First Semester Examination
Academic Session 2021/2022

February/March 2022

EAS663 – Dynamics and Stability of Structures

Duration : 2 hours

Please ensure that this examination paper contains **EIGHT (8)** printed pages before you begin the examination.

Instructions: This paper contains **FIVE (5)** questions. Answer **FOUR** questions.

All questions **MUST BE** answered on a new page.

(1). (a). Derive the non-linear elastic in plane bending of simply supported beam column as shown in **Figure 1**. Show that the maximum moment M_{max} may be closely approximated by using greater of $M_{max}=qL^2/8$ and $M_{max} = \frac{9qL^2}{128}\frac{(1-0.29N/N_{cr,y})}{(1-N/N_{cr,y})}$

where, $N_{cr,y}=\pi^2 EI_y/L^2$

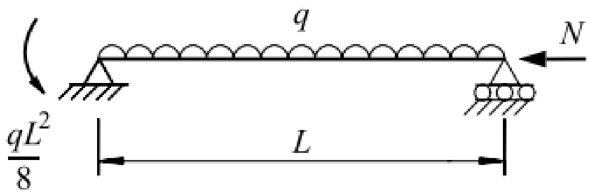


Figure 1

[12 marks]

(b). Derive the non-linear elastic in plane bending of propped cantilever beam as shown in **Figure 2**. Show that the maximum moment M_{max} may be closely approximated by using $\frac{M_{max}}{9qL^2/8} \approx \frac{(1-0.18N/N_{cr,y})}{(1-0.49N/N_{cr,y})}$

where, $N_{cr,y}=\pi^2 EI_y/L^2$

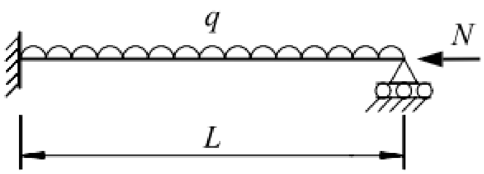


Figure 2

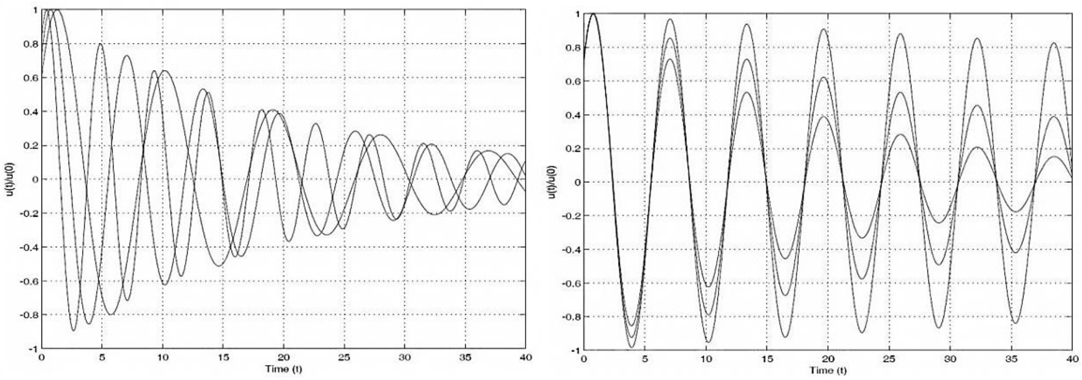
[13 marks]

(2). A SDOF building is modeled as a mass-spring-damper system. Effects of the damping and stiffness on the system are evaluated through a series of tests for the system with the properties as shown in **Table 1**.

Table 1

Test	Case	Damping coefficient	Stiffness	Mass
1	A	0.01	1.0	1.0
	B	0.05	1.0	1.0
	C	0.1	1.0	1.0
2	A	0.1	0.5	1.0
	B	0.1	1.0	1.0
	C	0.1	2.0	1.0

The results of the test are plotted in **Figure 3(a)** to **Figure 3(b)**



(a) (b)

Figure 3

(a). Identify which result in **Figure 3** showing the effect of damping and stiffness. Indicate Case A, Case B and Case C in each test. Give explanation to your answer.

[5 marks]

(b). A single degree of freedom building system as shown in **Figure 4** is excited by ground acceleration, 8 cos 5t m/s^2. Assume that the girder is rigid whereas the columns are flexible to the lateral deformation but rigid in the vertical direction. Using E = 35 GPa and I = 25(10^6) mm^4, 5 % damping and neglect the mass of columns, determine

(i). the natural period of building,

(ii). the steady state amplitude of vibration, and

(iii). the maximum shear force and bending moment in the column.

[20 marks]

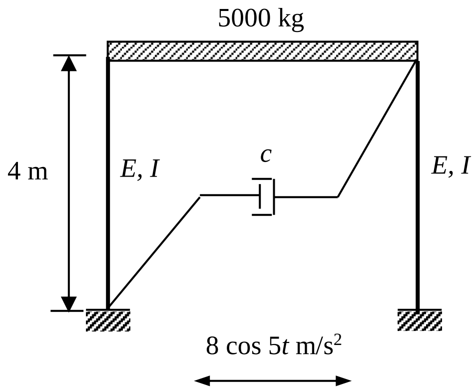


Figure 4

(3). The equations of motion in matrix form for the system shown in **Figure 5** are:

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 18 & -10 \\ -10 & 10 \end{bmatrix}\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} 450 & -200 \\ -200 & 200 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 30\sin 5t \\ 50\sin 2t \end{Bmatrix}$$

and the natural frequencies and mode shapes for modes 1 and 2 for the system are:

$$\omega_1 = 5.67 \text{ rad/s} \; ; \; \omega_2 = 16.11 \text{ rad/s}$$

$$\phi_1 = \begin{Bmatrix} 0.5184 \\ 1.0000 \end{Bmatrix} \; ; \; \phi_2 = \begin{Bmatrix} -0.3456 \\ 1.0000 \end{Bmatrix}$$

Solve the equations of motion and determine the steady state response of the system.

[25 marks]

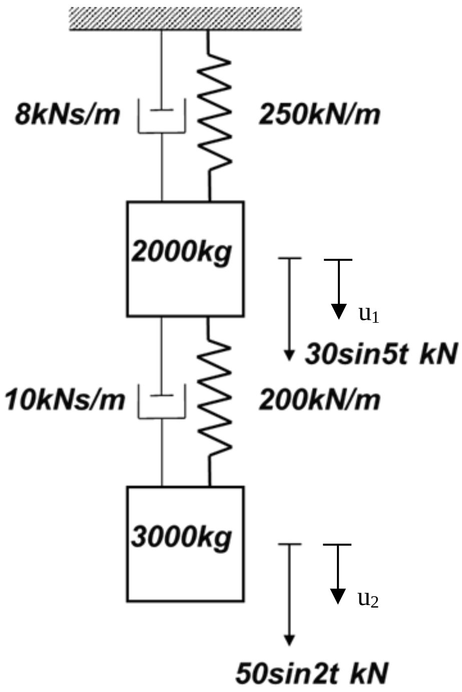


Figure 5

(4). (a). Define the meaning of critical load of a column. Obtain the critical load of the cantilever column in **Figure 6** by using Rayleigh-Ritz method. Assume the following two cases of deflected shapes for the slightly bent configuration at the state of neutral equilibrium:

Case I : $y = Ax^2$

Case II: $y = A\left[1 - \cos\frac{\pi x}{2L}\right]$

where *A*: amplitude of lateral displacement of the column at the free-end. The exact solution for the critical load is $P_{cr,\,exact} = 2.467\ EI/L^2$. Comment on the outcome of comparison of the solutions of critical load obtained for Case I and Case II with the exact solution.

[20 marks]

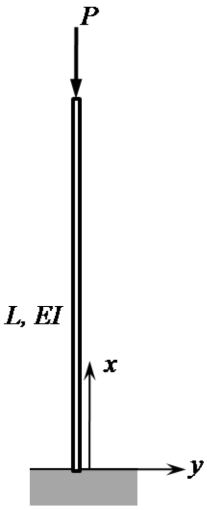


Figure 6

(b). Mid-span deflection of the simply supported beam subjected to a lateral load, *W* at mid-span and axial load, *P* as shown in **Figure 7** is given by the following equation:

$$\delta = \Delta \frac{1}{1-\left(P/P_{cr}\right)}$$

where $\Delta = WL^3/(48EI)$: the mid-span deflection in the absence of axial load and P_{cr} = critical axial load of the beam.

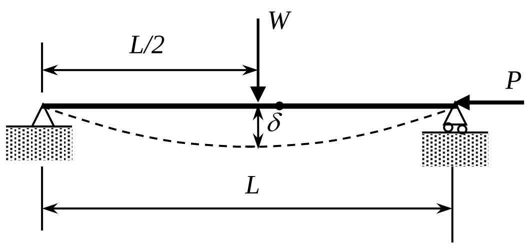


Figure 7

Using the above equation and suitable plot, explain the effect of constant *P* on the lateral stiffness of the beam.

[5 marks]

(6). (a). Using slope-deflection method, determine the critical load of column AB for the frame shown in **Figure 8**. Sketch the slightly deformed configuration at the state of neutral equilibrium. Refer Appendix A for the basic equations used in slope deflection method.

[20 marks]

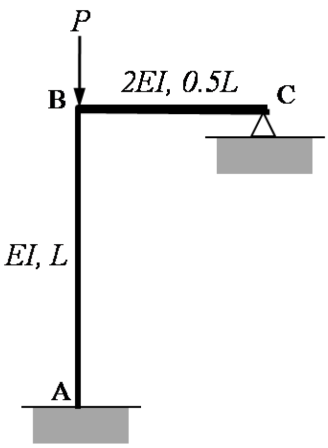


Figure 8

(b). Sketch **TWO (2)** possible modes of buckling for the braced frame shown in **Figure 9**. Indicate the one which will occur at lower buckling load.

(5 marks)

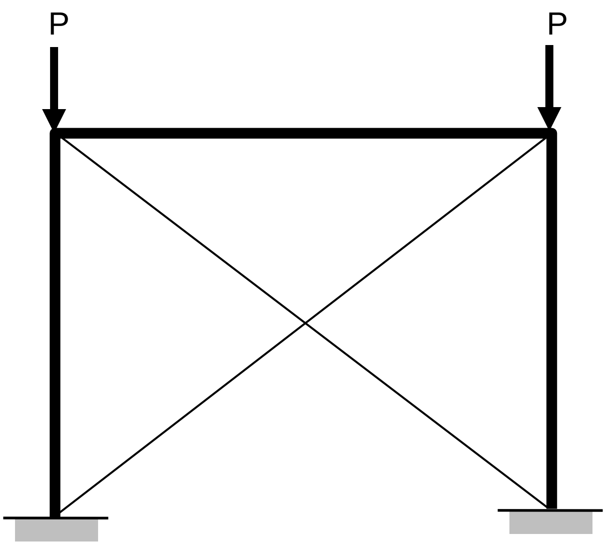


Figure 9

APPENDIX A

Slope deflection equations for a beam-column are given as follows:

$$M_A = \frac{EI}{L}\left(\alpha_n \theta_A + \alpha_f \theta_B\right)$$
$$M_B = \frac{EI}{L}\left(\alpha_f \theta_A + \alpha_n \theta_B\right)$$

where α_n and α_f are given as follows:

$$\alpha_n = \frac{\phi_n}{\phi_n^2 - \phi_f^2}\,; \;\; \alpha_f = \frac{\phi_f}{\phi_n^2 - \phi_f^2}$$

and

$$\phi_n = \frac{1}{(kL)^2}\left(1 - kL\cot kL\right); \quad \phi_f = \frac{1}{(kL)^2}\left(kL\csc kL - 1\right)$$

M_A, M_B, θ_A and θ_B are as shown in Fig.A1.

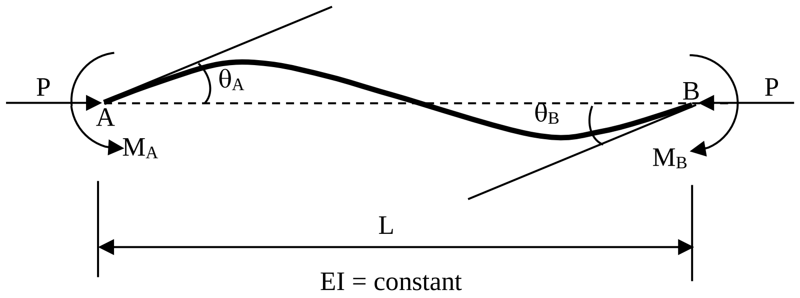


Fig.A1

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