



First Semester Examination
Academic Session 2021/2022

February/March 2022

EAS661 – Advanced Structural Mechanics

Duration : 2 hours

Please ensure that this examination paper contains **EIGHT (8)** printed pages before you begin the examination.

Instructions: This paper contains **FOUR (4)** questions. Answer **ALL** questions.

All questions **MUST BE** answered on a new page.

-2-

- (1). (a). Derive the governing differential equation for the linear elastic bar problem shown in **Figure 1** where, E : elastic modulus, A : cross-sectional area, w : uniformly distributed load per unit length. Next, using the derived governing differential equation, obtain the expression of axial displacement, u for the linear elastic bar problem. Both ends of the bars are fixed.

[15 marks]

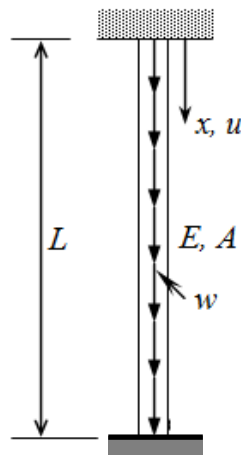


Figure 1

- (b). A small thin rectangular plate as shown in **Figure 2** is made of metal with $E=110$ GPa and $\nu=0.32$. Stress components σ_{xx} and σ_{yy} are uniformly distributed as shown in the figure. Determine:
- the magnitude of σ_{yy} so that the dimension $b=20$ mm remains the same under the load.
 - the corresponding state of stress and state of strain of a point within the rectangular plate.

[10 marks]

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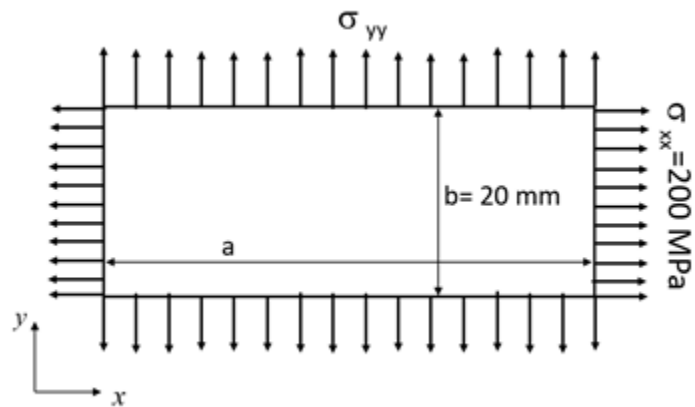


Figure 2

- (2). (a). **Figure 3** shows a spring-block assembly. The rigid block is constrained to translate only in vertical direction. Determine the vertical displacement v of the rigid block using principle of minimum potential energy. Use: $N = 5$ kN, $b = 100$ mm, $k_1 = 0.5$ kN/mm, $k_2 = 2.5$ kN/mm:

[7 marks]

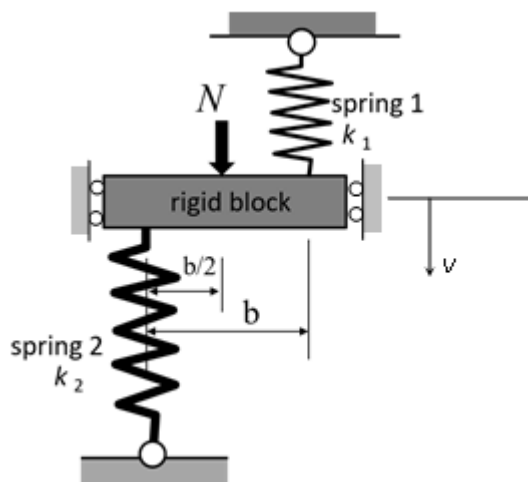


Figure 3

- (b). **Figure 4** shows a cantilever beam with length, L subjected to a uniformly distributed load of intensity, q and a vertical point load, P at point B. One linear elastic bar BC with axial rigidity EA and length $0.45L$ is attached to the free end B. Both ends of the linear elastic bar are pinned. The following expression for lateral displacement field, v has been suggested:

$$v = \Delta \left(1 - \cos \frac{\pi x}{2L} \right)$$

where Δ is a constant. Show that the above displacement field is kinematically admissible. Determine the shortening of the linear elastic bar BC by applying the principle of minimum potential energy. Flexural rigidity of the cantilever beam is EI .

[18 marks]

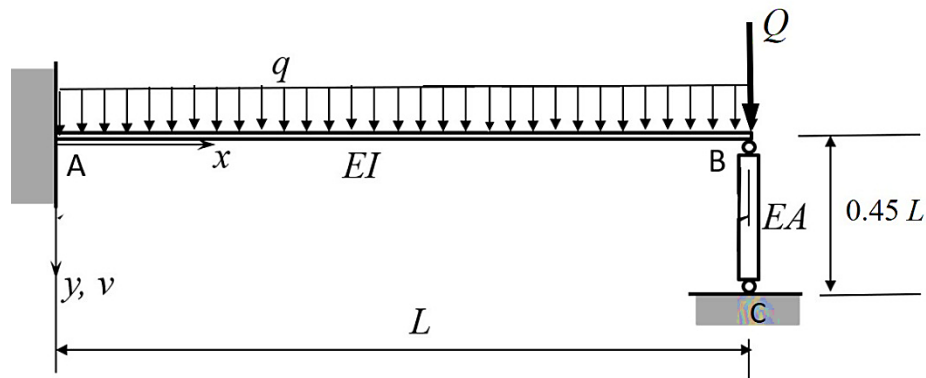
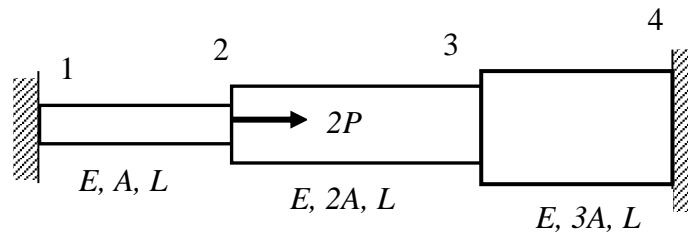


Figure 4

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- (3). (a). Write down the element stiffness matrices and global matrix for the three bar assembly which is loaded with force $2P$, and constrained at the two ends in terms of E , A and L as shown in **Figure 5**.

[15 marks]

**Figure 5**

- (b). **Figure 6** shows an assemblage of a spring system which is subjected to a nodal force of $2P = 100$ kN at node 4. The spring is fixed at node 1, 5 and node 6. Using the value of $k_1 = 200$ kN/m, $k_2 = k_4 = 150$ kN/m and $k_3 = k_5 = 200$ kN/m,

- (i). Derive the element stiffness matrix for each spring.
- (ii). Derive the global stiffness matrix for the system.
- (iii). Determine the deflection at node 2, 3 and 4 in unit metre.
- (iv). Determine the new global stiffness matrix, if the horizontal load at node 4 is reduced from $2P$ to P .

[10 marks]

...6/-

-6-

Given the stiffness of the spring element in dimensional space:

$$k = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

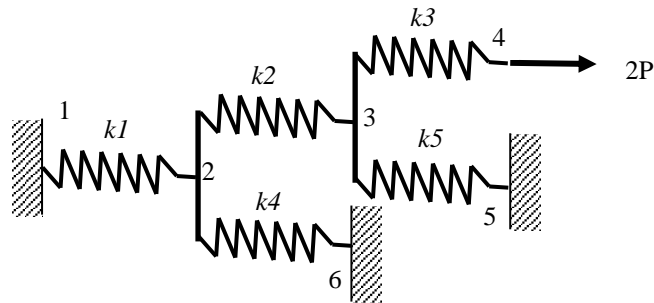


Figure 6

- (4). (a). Two plates shown in **Figures 7** and **8**, shall be analysed as a plane strain problem. Both plates are divided into several elements. Each node has been labelled accordingly. Calculate the bandwidth, $B = (R+1)$ NDOF for the plate assuming two degrees of freedom at each node.

[5 marks]

- (b). By rearranging the node labeling, determine the minimum value of R .

[5 marks]

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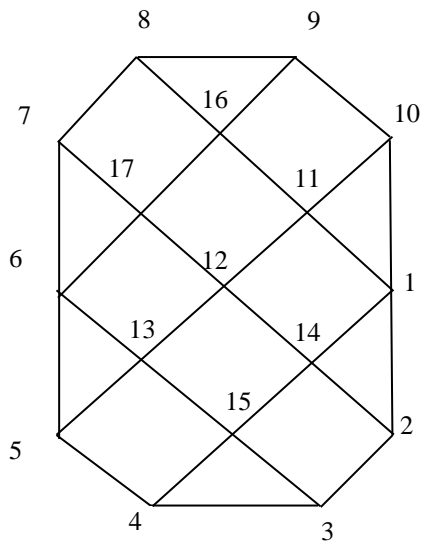


Figure 7

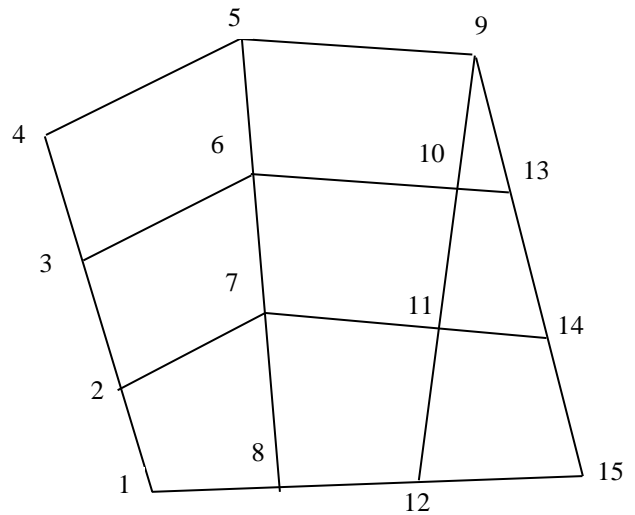


Figure 8

(c). **Figure 9** shows a frame structure labeled as node 1, 2 and 3 which are subjected to a nodal force of $P = 60 \text{ kN}$ at node 2 and uniformly distributed load of 5 kN/m . The frame is fixed at node 3 and roller supported at node 1. Given the value of $E = 207 \text{ GPa}$, $I = 3 \times 10^{-5} \text{ m}^4$ and $A = 0.005 \text{ m}^2$.

- (i). Derive the global stiffness matrix for the frame.
- (ii). Determine the deflection u_2 , v_2 and θ_2 in unit metre and rad, respectively.

[15 marks]

Given the stiffness of the beam element in dimensional space:

$$k = \frac{EI}{L^3} \begin{bmatrix} v_i & \theta_i & v_j & \theta_j \\ 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

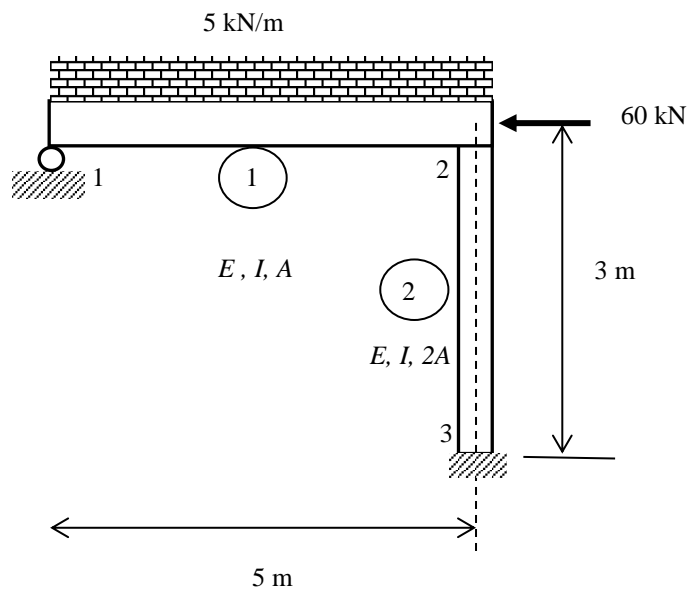


Figure 9

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