crack tip can be calculated as , where j represents the enriched element ahead of the the` crack tip. The analysis is set up to advance the crack by at least one enriched element after the loading cycle is stabilized [6].

The element with the fewest cycles is identified to be fractured, and its is represented as the number of cycles to grow the crack equal to its element length, . The most critical element is completely fractured with a zero constraint and a zero stiffness at the end of the stabilized cycle [6]. As the enriched element is fractured, the load is redistributed and a new relative fracture energy release rate must be calculated for the enriched elements ahead of the crack tips for the next cycle [5]. This capability allows at least one enriched element ahead of the crack tips to be fractured completely after each stabilized cycle and precisely accounts for the number of cycles needed to cause fatigue crack growth over that length. If , the enriched elements ahead of the crack tips will be fractured by increasing the cycle number count, *dN*, by one only[6].

2.5 Specifying the crack propagation direction

Many models are available for predicting the direction of crack extension based on either stress, strain, energy, or any combination of these. Appropriate models should be selected based on material and loading conditions. In ABAQUS/Standard we have to specify the crack propagation direction when the fracture criterion is satisfied it is a must [6]. There are three ways the crack can extend : at a direction normal to the direction of the maximum tangential stress, based on Maximum Energy Release Rate (MERR) criterion, and $K_{II} = 0$ criterion. It is set defined by defaoult the crack propagates normal to the direction of the maximum tangential stress.The model presented here were selected since they are presented in the literature [1].

$$\theta_C = \arccos(\frac{3K_{II}^2 + \sqrt{K_I^4 + 8K_I^2 K_{II}^2}}{8K_I^2 + 9K_{II}^2}) \qquad (2.8)$$

Where θc is the angle that will follow the crack for each of the crack increments. *θc* is measured with respect to a local polar coordinate system with its origin at the crack tip and aligned with the direction of the existing crack. The sign convention is such that *θc* < 0 when *KII* > 0 and vice-versa. Once the crack growth orientation is determined, a propagation increment Δa is added to the existing crack geometry and the analysis procedure is repeated.

Maximum Tangential Stress (MTS) was proposed by Erdogan and Sih [14] which states that crack extension will happen radially from the crack tip and perpendicular to the maximum applied tensile load [13]. These two criterion are met when the tangential stress, σ_θ, is maximized and the shear stress, $\tau_{r\theta}$ is zero. MERR states that crack extension will occur at an angle, θ, that maximizes G in Eq. (2.6) for a mixed Mode I-II condition. It is stated in literature [12] that crack extension occurs in a direction where $K_{II} = 0$ for isotropic, homogeneous materials and also the previous aouthors of the article proposed that the MTS and MERR models' solutions meet the $K_{II} = 0$ criterion once the crack has extended.

Each crack extension direction criterion will give slightly different results. Therefore, analysis of crack propagation in Abaqus should be run with each criterion described in this section and compared to experimental data for selection of the most appropriate one.

2.6 Methods of Fracture Analysis

A fracture analysis typically starts with an initial crack size, or crack initiation criteria based on stress or strain, and propagates the crack until a critical value is reached such as Kc or Gc [5]. Growth rates are calculated using a fatigue crack growth model such as the Walker equation, Eq. (2.11). Several methods have been developed in the literature and a few methods are selected and presented in the following sections.

2.6.1 Virtual Crack Closure Technique

To study the onset and propagation of cracking in quasi-static problems by using the virtual crack closure technique (VCCT). VCCT uses the principles of linear elastic fracture mechanics (LEFM) , so it is appropriate for problems in which brittle crack propagation occurs along predefined surfaces [10]. VCCT is based on the assumption that the strain energy released when a crack is extended by a certain amount is the same as the energy required to close the crack by the same amount.

We can include a VCCT crack in a static or quasi-static analysis procedure. Alternatively, VCCT crack can be include in an implicit dynamic analysis procedure to simulate the fracture and failure in a structure under high-speed impact loading. VCCT is available only for Abaqus/Standard (three-dimensional solid and shell and two-dimensional planar and axisymmetric models). The purpose of VCCT is to study a crack in parts containing geometry, orphan mesh elements, or a combination of the two [6]. For example, Figure 2.5 illustrates the similarity between crack extension from i to j and crack closure at j. The energy released when a crack is extended by a certain amount is the same as the energy required to close the crack[15].

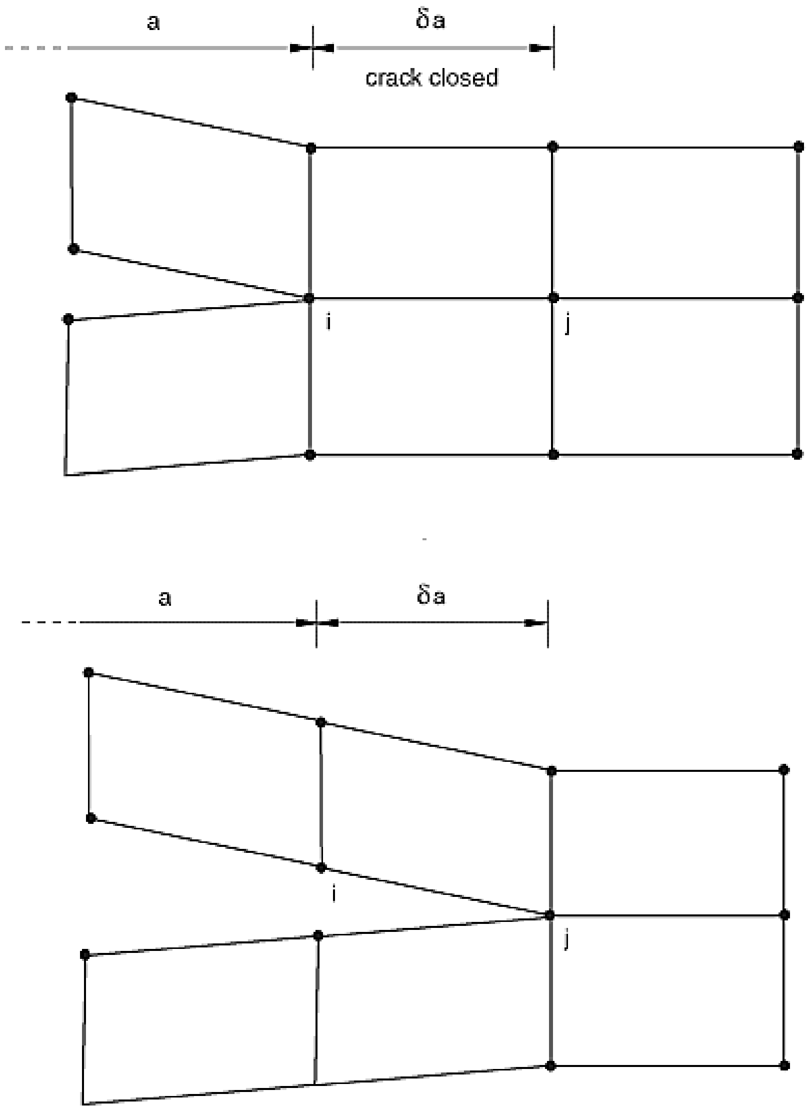

Figure 2.5 : Finite elemet representation of the VCCT method [6].

In the general case involving Mode I, II, and III the fracture criterion is defined as equation where G_{equiv} is the equivalent strain energy release rate calculated at a node, and G_{equivC} is the critical equivalent strain energy release rate calculated based on the user-specified mode-mix criterion and the bond strength of the interface.

The crack-tip node will debond when the fracture criterion reaches the value of 1.0. Abaqus provide three common mode-mix formulae for computing G_{equivC}: the BK law, the power law, and the Reeder law models [6].

$$f = \frac{G_{equiv}}{G_{equivC}} \geq 1.0 \qquad (2.9)$$

2.6.2 eXtended Finite Element Method

In comparison to the long-established finite element method, the X-FEM provides significant benefits in the numerical modelling of crack propagation. In the traditional formulation of the FEM, the existence of a crack is modelled by requiring the crack to follow element edges. n contrast, the crack geometry in the X-FEM need no longer be aligned with the element edges, which provides flexibility and versatility in modelling [6].

The method is based on the enrichment of the FE model with additional degrees of freedom (DOFs) that are tied to the nodes of the elements intersected by the crack [16]. In this manner, the discontinuity is included in the numerical model without modifying the discretization, as the mesh is generated without taking into account the presence of the crack. Therefore, only a single mesh is needed for any crack length and orientation. In addition, nodes surrounding the crack tip are enriched with DOFs associated with functions that reproduce the asymptotic LEFM fields [5]. This enables the modelling of the crack discontinuity within the crack-tip element and substantially increases the accuracy in the computation of the stress intensity factors (SIFs).

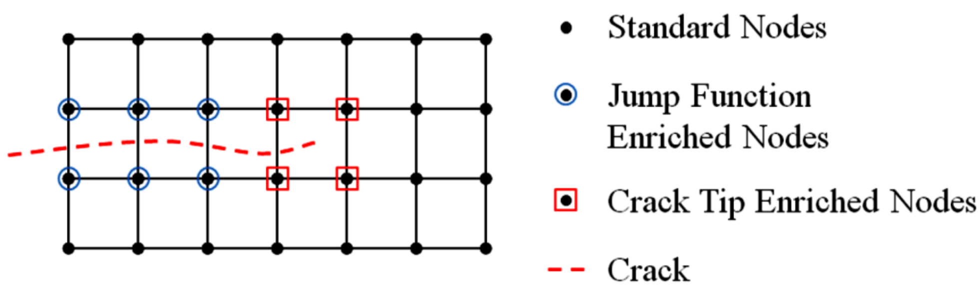

Figure 2.6 : XFEM nodes with enrichment and jump functions [12]**.**

We can study the onset and propagation of cracking in quasi-static problems using the extended finite element method (XFEM). XFEM allows to study crack growth along an arbitrary, solution-dependent path without needing to remesh the model [6]. XFEM is available only for three-dimensional solid and two-dimensional planar models. We can use XFEM to study a crack in parts containing geometry, orphan mesh elements, or a combination of the two. We can choose to study a crack that grows arbitrarily through the model or a stationary crack. We can specify the initial location of the crack or we can allow Abaqus to determine the location of the crack during the analysis based on the value of the maximum principal stress or strain calculated in the crack domain [6].

There are a few authors proposed the procedure for fatigue crack growth analysis using XFEM such as shown in Figure 2.7. The basic procedure is as follows according to literature [12] :

1. Build and mesh a finite element model.

2. Define the crack location by nodal level set values.

3. Apply quasi-static load from minimum to maximum value.

4. Determine $\Delta K's$ or $\Delta G's$.

5. Determine crack extension direction.

6. Calculate incremental crack growth length.

7. Determine the cycles required to grow the crack this incremental length based on a fatigue crack growth model.

8. Add cycles to the previous cycle count.

9. Redefine crack location by adjusting nodal level set values without remeshing.

10. Repeat steps 3 through 9 until critical values are reached.

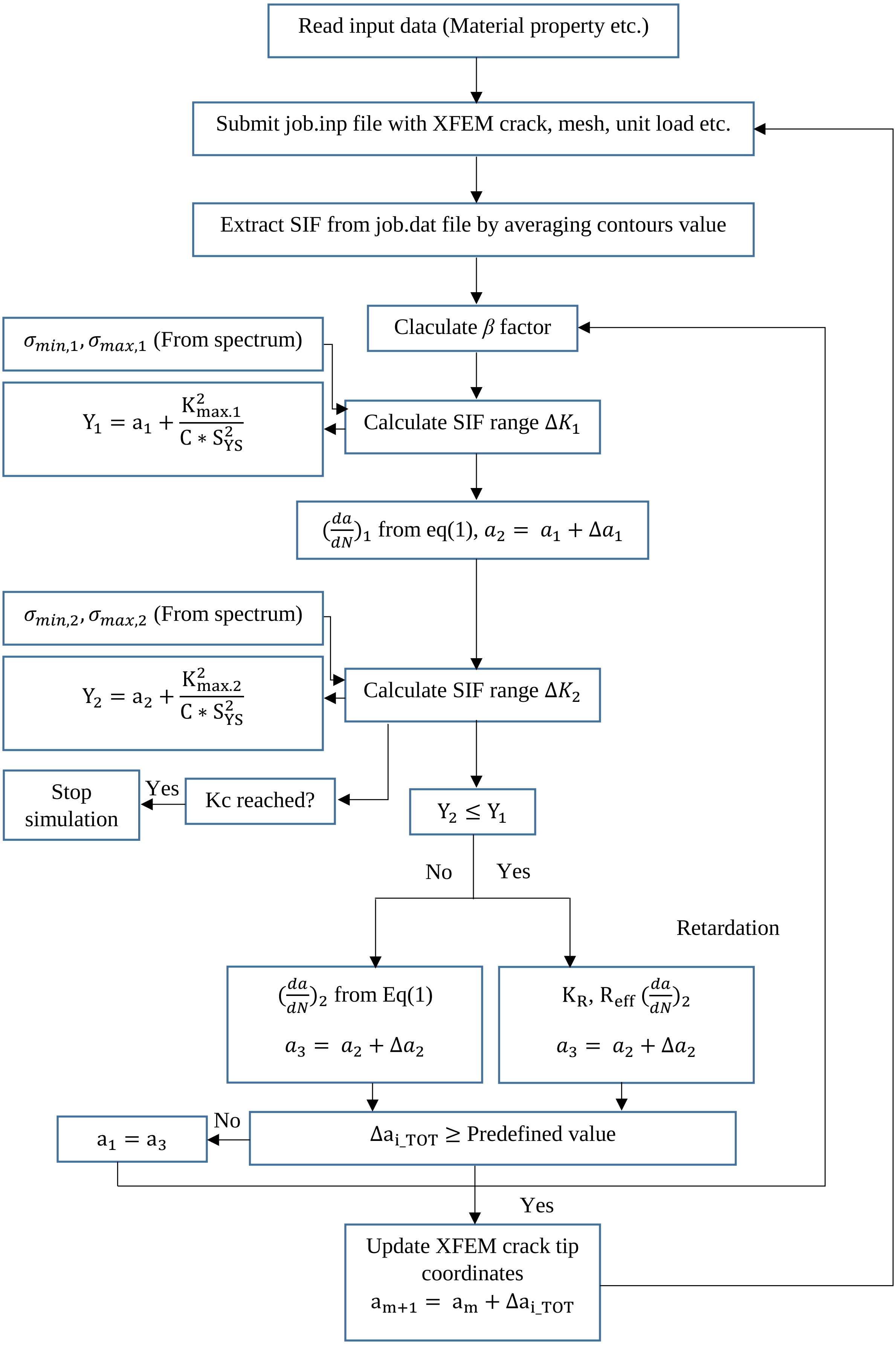

Figure 2.7 : Proposed Fatigue Crack Growth Algorithm by [17].

2.6.3 eXtended Finite Element Method with Phantom Nodes.

Phantom nodes, which are superposed on the original real nodes, are introduced to represent the discontinuity of the cracked elements, as illustrated in Figure 2.8. When the element is intact, each phantom node is completely constrained to its corresponding real node[13]. When the element is cut through by a crack, the cracked element splits into two parts. Each part is formed by a combination of some real and phantom nodes depending on the orientation of the crack [13].

The phantom node method was first proposed by Hansbo and Hansbo [18]. In the phantom node method, a material domain with internal discontinuity namely real nodes and ghost nodes can be modelled by one element with two pair of nodes [18]. When the stresses of the element reach the material strength, a discontinuity is modelled by forming two superposing elements with the help of ghost nodes. Each of the two elements contains as it were portion of the domain space. Since all the modes of the components are at the external boundaries of the domain, the area of the irregularity does not got to be know.When modelling a strong discontinuity inside an element, the phantom node method has been proven to be equivalent to the eXtended Finite Element Method (XFEM) with only the Heaviside enrichment function [6].

Both methods essentially use extra Degrees Of Freedom (DOFs) to interpolate the new crack surfaces. The difference between the two is that the phantom node method keeps the nodal DOFs as displacement DOFs and stores the extra DOFs needed as the displacement DOFs of the ghost nodes, while XFEM keeps the number of nodes constant and stores the extra DOFs needed as enriched DOFs at each node [3]. The advantage is it is easier to be implemented in existing FEM programmes because each node has only the standard displacement DOFs and only the standard FEM shape functions are needed to interpolate them [18].

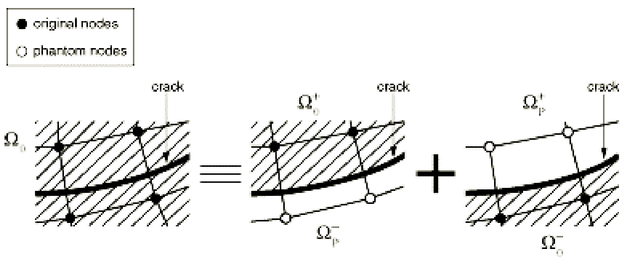

Figure 2.8 : The concept of Phantom Node Method [6].

2.6.4 eXtended Finite Element Method (XFEM) Approach for Modelling Crack Propagation

2.6.4.1 Approaches to low-cycle fatigue analysis

The traditional approach for determining the fatigue limit for a structure is to establish the curves (load versus number of cycles to failure) for the materials in the structure. Such an approach is still used as a design tool in many cases to predict fatigue resistance of engineering structures. However, this technique is generally conservative, and it does not define a relationship between the cycle number and the degree of damage or crack length.

One alternative approach is to predict the fatigue life by using a crack/damage evolution law based on the inelastic strain/energy when the structure's response is stabilized after many cycles. Because the computational cost to simulate the slow progressive damage in a material over many load cycles is prohibitively expensive for all but the simplest models, numerical fatigue life studies usually involve modeling the response of the structure subjected to a small fraction of the actual loading history. This response is then extrapolated over many load cycles using empirical formulae to predict the likelihood of crack initiation and propagation. Since this approach is based on a constant crack/damage growth rate, it may not realistically predict the evolution of the crack or damage.

2.6.4.2 Low-cycle fatigue analysis in Abaqus/Standard

The direct cyclic analysis capability in Abaqus/Standard provides a computationally effective modeling technique to obtain the stabilized response of a structure subjected to periodic loading and is ideally suited to perform low-cycle fatigue calculations on a large structure. The capability uses a combination of Fourier series and time integration of the nonlinear material behavior to obtain the stabilized response of the structure directly.

The direct cyclic low-cycle fatigue procedure models the progressive damage and failure both in bulk materials and at material interfaces. The former can be based on either a continuum damage mechanics approach or the principles of linear elastic fracture mechanics with the extended finite element method.

The response is obtained by evaluating the behavior of the structure at discrete points along the loading history as shown in Figure 2.9. The solution at each of these points is used to predict the degradation and evolution of material properties that will take place during the next increment, which spans a number of load cycles. The degraded material properties are then used to compute the solution at the next increment in the load history. Therefore, the crack/damage growth rate is updated continually throughout the analysis.

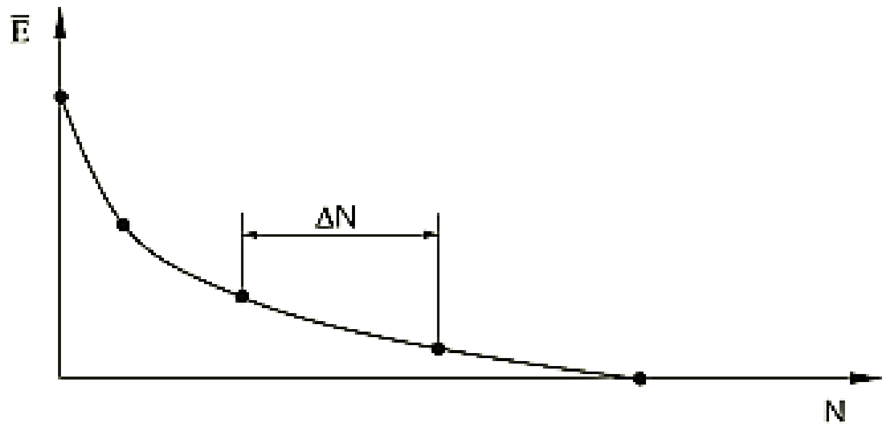

Figure 2.9 : Elastic stiffness degradation as a function of the cycle number [6].

The elastic material stiffness at a material point remains constant and contact conditions remain unchanged when the stabilized solution is computed at a given point in the loading history. Each of the solutions along the loading history represents the stabilized response of the structure subjected to the applied period loads, with a level of material damage at each point in the structure computed from the previous solution. This process is repeated up to a point in the loading history at which a fatigue life assessment can be made.

In bulk material, there are two approaches to modeling the progressive damage and failure. One approach is based on continuum damage mechanics. This approach is more appropriate for ductile material, in which the cyclic loading leads to stress reversals and the accumulation of plastic strains, which in turn cause the initiation and propagation of cracks. The other approach is based on the principles of linear elastic fracture mechanics with the extended finite element method. This approach is more appropriate for brittle material or material with small scale yielding, in which the cyclic loading leads to material strength degradation causing fatigue crack growth along an arbitrary path. The onset and growth of the crack are characterized by the relative fracture energy release rate at the crack tip based on the Paris law.

At interfaces of laminated composites the cyclic loading leads to interface strength degradation causing fatigue delamination growth. The onset and growth of delamination are also characterized by the relative fracture energy release rate at the crack tip based on the Paris law.

Both the progressive damage mechanism in the bulk material and the progressive delamination growth mechanism at interfaces can be considered simultaneously, with the failure occurring first at the weakest link in a model.

2.6.4.3 Determining whether to use the Fourier coefficients from the previous step

A low-cycle fatigue step using the direct cyclic approach can be the only step in an analysis, can follow a general or linear perturbation step, or can be followed by a general or linear perturbation step. Multiple low-cycle fatigue analysis steps can be included in a single analysis. In such a case the Fourier series coefficients obtained in the previous step can be used as starting values in the current step. By default, the Fourier coefficients are reset to zero, thus allowing application of cyclic loading conditions that are very different from those defined in the previous low-cycle fatigue step.

As in a direct cyclic analysis, you can specify that a low-cycle fatigue step in a restart analysis should use the Fourier coefficients from the previous step, thus allowing continuation of an analysis to simulate more loading cycles. In a low-cycle fatigue analysis a restart file is written at the end of the stabilized cycle.

Consequently, a restart analysis that is a continuation of a previous low-cycle fatigue analysis will start with a new loading cycle at $t = 0$.

2.6.4.5 Progressive damage and damage extrapolation in bulk ductile material based on continuum damage mechanics approach

Low-cycle fatigue analysis in Abaqus/Standard allows modeling of progressive damage and failure for ductile materials in any elements whose response is defined in terms of a continuum-based constitutive model. This includes cohesive elements modeled using a continuum approach. The inelastic definition in a material point must be used in conjunction with the linear elastic material model. After damage initiation the elastic material stiffness is degraded progressively in each cycle based on the accumulated stabilized inelastic hysteresis energy.

It is impractical and computationally expensive to perform a cycle-by-cycle simulation for a low-cycle fatigue analysis; Instead, to accelerate the low-cycle fatigue analysis, each increment extrapolates the current damaged state in the bulk material forward over many cycles to a new damaged state after the current loading cycle is stabilized.

2.6.4.6 Damage initiation and evolution

Damage initiation refers to the beginning of degradation of the response of a material point. In a low-cycle fatigue analysis the damage initiation criterion is characterized by the accumulated inelastic hysteresis energy per cycle. and material constants are used to determine the number of the cycle in which damage is initiated, .

At the end of a stabilized loading cycle, , Abaqus/Standard checks to see if the damage initiation criterion is satisfied in any material point; material stiffness at a material point will not be degraded unless this criterion is satisfied.

Once the damage initiation criterion is satisfied at a material point, the damage state is calculated and updated based on the inelastic hysteresis energy for the stabilized cycle. Abaqus/Standard assumes that the degradation of the elastic stiffness can be modeled using the scalar damage variable, . The rate of the damage in a material point per cycle, is calculated based on the accumulated inelastic hysteresis energy, the characteristic length associated with an integration point, and material constants.

Typically, a material has completely lost its load carrying capacity when . You can remove an element from the mesh if all of the section points at all integration locations of the element have lost their load carrying capability.

2.6.4.7 Damage extrapolation technique in the bulk material

If the damage initiation criterion is satisfied in any material point at the end of a stabilized cycle,D_N , Abaqus/Standard extrapolates the damage variable from the current cycle forward to the next increment over a number of cycles, ΔN. The new damage state,$D_{N+\Delta N}$, is given by where L is the characteristic length associated with an integration point, and C_3 and C_4 are material constants

$$D_{N+\Delta N} = D_N + \frac{\Delta N}{L} c_3 \Delta\omega^{C_4} \quad (2.10)$$

2.6.4.8 Discrete crack propagation along an arbitrary path based on the principles of linear elastic fracture mechanics with the extended finite element method

Low-cycle fatigue analysis in Abaqus/Standard allows the modeling of discrete crack growth along an arbitrary path based on the principles of linear elastic fracture mechanics with the extended finite element method. You complete the definition of the crack propagation capability by defining a fracture-based surface behavior and specifying the fracture criterion in enriched elements.

The fracture energy release rates at the crack tips in enriched elements are calculated based on the modified virtual crack closure technique (VCCT). VCCT uses the principles of linear elastic fracture mechanics. Therefore, VCCT is appropriate for problems in which brittle fatigue crack growth occurs, although nonlinear material deformations can occur somewhere else in the bulk materials. To accelerate the low-cycle fatigue analysis, the damage extrapolation technique is used, which advances the crack by at least one element length after each stabilized cycle.

2.6.4.9 Onset and growth of fatigue crack

The onset and growth of fatigue crack at an enriched element are characterized by using the Paris law, which relates the relative fracture energy release rate,ΔG , to crack growth rates. Two criteria must be met to initiate fatigue crack growth: one criterion is based on material constants, ΔG , and the current cycle number,N ; the other criterion is based on the maximum fracture energy release rate,G_{max} , which corresponds to the cyclic energy release rate when the structure is loaded up to its maximum value. Once the