STEADY BOUNDARY LAYER FLOW OF NANOFLUID WITH MICROORGANISM

NUR ARDIANA BINTI AMIRSOM

UNIVERSITI SAINS MALAYSIA

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by

NUR ARDIANA BINTI AMIRSOM

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LIST OF SYMBOLS

a	positive constant (s^{-1})
A	reciprocal magnetic Prandtl number
b	positive constant (s^{-1})
$\tilde{b}$	chemotaxis constant (m)
B	positive constant (m^2)
B_0	magnetic field and strength $\left(kg\,A^{-1}s^{-2}\right)\cdot Tesla$
Bi	Biot number (-)
c_2	mass diffusivity parameter (-)
c_p	specific heat at constant pressure of the fluid $\left(J\,kg^{-1}K^{-1}\right)$
C	nanoparticles volume fraction (-)
C_s	mass slip
$C_{sf}(\overline{x},\overline{y})$	mass slip factor
C_f	convective surface mass (-)
C_∞	ambient concentration (-)
c_s	heat capacity of the solid surface $\left(J\,kg^{-1}K^{-1}\right)$
C_w	nanoparticles volume fraction at wall (-)
C_∞	nanoparticles volume fraction at ambient (-)
C_{f_x}	skin friction coefficient ($m\,s^{-1}$)
D_B	Brownian diffusion coefficient ($m^2\,s^{-1}$)
$D_B(C)$	variable Brownian diffusion coefficient ($m^2\,s^{-1}$)
D_T	thermophoretic diffusion coefficient ($m^2\,s^{-1}$)
D_n	diffusivity of microorganism ($m^2\,s^{-1}$)
$D_n(C)$	variable diffusivity of microorganisms ($m^2\,s^{-1}$)
Ec	Eckert number (-)
$f(\eta)$	velocity at x - component (-)
g	acceleration due to gravity ($m\,s^{-1}$)
$g(\eta)$	velocity at y - component (-)
h_1	dimensional positive constant (K^{-1})
h_2	temperature dependent viscous parameter (-)
h_3	dimensional positive constant (K^{-1})
h_4	temperature dependent thermal conductive parameter (-)
h_5	positive constant (-)
h_6	mass diffusivity parameter (-)
h_8	microorganism diffusivity parameter (-)

h_f	heat transfer coefficient $\left(W\, m^{-2}\, K^{-1}\right)$
h_m	mass transfer coefficient ($m\, s^{-1}$)
h_n	microorganism transfer coefficient ($m\, s^{-1}$)
k	thermal conductivity of nanofluid $\left(W\, m^{-1}\, K^{-1}\right)$
$k(T)$	variable thermal conductivity of nanofluid $\left(W\, m^{-1}\, K^{-1}\right)$
K	consistency coefficient of the fluid $\left(kg\, s^{n-2} m^{-n}\right)$
K_0	permeability of porous medium (m^2)
L	characteristic length (-)
Lb	bioconvection Lewis number (-)
Le	Lewis number (-)
M	magnetic parameter (-)
Me	melting (phase change) parameter (-)
n	density of motile microorganism (-)
n_s	microorganism slip
$n_{s\,f}(\bar{x}, \bar{y})$	microorganism slip factor
m	power law parameter (-)
n_f	convective surface motile microorganism (-)
N	linear / non-linear positive constant
Nb	Brownian motion parameter (-)
Nc	convective microorganism parameter (-)
Nd	convective mass parameter (-)
Nt	thermophoresis parameter (-)
Nr	buoyancy ratio (-)
Pe	Péclet number (-)
$\Pr$	Prandtl number (-)
p	positive constant
q	positive constant
$\dot{q}$	volumetric rate of heat generation ($W\, m^{-3}$)
Q	local internal heat generation parameter (-)
Rb	bioconvection Rayleigh number (-)
Sc	Schmidt number (-)
Sb	bioconvection Schmidt number (-)
s	variable surface temperature
S	blowing parameter (-)
T	nanofluid temperature (K)
T_f	convective surface temperature (K)
T_∞	free stream temperature
T_0	solid surface temperature (K)
T_w	wall surface temperature (K)
T_{Me}	melting surface temperature (K)
T_s	thermal slip

$T_{sf}(\bar{x},\bar{y})$	thermal slip factor
$\bar{u}_e$	external fluid velocity
$\bar{u}_{slip}$	velocity slip ($m\,s^{-1}$)
$\bar{u}_w$	velocity along stretching surface ($m\,s^{-1}$)
$\bar{u}$	velocity component along the $\bar{x}$ - direction ($m\,s^{-1}$)
u	non-dimensional velocity component along the $\bar{x}$ - direction (-)
u_1	positive constant
U_s	velocity slip at $\bar{x}$ direction,
$U_{sf}(\bar{x},\bar{y})$	velocity slip factor at $\bar{x}$ direction,
$\bar{v}$	velocity components along the $\bar{y}$ – direction ($m\,s^{-1}$)
v	non-dimensional velocity component along the y - direction (-)
V_s	velocity slip at $\bar{y}$ direction
$V_{sf}(\bar{x},\bar{y})$	velocity slip factor at $\bar{y}$ direction
$\bar{w}$	velocity component along the $\bar{z}$ - direction ($m\,s^{-1}$)
w	non-dimensional velocity component along the z – direction (-)
W_c	maximum cell swimming speed ($m\,s^{-1}$)
$\bar{x}$	coordinate along the surface (m)
x	non-dimensional coordinate along the surface (-)
$\bar{y}$	coordinate along the surface (m)
y	non-dimensional coordinate along the surface (-)
$\bar{z}$	coordinate along the surface (m)
z	non-dimensional coordinate along the surface (-)

Greek letters

α	thermal diffusivity ($m^2 s^{-1}$)
α_m	thermal diffusivity of the porous medium ($m^2 s^{-1}$)
α_1	magnetic diffusivity
β	volumetric coefficient of the thermal expansion (K^{-1})
η	similarity variable (-)
ε	variable thermal conductivity (-)
γ	average volume fraction of microorganisms (-)
$\theta(\eta)$	temperature (-)
ρ	constant fluid density
ρ_f	density of the base fluid
ρ_p	density of the nanoparticle ($kg\,m^{-3}$)
ρ_{f_∞}	density of base fluid ($kg\,m^{-3}$)
ρ_{m_∞}	density of motile microorganism ($kg\,m^{-3}$)
τ	ratio of heat capacity of the nanoparticle material to heat capacity of fluid (-)
$\phi(\eta)$	dimensionless nanoparticles volume fraction (-)

$\chi(\eta)$	dimensionless motile microorganism (-)
ψ	stream function (-)
δ	first order slip parameter (-)
λ	molecular mean free path (m)
μ	dynamic viscosity of the fluid
$\mu(T)$	temperature dependent viscosity
μ_m	magnetic permeability
υ_f	kinematic viscosity of the fluid $\left(m^2 s^{-1}\right)$
ρ_f	density of the base fluid $\left(kg\, m^{-3}\right)$
λ	stretching rate ratio along the $\overline{x}$ - direction to the $\overline{y}$ - direction
λ_L	latent heat transfer of the fluid

Subscripts/superscripts

w	condition at the wall
∞	free stream/ambient condition
'	differentiation with respect to η, e.g $f'(\eta), f''(\eta), f'''(\eta)$

LIST OF ABBREVIATIONS

ODEs	Ordinary Differential Equations
PDEs	Partial Differential Equations
RKF45	Runge-Kutta-Fehlberg Fourth-Fifth Order Numerical Method
BVP	Boundary Value Problem
MHD	Magnetohydrodynamic
EHD	Electrohydrodynamic

ALIRAN LAPISAN SEMPADAN MANTAP BAGI NANOBENDALIR DENGAN MIKROORGANISMA

ABSTRAK

Dalam tesis ini, analisis dua dan tiga dimensi laminar perolakan aliran lapisan sempadan nanobendalir dengan mikroorganisma dikaji. Ia melibatkan aliran lapisan sempadan laminar dua dan tiga dimensi dengan pemindahan haba dan jisim di bawah pelbagai bentuk fizikal serta geometri. Parameter - parameter yang terlibat dalam kajian ini terdiri daripada pengaruh medan magnet, penjanaan / penyerapan haba, gelincir halaju, gelincir haba, gelincir jisim , gelincir mikroorganisma, kelikatan, konduktif terma, kemeresapan jisim, dan kemeresapan mikroorganisma. Kadar pemindahan haba lebur, peniupan Stefan dan beberapa sempadan telah diambil kira. Bendalir ini dicirikan sebagai Newtonan, tak Newtonan, likat, mampat, magnetohidrodinamik dan mempunyai sifat fizikal yang tetap atau berubah-ubah. Lapisan sempadan yang mantap telah dipertimbangkan dan transformasi yang sesuai digunakan untuk mengubah persamaan pembezaan separa ke persamaan pembezaan biasa tak linear. Persamaan pembezaan biasa tersebut telah diselesaikan secara berangka menggunakan bvp4c dalam matlab untuk pelbagai nilai parameter kawalan. Graf telah diplotkan untuk memaparkan kesan parameter kawalan pada halaju tak berdimensi, kelikatan, suhu, kepekatan (pecahan isipadu nano zarah), mikroorganisma serta faktor geseran kulit, kadar haba, kadar pemindahan jisim dan kadar pergerakan mikroorganisma. Penyelesaian berangka untuk faktor geseran kulit, kadar haba, kadar pemindahan jisim dan kadar pergerakan mikroorganisma telah disediakan dalam jadual untuk pelbagai nilai parameter. Bidang aliran dan kuantiti fizikal yang lain sangat dipengaruhi oleh parameter kawalan. Perbandingan dengan kerja yang telah diterbitkan sebelum ini dijalankan dan hasilnya didapati dalam persetujuan yang baik.

STEADY BOUNDARY LAYER FLOW OF NANOFLUID WITH MICROORGANISM

ABSTRACT

In this thesis, an analysis of two and three dimensional laminar convective boundary layer flow of a nanofluid with microorganism is investigated. It involves two and three dimensional laminar convective external boundary layer flow with heat and mass transfer under various physical configurations as well as geometries. The parameters that involved in this research are consist of magnetic field, heat generation/absorption, velocity slip, thermal slip, mass slip, microorganisms slip, viscosity, thermal conductive, mass diffusivity, and microorganism diffusivity. Melting heat transfer rate, Stefan blowing and multiple boundary conditions are taken into account. The fluid is characterized to be Newtonian, non- Newtonian, viscous, incompressible, magnetohydrodynamic and has constant or variable physical properties. Steady boundary layers are considered and appropriate transformations are used to transform the partial differential equations into nonlinear ordinary differential equations. The transformed equations are solved numerically using the bvp4c in Matlab for various values of the controlling parameters. Graphs are plotted to display the effect of the controlling parameters on the dimensionless velocity, viscosity, temperature, concentration (nanoparticle volume fraction), microorganism as well as skin friction factor, rate of heat, rate of mass transfer and rate of motile microorganism. The numerical solution for the skin friction factor, rate of heat, rate of mass transfer and rate of motile microorganism are generated for various values of the parameters. The flow field and other quantities of physical interest are found to be significantly influenced by the controlling parameters. A comparison with previously published work is carried out and the results are found to be in a good agreement.

CHAPTER 1

INTRODUCTION

1.1 Introduction

This thesis deals with the mathematical modeling of two-dimensional and three-dimensional steady boundary layer flows in a nanofluid with magnetohydrodynamic effects and microorganisms. The fluid is assumed to be incompressible and non-Newtonian / Newtonian. As heat and mass transfer are important in a multitude of industrial and manufacturing processes, knowledge of the mechanics and behaviour of heat and mass transfer in Newtonian/non-Newtonian nanofluids are also important. The dominant heat and mass transfer mode in many industrial and manufacturing processes is convection. The flows occur in many industrial processes and the development of suitable mathematical models are important to help obtain understanding of such flows. In the past decade, there are a lot of work done on the convective heat transport in nanofluids but nanofluids containing gyrotactic microorganisms (direction of swimming in response to a motion/acceleration) have not been extensively investigated. Thus, the study of nanofluid with microorganism gain great interest among researchers. Hence this is the focus of the study in which the presence of motile microorganisms in the system will increase the rate of mass transfer, heat transfer and also improves the nanofluid stability. Analysis through graphs and tables by combining the accuracy and efficiency of the model gives a better understanding of heat, mass and microorganism transfer phenomena. The details are discussed in the next sections. This chapter only focuses on a description of the problem statement, objective and methodology used in this thesis.

1.2 Problem Statement

Boundary layer flows which model magneto-bioconvective of nanofluids Newtonian / non-Newtonian is important for industrial applications. The transport phenomena is governed by a partial differential equations system with the relevant boundary conditions. It is important that reliable mathematical models of the phenomena are developed, investigated and solutions found especially in an economical manner. The present study focuses on the problems of MHD boundary layer flow of Newtonian and non-Newtonian fluid along a stretching sheet, vertical plate, horizontal plate and needle. Both two-dimensional and three - dimensional cases are considered. The effects of selected governing parameters are studied. The parameters magnetic field, blowing, melting (phase change), Brownian motion, thermophoresis, reciprocal of magnetic Prandtl number, Lewis number, Schmidt number, Péclet number, influence of stretching rate ratio, x - direction momentum slip, y - direction momentum slip, thermal slip, mass slip, microorganism slip, bioconvection Schmidt number, bioconvection Péclet number, bioconvection Lewis number, temperature-dependent thermal conductive, first and second order slip, mass diffusivity, microorganism diffusivity are investigated. There are 5 problems involving bioconvection that have been studied in this thesis. The main motivation of choosing these problems is the fact that bioconvection have been extensively used for industrial processes. The following studies are :

1. Three - dimensional bioconvection nanofluid flow from a bi - axial stretching sheet with anisotropic slip, thermal jump and mass slip effects.
2. MHD bioconvection nanofluid flow with melting heat transfer and Stefan blowing.
3. MHD bioconvection nanofluid flow with second order slip and variable thermophysical properties.
4. Non - Newtonian bioconvection nanofluid flow for multiple convective boundary layer with internal heat generation and Stefan blowing.

5. Non - Newtonian MHD bioconvection nanofluid flow past a needle with Stefan blowing.

The effects are discussed based on dimensionless velocity, temperature, nanoparticle volume fraction (concentration) and microorganism. Skin friction, Nusselt number, Sherwood number and microorganism transfer rate are also discussed. An in-depth study of models related to the physical situations described above are important in the light of various applications.

1.3 Objectives

The objectives of this thesis are

(i) To formulate mathematical models based on conservation of mass, momentum, energy, nanoparticle volume fraction and microorganism with various boundary layer equation for steady MHD boundary layer flow of Newtonian / non - Newtonian nanofluids with microorganism.

(ii) To generate the numerical results of the mathematical models.

(iii) To discuss the effects of controlling parameters on physical quantities of engineering interest.

1.4 Scope and Limitation

The scope of this study is limited to problems involving two and three dimensions, Newtonian / non - Newtonian fluid, MHD boundary layer flows, steady, laminar and flow of bionanofluid. The physics of the problem is investigated by studying the effects of selected parameters on velocity, temperature, nanoparticle volume fraction and density of microorganism. Those effects are very important to achieve the clear understanding of the nanofluid flow with microorganism. For the problems in this thesis, only first solutions of the system of ODEs are considered. This is because first solution is stable and physically realizable. According to Bachok et al. (2010), Roşca et al. (2016), Sharma et al. (2014), Weidman et al. (2006), second solution is unstable solution and physically not realizable in the fluid mechanics area. Other than that, Ghosh et al. (2016) also mentioned that dual solution exist for shrinking together with suction cases. In this thesis, we only focus on stretching case, not shrinking case. Since many papers propose that second solution is an unstable solution and unrealistic, it is not necessary to find and discuss in detail the multiple solution for this research. It might give a contribution in mathematics but does not give any physical meaning regarding the models (Bachok et al., 2010).

1.5 Methodology

New mathematical models of the problem related to steady MHD boundary layer bionanofluid with boundary conditions over various geometries are first formulated. Bionanofluid is the combination of nanofluids with microorganisms. All the problems in this thesis are based on the equations of conservation of mass, momentum, energy, nanoparticle volume fraction and microorganism in dimensional form of governing partial differential equations (PDEs). This system of PDEs as well as boundary conditions are transformed into ordinary differential equations (ODEs) and associated conditions by using similarity transformation. The purpose of using similarity transformation is to reduce the number of independent variables to a single dependent variable and then form a boundary value problem involving ODEs. The ODE system with appropriate boundary conditions, which form the boundary value problem, are then solved numerically by using bvp4c in MATLAB. To show the validity and accuracy of the results, the results obtained are then compared with results in the previous literature. Lastly, the physical quantities which is skin friction, heat transfer rate, mass transfer rate and microorganism transfer rate are investigated and discussed for each problem in Chapters 4-8.

1.6 Bvp4c Function

The bvp4c ("*boundary value problem with fourth-order accuracy*") function is a function built in MATLAB that implements the Lobatto IIIa formula which is a type of collocation method to solve the boundary value problem (Afonso and Vasconcelos, 2015). This collocation method is then reduced to the Simpson method when applied to the quadrature problem (Shampine et al., 2000; Shampine et al., 2003; Kierzenka and Shampine, 2001). This Lobatto IIIa method comes from the Runge-Kutta family and it is classified as an implied iteration method. In general, the Lobatto IIIa method uses the concept of approximation to obtain numerical solutions at two end points which are x_n and x_{n+1} for each sub-interval of integration $\left[x_n, x_{n+1}\right]$. Explanation of the Lobatto IIIa method is restricted here and the readers may refer to Jay (1996) to obtain a further understanding of this Lobatto IIIa method. Hence, the solution of a boundary value problem can be expressed as (Shampine et al., 2000):

$$y' = f(x, y, p), \; a \leq x \leq b, \tag{1.1}$$

which is subject to the boundary condition of two non linear common points, that is

$$g(y(a), y(b), p) = 0, \tag{1.2}$$

with p as an unknown vector parameter. The approximate solution ($S(x)$) is a continuous function which is a cubic polynomial at each sub-interval [x_n, x_{n+1}] for a range a = $x_0 < x_1 < \ldots < x_N$ = b. The approximate solution fulfills the boundary conditions of Equation (1.1) and can be written as follows:

$$g(S(a), S(b)) = 0, \tag{1.3}$$

and $S(x)$ also satisfies the differentiatial equations that are collocated at both endpoints and midpoints for each sub-interval, which can be explained as follows using the Equation (1.1):

$$S'(x_n) = f(x_n, S(x_n)),$$
$$S'\left(\frac{(x_n + x_{n+1})}{2}\right) = f\left(\frac{(x_n + x_{n+1})}{2}, S\left(\frac{(x_n + x_{n+1})}{2}\right)\right), \qquad (1.4)$$
$$S'(x_{n+1}) = f(x_{n+1}, S(x_{n+1})).$$

The conditions as in Equation (1.3) and Equation (1.4) produce a system of nonlinear algebraic equations for coefficients defining $S(x)$. Thus, the solution of $y(x)$ is approached over all intervals $[a, b]$ and boundary conditions are always taken into account. Further, the linear method is used to solve the non-linear algebra equation system by iteration. According to Shampine et al. (2000), $S(x)$ is a fourth order approximation to an isolated solution $y(x)$, that is

$$\| y(x) - S(x) \| \leq C h^4, \qquad (1.5)$$

with h is the maximum of the step sizes $(h_n = x_{n+1} - x_n)$ and C is a constant, and the boundary as in Equation (1.5) is true for all x in $[a, b]$. In addition, the basic syntax for bvp4c functions is

sol=bvp4c (OdeBVP, OdeBC, solinit, options) (1.6)

Four input arguments in Equation (1.6) are :

1. OdeBVP

A function that evaluates the ordinary differential equation in an equivalent form with the first level differential equation, with the basic form of *dydx* = OdeBVP (*x*, *y*), where *x* is a scalar, whereas *y* and *dydx* are column vectors.

2. OdeBC

A function that assesses the residuals in boundary terms, with a basic form res=OdeBVP (*ya*, *yb*) where *ya* and *yb* are column vectors representing *y* (*a*) and *y* (*b*), while res is the column vector for the residuals that fulfilled the boundary conditions.

3. solinit

A structure that contains the initial guess of the solution. This structure can be developed using the bvpinit function, i.e solinit = bvpinit (*x*, *y*init), where *x* is the vector of the initial guess for the netting point beginner, while *y*init is an initial guess for a solution in the form constant or function for *x*.

4. options

The solver that controls the error in the solution, with the form options = bvpset ('stats', 'off', 'RelTol', 1e-7), with RelTol as well 1e-7 refers to the relative error tolerance set to 10^{-7}.

All problems in this thesis have been solved via bvp4c function with Matlab. Here, we show how a BVP problem is formulated and solved with bvp4c. In this section, the ordinary differential equation in Chapter 6 are used to show how bvp4c in Matlab is used. First, the ODEs system in Equations (6.10) - (6.14) in Chapter 6 must be reduced as a system of first order ODEs. Next, some new variables are introduced, namely:

$$\begin{aligned} &f = y(1), \quad &&\theta = y(4), \quad &&\phi = y(6), \quad &&\chi = y(8),\\ &f' = y(2), \quad &&\theta' = y(5), \quad &&\phi' = y(7), \quad &&\chi' = y(9).\\ &f'' = y(3), \end{aligned} \tag{1.7}$$

By applying Equation (1.2), Equations (6.10) – (6.14) are transformed to first order ODEs by using the above equations, yields:

$y(2)$

$y(3)$

$$\frac{1}{\left(1+h_2-\theta h_2\right)}((-y(1)*y(3))+(y(2)^2)+M*y(2)+h_2*y(5)*y(3)), \tag{1.8}$$

$y(5)$

$$\frac{1}{1+h_4*f(4)}*\begin{pmatrix} -(h_4+Nt)*(y(5)^2)+(1+h_6*y(6))*Nb*y(5)*y(7)\\ -Ec*Pr*(1+h_2-h_2*y(4))*(y(3)^2)-Pr*y(1)*y(5) \end{pmatrix} \tag{1.9}$$

$y(7)$

$$\frac{1}{1+h_6*y(6)}*\left(\begin{array}{l}(-Sc*y(1)*y(7)-h_6*(y(7)^2)-\dfrac{Nt}{Nb}\\ *\left(\dfrac{1}{1+h_4*f(4)}*\left(\begin{array}{l}-(h_4+Nt)*(y(5)^2)\\ +(1+h_6*y(6))*Nb*y(5)*y(7)\\ -Ec*Pr*(1+h_2-h_2*y(4))*(y(3)^2)\\ -Pr*y(1)*y(5)\end{array}\right)\right)\end{array}\right), \quad (1.10)$$

$y(9)$

$$\begin{array}{l}\dfrac{1}{1+h_8*y(6)}*\\ (-Sb*y(1)*y(9)+Pe*y(8)*(\dfrac{1}{1+h_6*y(6)}*((-Sc*y(1)*y(7)-h_6*(y(7)^2)\\ -\dfrac{Nt}{Nb}*(\dfrac{1}{1+h_4*f(4)}*(-(h_4+Nt)*(y(5)^2)+(1+h_6*y(6))*Nb*y(5)*y(7)\\ -Ec*Pr*(1+h_2-h_2*y(4))*(y(3)^2)-Pr*y(1)*y(5)))))\\ -(h_8-Pe)*y(7)*y(9)\end{array} \quad (1.11)$$

whereas boundary condition in Equation (6.14) rewritten as:

$$y0(2)-1-\delta*y0(3)-\gamma*\frac{1}{(1+h_2-\theta h_2)}\left(\begin{array}{l}-y(1)*y(3))+(y(2)^2)\\ +M*y(2)+h_2*y(5)*y(3)\end{array}\right)$$

$$\begin{array}{l}y0(4)\\ (1+h_4*y0(4))*Me*y0(5)+Pr*y0(1)\\ y0(6)-1\\ y0(8)-1\end{array} \quad (1.12)$$

$yinf(2)$
$yinf(4)-1$
$yinf(6)$
$yinf(8)$

The details of the bvp4c program code for double loop in this problem is shown in Appendix A.

1.7 Outline of thesis

This thesis contains 8 chapters. Chapter 1 focuses on the introduction, problem statement, objective and methodology as well as scopes. Then, Chapter 2 discusses the basic concepts related to the area of study which are Newtonian, non-Newtonian, nanofluids, bioconvection, heat transfer, mass transfer, magnetohydrodynamic and lastly boundary conditions.

The literature review of most recent studies related to this thesis plays an important role because it acts as a reference and a catalyst to get some fresh idea in this study. These literature reviews are placed in Chapter 3. In Chapter 4, three - dimensional bioconvection nanofluid flow with anisotropic slip, thermal slip and mass slip are considered and discussed.

Next, the governing equation that are involved in MHD bioconvection nanofluid with melting heat transfer rate and Stefan blowing are discussed in Chapter 5. Chapter 6 covers bioconvection nanofluid with second order slip and variable thermophysical properties. Chapter 7 concentrates on non-Newtonian bioconvection nanofluid flow for multiple convective boundary layer with internal heat generation and Stefan blowing. Chapter 8 discusses non-Newtonian MHD bioconvection nanofluid flow with Stefan blowing. Lastly, the conclusion and some areas for future work research are discussed in Chapter 9.

CHAPTER 2

BASIC CONCEPTS

2.1 Introduction

A fluid is a substance which deforms continuously under the application of a shear stress. There are two types of fluids which are Newtonian and non-Newtonian fluids. Newtonian fluids obey Newton's laws, but a non-Newtonian fluid does not. Heat and mass transfer of Newtonian and non-Newtonian fluids play an important role in fluid dynamics and heat transfer. This has various applications in science and engineering. In this chapter, basic concepts of fluid dynamics and heat transfer are reviewed.

2.2 Newtonian and Non-Newtonian fluids

All fluids can be classified into two basic types which are Newtonian and non-Newtonian fluids. This thesis involves both fluids. Sir Isaac Newton has explained that the shear stress developed by fluid friction is given by the product of the coefficient of viscosity and the velocity gradient. The Newtonian fluid model states that the viscous stress in a Newtonian fluid is proportional to the rate of strain. Newtonian fluid model applies to both compressible and incompressible fluids. Compressible fluids are fluids with variable density. Incompressible fluid are fluids with constant density where the change in density with pressure is so small as to be negligible. This is usually the case with liquids. In this thesis, only the incompressible case is considered which is water (liquids). Examples of Newtonian fluids include air, water, sugar solutions, glycerin, kerosene, gasoline, silicon oils, mineral oil, light-hydrocarbon oils, and other gases are Newtonian (Massey and Smith, 2006).

Fluids which do not follow the laws of Newton are called non-Newtonian fluids. A non-Newtonian fluid is a fluid whose viscosity changes according to pressure or force. The relation between the shear stress and the shear rate is nonlinear and may depend on time. Therefore, a constant coefficient of viscosity cannot be defined. The most common examples of a non-Newtonian fluid that have been widely used in daily life include shampoo, blood, ketchup and paint (Qayyum et al., 2017).

2.3 Magnetohydrodynamics

MHD is the study of the interaction between the magnetic field and electrically conducting fluids. Examples of such magneto-fluids include plasmas and liquid metals. It is known that if a conducting fluid is placed in a constant magnetic field, every motion of the fluid generates a force called electromotive force which then produces electric currents called Lorentz force (Makinde and Animasaun, 2016). These currents result in mechanical forces which then change the state of motion of the fluid. The existence and influence of Lorentz force during chemical reaction between the bulk fluid and a highly concentrated catalyst is important. Not only that, the presence of MHD in the momentum equation is valid only when magnetic Reynolds number of the flow is so small and thus it is appropriate to neglect induced magnetic field (Makinde and Animasaun, 2016). Magnetohydrodynamic is also useful in planetary magnetospheres and aeronautics as well as electrical engineering (Makinde and Animasaun, 2016).

The flow of magnetohydrodynamic is assumed as incompressible electrically conducting fluid, the body force or known as Lorentz force is considered. It can be defined as:

$$\mathbf{F}_{mag} = \mathbf{E} + \mathbf{J} \times \mathbf{B} \qquad (2.1)$$

where **E** = electric field, **J** = current density and **B** = total magnetic field. Furthermore, according to Ohm's law, the current density **J** can be written as:

$$\mathbf{J} = \sigma\,(\mathbf{E} + \mathbf{V} \times \mathbf{B}), \qquad (2.2)$$

Where σ is the electrical conductivity . It also assumed that the effects of polarization are neglected (**E** = **0**) and therefore Lorentz force $\mathbf{F}_{\text{mag}} = \mathbf{J}\times\mathbf{B}$, and current density $\mathbf{J} = \sigma(\mathbf{V}\times\mathbf{B})$. In order to solve for $\mathbf{F}_{\text{mag}}$ and **J**, we used Maxwell equation which consist of a set of four PDEs namely:

Faraday's law for induction: $\nabla\times\mathbf{E} = -\dfrac{\partial\mathbf{B}}{\partial x}$, (2.3)

Amphere-Maxwell law: $\nabla\times\mathbf{B} = \mu_m\,\mathbf{J}$, (2.4)

Gauss' law for electric: $\nabla\cdot\mathbf{E} = 0$ (2.5)

Gauss' law for magnetic: $\nabla\cdot\mathbf{B} = 0$. (2.6)

Here, μ_m is magnetic permeability. The components of **B**, **E** and **J** are $\mathbf{B} = (0,\ B_0 + b^*,\ 0)$, $\mathbf{E} = \left(E_x, E_y, E_z\right)$ and $\mathbf{J} = \left(J_x, J_y, J_z\right)$ respectively. The total magnetic field (**B**), is a sum of applied magnetic field ($\mathbf{B}_0$) and induced magnetic field, (**b***). Here, since the magnetic Reynolds number is assumed to be small, **b*** can be ignored. Therefore, the current density $\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V}\times\mathbf{B})$ with $\mathbf{V} = \left(\bar{u}, \bar{v}, 0\right)$ is given by:

$$\mathbf{V}\times\mathbf{B} = \sigma\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \bar{u} & \bar{v} & 0\\ 0 & \mathbf{B_0} & 0\end{vmatrix} = \sigma\mathbf{B_0}\bar{u}\,\mathbf{k}. \tag{2.7}$$

Hence, the Lorentz force $\mathbf{F}_{\text{mag}} = \mathbf{J}\times\mathbf{B}$ can be written as:

$$\mathbf{J}\times\mathbf{B} = \sigma\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & \sigma\mathbf{B_0}\bar{u}.\\ 0 & \mathbf{B_0} & 0\end{vmatrix} = -\sigma\mathbf{B}_0^2\bar{u}\,\mathbf{i} \tag{2.8}$$

2.4 Induced Magnetic Field

Electrohydrodynamics (EHD) is the study of electrically charged fluid (Castellanos, 1998). It involves the motion of ionized particles/molecules as well as their interactions with electric fields and surrounding fluid. Due to the induced external electric field, the positive and negative electric charge shift slightly in opposite directions within an insulator, then encourage electric polarization to occur. Therefore, one side of the atom becomes positive and the opposite side becomes negative. This polarization effect of the electric field vector $\mathbf{E} = \left(\bar{E}_x, \bar{E}_y, \bar{E}_z\right)$ is taken under consideration. External magnetic that are applied in the flow is called induced magnetic field. If the magnet is removed then the magnetic field will not be the induced magnetic field. When a magnetic field of uniform strength H_0 is applied transversely along the direction of the main flow stream and the magnetic Reynolds number is large enough then the effect of induced magnetic field cannot be ignored. Hence considering $\mathbf{H} = (\bar{H}_1, \bar{H}_2, 0)$ to be the induced magnetic vector at one point $(\bar{x}, \bar{y}, \bar{z})$ in the fluid flow, the governing equations are (Davies, 1963; Ahmed, 2011; Ali et al., 2011; Cowling, 1957).

Gauss Law of Magnetism:

$$\nabla \bullet \mathbf{H} = 0, \tag{2.9}$$

$$\Rightarrow \left(\frac{\partial}{\partial \bar{x}} i + \frac{\partial}{\partial \bar{y}} j \right) \bullet \left(\bar{H}_1 i + \bar{H}_2 j \right) = 0$$

$$\Rightarrow \frac{\partial \bar{H}_1}{\partial \bar{x}} + \frac{\partial \bar{H}_2}{\partial \bar{y}} = 0.$$

Conservation of momentum:

$$\left(\nabla \bullet \mathbf{V}\right)\mathbf{V} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 \mathbf{V} + \frac{\mu}{4\pi\rho}\left(\nabla \bullet \mathbf{H}\right)\mathbf{H}, \tag{2.10}$$

$$\Rightarrow \left[\left(\frac{\partial}{\partial \bar{x}} i + \frac{\partial}{\partial \bar{y}} j \right) \bullet \left(\bar{u} i + \bar{v} j \right) \right] \left(\bar{u} i + \bar{v} j \right) = -\frac{1}{\rho} \left(\frac{\partial p}{\partial \bar{x}} i + \frac{\partial p}{\partial \bar{y}} j \right)$$
$$+ \frac{\mu}{\rho} \left(\frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2} \right) \left(\bar{u} i + \bar{v} j \right)$$
$$+ \frac{\mu}{4\pi\rho} \left[\left(\frac{\partial}{\partial \bar{x}} i + \frac{\partial}{\partial \bar{y}} j \right) \bullet \left(\bar{H}_1 i + \bar{H}_2 j \right) \right] \left(\bar{H}_1 i + \bar{H}_2 j \right),$$

$$\Rightarrow\left(\bar u\frac{\partial\bar u}{\partial\bar x}+\bar v\frac{\partial\bar u}{\partial\bar y}\right)i+\left(\bar u\frac{\partial\bar v}{\partial\bar x}+\bar v\frac{\partial\bar v}{\partial\bar y}\right)j=-\frac{1}{\rho}\left(\frac{\partial p}{\partial\bar x}i+\frac{\partial p}{\partial\bar y}j\right)$$
$$+\frac{\mu}{\rho}\left[\left(\frac{\partial^2\bar u}{\partial\bar x}+\frac{\partial^2\bar u}{\partial\bar y}\right)i+\left(\frac{\partial^2\bar v}{\partial\bar x}+\frac{\partial^2\bar v}{\partial\bar y}\right)j\right]$$
$$+\frac{\mu}{4\pi\rho}\left[\left(\bar H_1\frac{\partial\bar H_1}{\partial\bar x}+\bar H_2\frac{\partial\bar H_1}{\partial\bar y}\right)i+\left(\bar H_1\frac{\partial\bar H_2}{\partial\bar x}+\bar H_2\frac{\partial\bar H_2}{\partial\bar y}\right)j\right],$$

$$\Rightarrow\left(\bar u\frac{\partial\bar u}{\partial\bar x}+\bar v\frac{\partial\bar u}{\partial\bar y}\right)i+\left(\bar u\frac{\partial\bar v}{\partial\bar x}+\bar v\frac{\partial\bar v}{\partial\bar y}\right)j$$
$$=\left[-\frac{1}{\rho}\frac{\partial p}{\partial\bar x}+\frac{\mu}{\rho}\left(\frac{\partial^2\bar u}{\partial\bar x^2}+\frac{\partial^2\bar u}{\partial\bar y^2}\right)+\frac{\mu}{4\pi\rho}\left(\bar H_1\frac{\partial\bar H_1}{\partial\bar x}+\bar H_2\frac{\partial\bar H_1}{\partial\bar y}\right)\right]i$$
$$\left[-\frac{1}{\rho}\frac{\partial p}{\partial\bar y}+\frac{\mu}{\rho}\left(\frac{\partial^2\bar v}{\partial\bar x^2}+\frac{\partial^2\bar v}{\partial\bar y^2}\right)+\frac{\mu}{4\pi\rho}\left(\bar H_1\frac{\partial\bar H_2}{\partial\bar x}+\bar H_2\frac{\partial\bar H_2}{\partial\bar y}\right)\right]j.$$

$\bar x$ - momentum equation:

$$\Rightarrow\bar u\frac{\partial\bar u}{\partial\bar x}+\bar v\frac{\partial\bar u}{\partial\bar y}=-\frac{1}{\rho}\frac{\partial p}{\partial\bar x}+\frac{\mu}{\rho}\left(\frac{\partial^2\bar u}{\partial\bar x^2}+\frac{\partial^2\bar u}{\partial\bar y^2}\right)+\frac{\mu}{4\pi\rho}\left(\bar H_1\frac{\partial\bar H_1}{\partial\bar x}+\bar H_2\frac{\partial\bar H_1}{\partial\bar y}\right),$$

According to Tham et al. (2013), at $y\to\infty$,

$$\bar u=\bar u_e=\frac{3}{2}\sin x,\ \bar H=\bar H_e,\frac{\partial\bar u_e}{\partial\bar x}=\frac{\partial^2\bar u_e}{\partial\bar x^2}=\frac{\partial\bar H_e}{\partial\bar x}=0,$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial\bar x}=\bar u_e\frac{d\bar u_e}{d\bar x}-\frac{\mu}{4\pi\rho}\bar H_e.$$

$$\Rightarrow\bar u\frac{\partial\bar u}{\partial\bar x}+\bar v\frac{\partial\bar u}{\partial\bar y}=\left(\bar u_e\frac{d\bar u_e}{d\bar x}-\frac{\mu}{4\pi\rho}\bar H_e\frac{d\bar H_e}{d\bar x}\right)+\frac{\mu}{\rho}\left(\frac{\partial^2\bar u}{\partial\bar x^2}+\frac{\partial^2\bar u}{\partial\bar y^2}\right)$$
$$+\frac{\mu}{4\pi\rho}\left(\bar H_1\frac{\partial\bar H_1}{\partial\bar x}+\bar H_2\frac{\partial\bar H_1}{\partial\bar y}\right),\tag{2.11}$$

$\bar y$ - momentum equation:

$$\bar u\frac{\partial\bar v}{\partial\bar x}+\bar v\frac{\partial\bar v}{\partial\bar y}=-\frac{1}{\rho}\frac{\partial p}{\partial\bar y}+\frac{\mu}{\rho}\left(\frac{\partial^2\bar v}{\partial\bar x^2}+\frac{\partial^2\bar v}{\partial\bar y^2}\right)+\frac{\mu}{4\pi\rho}\left(\bar H_1\frac{\partial\bar H_2}{\partial\bar x}+\bar H_2\frac{\partial\bar H_2}{\partial\bar y}\right).\tag{2.12}$$

Conservation of induced magnetic equation:

$$\nabla\times(\mathbf{V}\times\mathbf{H})+\mu_e\nabla^2\mathbf{H}=0,\tag{2.13}$$

$$\Rightarrow\left(\frac{\partial}{\partial\bar x}+\frac{\partial}{\partial\bar y}\right)\times\begin{vmatrix}x & y & z\\ \bar u & \bar v & 0\\ \bar H_1 & \bar H_2 & 0\end{vmatrix}+\mu_e\left(\frac{\partial^2}{\partial\bar x^2}+\frac{\partial^2}{\partial\bar y^2}\right)\left(\bar H_1x+\bar H_2y\right)=0,$$

$$\Rightarrow \left(\frac{\partial}{\partial \bar{x}}+\frac{\partial}{\partial \bar{y}}\right)\times\left(\bar{H}_2\bar{u}-\bar{H}_1\bar{v}\right)z+\mu_e\left(\frac{\partial^2}{\partial \bar{x}^2}+\frac{\partial^2}{\partial \bar{y}^2}\right)\left(\bar{H}_1 x+\bar{H}_2 y\right)=0,$$

$$\Rightarrow \begin{vmatrix} x & y & z \\ \frac{\partial}{\partial \bar{x}} & \frac{\partial}{\partial \bar{y}} & 0 \\ 0 & 0 & \left(\bar{H}_2\bar{u}-\bar{H}_1\bar{v}\right) \end{vmatrix}+\mu_e\left(\frac{\partial^2}{\partial \bar{x}^2}+\frac{\partial^2}{\partial \bar{y}^2}\right)\left(\bar{H}_1 x+\bar{H}_2 y\right)=0,$$

$$\Rightarrow \frac{\partial}{\partial \bar{y}}\left(\bar{H}_2\bar{u}-\bar{H}_1\bar{v}\right)x-\frac{\partial}{\partial \bar{x}}\left(\bar{H}_2\bar{u}-\bar{H}_1\bar{v}\right)y$$
$$+\mu_e\left[\left(\frac{\partial^2\bar{H}_1}{\partial \bar{x}^2}+\frac{\partial^2\bar{H}_1}{\partial \bar{y}^2}\right)x+\left(\frac{\partial^2\bar{H}_2}{\partial \bar{x}^2}+\frac{\partial^2\bar{H}_2}{\partial \bar{y}^2}\right)y\right]=0,$$

$$\Rightarrow \left[\bar{H}_2\frac{\partial \bar{u}}{\partial \bar{y}}+\bar{u}\frac{\partial \bar{H}_2}{\partial \bar{y}}-\bar{H}_1\frac{\partial \bar{v}}{\partial \bar{y}}-\bar{v}\frac{\partial \bar{H}_1}{\partial \bar{y}}+\mu_e\left(\frac{\partial^2\bar{H}_1}{\partial \bar{x}^2}+\frac{\partial^2\bar{H}_1}{\partial \bar{y}^2}\right)\right]x$$
$$+\left[-\bar{H}_2\frac{\partial \bar{u}}{\partial \bar{x}}-\bar{u}\frac{\partial \bar{H}_2}{\partial \bar{x}}+\bar{H}_1\frac{\partial \bar{v}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{H}_1}{\partial \bar{x}}+\mu_e\left(\frac{\partial^2\bar{H}_2}{\partial \bar{x}^2}+\frac{\partial^2\bar{H}_2}{\partial \bar{y}^2}\right)\right]y=0.$$

Using continuity equation, $\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0$ and $\frac{\partial H_1}{\partial \bar{x}}+\frac{\partial H_2}{\partial \bar{y}}=0$, obtain

$\bar{x}$ - induced magnetic equation :

$$\bar{u}\frac{\partial \bar{H}_1}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{H}_1}{\partial \bar{y}}-\bar{H}_1\frac{\partial \bar{u}}{\partial \bar{x}}-\bar{H}_2\frac{\partial \bar{u}}{\partial \bar{y}}=\mu_e\left(\frac{\partial^2\bar{H}_1}{\partial \bar{x}^2}+\frac{\partial^2\bar{H}_1}{\partial \bar{y}^2}\right), \tag{2.14}$$

$\bar{y}$ - induced magnetic equation :

$$\bar{u}\frac{\partial \bar{H}_2}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{H}_2}{\partial \bar{y}}-\bar{H}_1\frac{\partial \bar{v}}{\partial \bar{x}}-\bar{H}_2\frac{\partial \bar{v}}{\partial \bar{y}}=\mu_e\left(\frac{\partial^2\bar{H}_2}{\partial \bar{x}^2}+\frac{\partial^2\bar{H}_2}{\partial \bar{y}^2}\right). \tag{2.15}$$

2.5 Nanofluid

Nanofluid was first introduced by Choi (1995). It refers to a fluid that contain nanoparticles and base fluid (solvent). Nanoparticles is a solid particle having diameter between 1 and 100nm and can flow smoothly through the microchannels easily (Fauzi and Amin, 2012). These particles are made up of metals (*Al, Cu, Ag, Fe*), metal oxides $\left(Al_2O_3, CuO, SiO_2, TiO_2\right)$, carbides, nitrides (*SiC*, *AlN*, *SiN*) or non - metal (graphite, carbon nanotubes) (Mustafa et al., 2012). The chemical and physical properties of nanoparticles are unique.

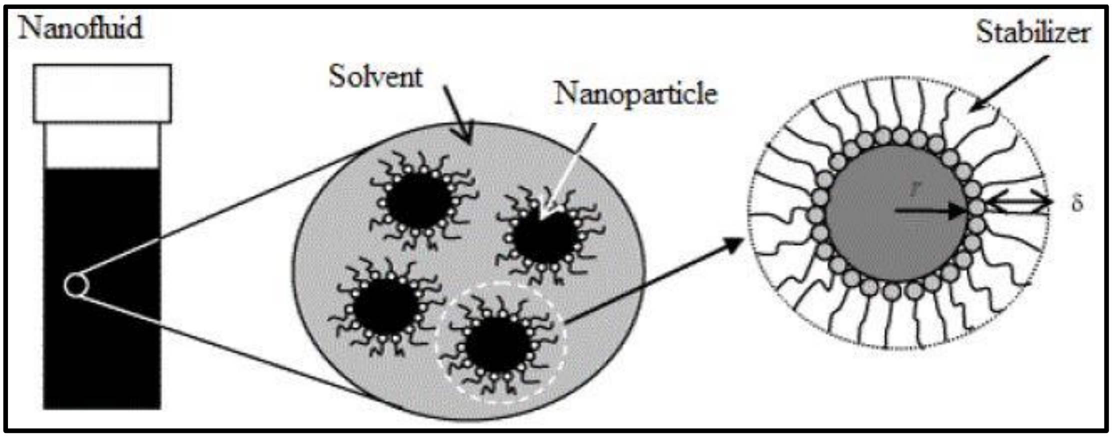

Figure 2.1: Nanoparticle in the base fluid (Bucak, 2011)

By referring Figure 2.1, addition of these particles in a base fluid results in an increase in the thermal conductivity of base fluids. Thermal conductivity and convective heat transfer coefficient of nanofluid are better compared to the base fluid. (Rahman et al., 2014). Base fluid usually consists of water, lubricants and oils as well as ethylene glycol. Water-based nanofluid (Pr = 6.8) is chosen in this research as a solvent. According to Anoop et al. (2009), gyrotactic microorganisms can thrive and remain active in water-based nanofluid (Pr = 6.8). Many researchers have found that thermal conductivity and viscosity of nanofluids are strongly dependent on the nanoparticles volume fraction. Increase in the volume fraction of nanoparticles will increase the viscosity and thermal conductivity of nanofluids. Nanofluids give benefits to many industrial and medical systems including optics, coatings, ceramics and drug delivery (Uddin et al., 2016a).

2.5.1 Advantages of nanofluid

The advantages of nanofluid are (Saidur et al., 2011; Mohammed et al., 2011):

1. Absorption of solar energy will be maximized with increasing size, shape, material, and volume fraction of the nanoparticles.

2. The suspended nanoparticles increase the surface area and the heat capacity of the fluid on account of the very small particle size.
3. To create a suitability for different applications, the characteristics of fluid may be altered by changing the concentration of nanoparticles.
4. The dispersion of nanoparticles flattens the transverse temperature gradient of the fluid.
5. High dispersion stability with predominant Brownian motion of particles.

2.5.2 Applications of Nanofluids

The increasing demand of nanofluids in industrial applications has lead to increased attention from many researchers. There are many applications of nanofluid in our daily life. These include

a) Fuel

Combustion of diesel fuel mixed with aqueous aluminum nanofluid increases the total combustion heat and decreases the concentration of smoke and nitrous oxide in the exhaust emission from the diesel engine. During combustion, decomposition of hydrogen is increased from water due to the high oxidation activity of pure aluminium (Gupta et al., 2012).

b) Coolant

Nanofluids are used in radiators as a coolant. Due to the higher efficiency of the coolant, truck engines can be operated at higher temperatures allowing more horsepower (Gupta et al., 2012).

c) Brake Fluids

During braking, the heat generated causes the brake fluid to reach its boiling point and the vapor created causes a delay in the dispersing of the heat resulting from braking. It will cause a breakdown and poses a safety hazard. The use of nanofluid will maximize the performance in heat transfer and remove any safety risk (Gupta et al., 2012).

d) Domestic refrigerator

1, 1, 1, 2 - Tetrafluoroethane (HFC134a) is often used as a refrigerant. Traditional mineral oil is often avoided as a lubricant because of the strong chemical polarity of HFC-134a in refrigeration equipment. Thus, nanoparticles can be utilized to enhance the working fluid properties and energy efficiency of the refrigerating system which will result in a reduction in CO_2 emission (Gupta et al., 2012).

e) Solar Devices

Otanicar et al. (2010) have shown an increase in efficiency of up to 5% on solar thermal collector using nanofluid as absorption mechanism. The results show an initial rapid increase in efficiency with volume fraction and the number of fragments efficiency continues to increase.

f) Biomedical

Nanofluids were also developed for medical treatments, including cancer therapy. According to Jordan et al. (1999), iron-based nanofluids can be used to produce higher temperatures around the tumor cells. This enables the killing of cancerous cells without affecting the nearby healthy tissues. Nanofluids can also be used for safer surgery by cooling the surgical region. This enhances a patient's chance of survival and reduces the risk of organ damage (Mishra, 2018). Other than that, nanofluid is also used in nanodrug application. Nanodrug delivery systems monitors and control target cell in response to pharmaceutical stimuli (Ravi and Vinod, 2014). Magnetic nanofluids can be used to lead the particles up the blood flow to the tumor with magnets. Thus, doctors can deliver high local dose drug or radiation without damaging nearby healthy tissue. Damaged healthy tissues is a side effect of traditional cancer treatment methods.

Magnetic nanoparticles provide a mean for handling and manipulation of the nanofluid by magnetic force compared to other metal type of nanoparticles (Ravi and Vinod, 2014).

2.6 Bioconvection

Microorganisms have been extensively used to produce industrial and commercial products such as ethanol and fertilizers. Bioconvective microorganisms are used in water treatment plants. Hydrogen gas and biodiesel are produced by these microorganismsms and these products are renewable energy sources. As is well-known, renewable energy is now very important. The reason why we need to study the swimming patterns and mass transfer property of the microorganisms is so that the bioconvection can be made a more effective process (Tausif et al., 2016).

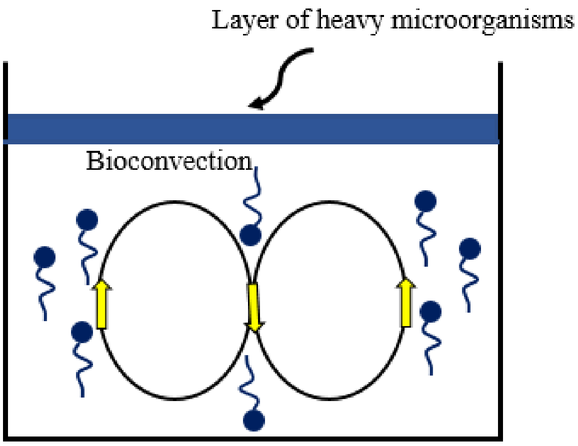

Figure 2.2: Bioconvection process (Khan and Makinde, 2014).

Bioconvection is an interesting phenomenon of fluid mechanics that is driven by the swimming motion of microorganisms, which are dense than water and swim (on average) in an upward direction which are then an accumulation of microorganisms is formed (Tausif et al., 2016). Due to that, the upper surface of suspensions becomes too dense and unstable, causing the microorganisms to fall and result in bioconvection.

This bioconvection process are shown in Figure 2.2 in which return up swimming microorganisms will maintain this bioconvection pattern.

Up swimming microorganisms respond to certain stimuli by tending to swim in certain directions (Hill and Pedley, 2005). These responses are gravitaxis (swimming in the opposite sense to gravity), gyrotaxis (direction of swimming is determined by the balance of gravitational and viscous torques), phototaxis (swimming towards or away from light), and chemotaxis (swimming in response to a chemical). Bioconvection patterns created by different strains of microorganism can help to identify different organisms. While bioconvection patterns formed by them might seem similar, the mechanisms of orientation are different (Hill and Pedley, 2005). There are two typical types of up swimming microorganisms that are usually used in bioconvection experiments: bacteria and algae (Hill and Pedley, 2005).

Bioconvection has various applications. Most of the applications of bioconvection are in bio microsystems and biotechnology due to mass transport enhancement and mixing. These are important issues in many micro-systems (Sokolov et al., 2009 ; Tsai et al. 2009). Bioconvection is also known to be useful in the biomedicine industry such as nanodrug delivery, cancer therapeutics and magnetic nanofluid. Bioconvection plays a key role in the field of mechanical applications where electric fields are used to control bioconvection phenomenon for producing energy source or mechanical power source. Combination of bioconvection and nanofluids (bionanofluid) becomes significant for micro-fluidic applications. The application of bioconvection is relevant in the biomedicine industry such as nanodrug delivery and cancer therapeutics (Ravi and Vinod, 2014). However, most of the research that are related to bioconvection is concerned with non-Newtonian fluids. When the motion of microorganisms are considered as a group (rather than individuals), then the motion can be described mathematically and this is where mathematical modelling using differential equations comes into play.

The main motivation in this thesis is the fact that bioconvection have been extensively used for industrial processes. In this work, an investigation on microorganisms together with nanofluid as well as magnetohydrodynamic (MHD) effects and other parameters with appropriate boundary conditions are carried out. Advantages of the inclusion of microorganisms to a suspension include enhanced mass transfer, microscale mixing, and anticipated improved stability of the nanofluid (Kuznetsov, 2011b). There are similarities and differences between nanoparticles and motile microorganisms. The details are discussed in the next sections.

2.7 Introduction to Boundary layer

The change in viscosity, thermal diffusivity, mass diffusivity and microorganism diffusivity occurs in a very thin layer of fluid adjacent to the solid surface is called boundary layer. The flow outside the boundary layer is known as free stream are not significant (Holland and Brag, 1995).

In 1904, Ludwig Prandtl introduced the current boundary layer theory when he presented his paper entitled "On the motion of fluids of very small viscosity" in Heidelberg, Germany (Acheson, 1990). In his paper, he introduced a concept about the existence of a thin layer attached to the surface of the body immersed in the fluid flow field. This boundary layer has a large and normal velocity against the surface wall of the body ($\partial u / \partial y$). u is non-dimensional velocity component along the $\overline{x}$ - direction while y is non-dimensional coordinate along the surface.

Consequently, a very small viscosity (μ) can play an important role in this region as the viscous shear stress $\left(\tau = \mu \partial u / \partial y\right)$ seeks to achieve great value. Hence, Prandtl concludes that the influence of viscosity within the boundary layer is sufficient in the flow field analysis. Prandtl showed the importance of viscosity effects in the fluid flow at the Reynolds high value (Re >>1) and showed how the Navier-Stokes equation can be simplified using the concept of the boundary layer. The concept of the boundary layer explains that the flow of fluid with a high Reynolds number value can be divided into two different parts (Schlichting and Gersten, 2017). The first part is an inviscid part of the flow (far from the surface of the body) while the second part is called viscous flow (close to the surface of the body) (see Figure 2.3). In an inviscid flow region, the viscosity behavior is not important because there is no velocity gradient. Therefore, the fluid flow in that region is classified as a potential flow and no friction (Schlichting and Gersten, 2017). The boundary layer is divided into four types which are velocity boundary layer, thermal boundary layer, concentration boundary layer and microorganism boundary layer (Özişik, 1985). All these layers are relevant in this thesis. Consider laminar fluid flow over flat plate surface as shown in Figure 2.3.

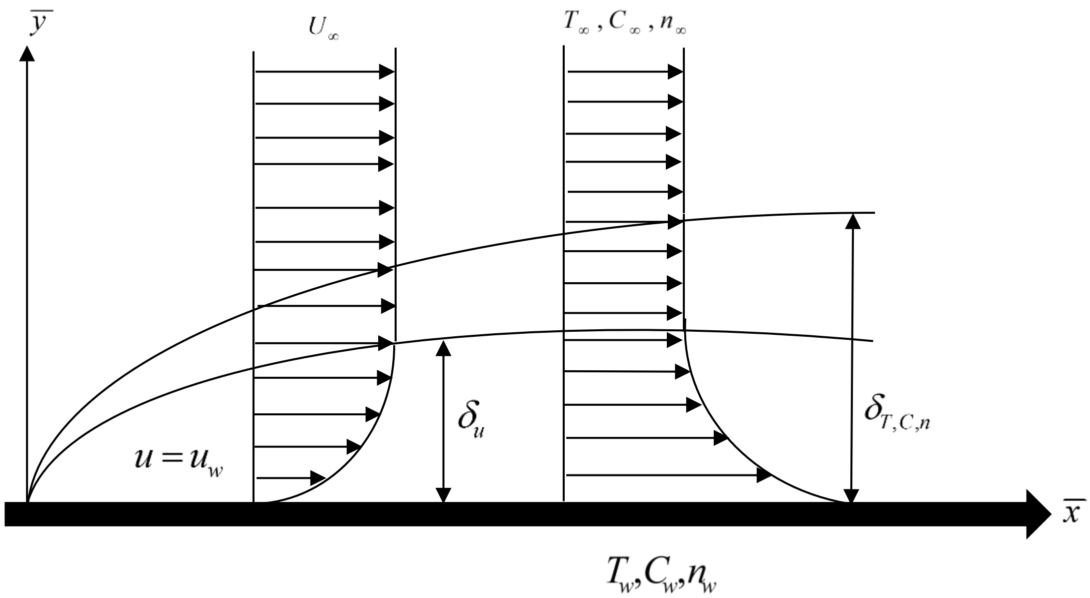

Figure 2.3: Boundary layer over flat plate surface

According to Prandtl's boundary layer flow theory, viscosity effects are limited to a very small region around a body. When the region is passed, then the induced flow velocity through the effects of fluid viscosity will reach its original value again (Darus, 1990). Here are some key assumptions to derive the Navier-Stokes differential equation to the Prandtl boundary layer equation (Schlichting and Gersten, 2017):

1. Boundary layer thickness (δ) is very small compared to the body length characteristic (*L*) and can be defined as $\delta << L$. Therefore, the solution of the boundary layer equation is asymptotic for the very high Reynolds number.

2. Fluid that satisfy the no-slip condition on the body surface (in the boundary layer) through the following definition: *u* (*x*, 0) = *v* (*x*, 0). For velocity in the inviscid flow can be defined as $u(x,\infty)=U_\infty$ and $v(x,\infty)=V_\infty$, where u and v are the velocity components in *x* and *y* directions respectively, while U_∞ and V_∞ are the free stream velocities.

3. The effect of flow field viscosity is only limited to the boundary layer region. In the inviscid flow region, the fluid flow can be described using an inviscid solution such as that from Euler's equation.

4. The order of magnitude of *u* in the boundary layer is $u = O(U_\infty)$.

Prandtl boundary layer equations can be characterized as a partial parabolic differential equations thus making the process of solution and analysis for various fluid flow and heat transfer problems easier (Cebeci and Bradshaw, 1988).

The flow approaches at the end surface with a flow velocity in the direction and the surrounding temperature. When the fluid flows with velocity through the flat plate surface ($y = 0$), the molecules or the fluid elements touching the plate have zero velocity (Kakaç and Yener, 1994 ; Long and Sayma, 2009). These fluid elements then slow down the movement of fluid elements near them to a distance $y = \delta h$ from the surface of the plate. The quantity of δh refers to the thickness of the coating boundary which is usually defined as the value of y with u = 0.99 for a two dimensional flow case (Long and Sayma, 2009). This effect can be ignored when the velocity of the elements in the fluid is approaching the free flow velocity. Boundary layer velocity profile refers to the value of u which changes with distance y (for case of two dimensions) or value u, v which varies with distance x, y from the boundary layer surface (Long and Sayma, 2009). Thermal boundary layer is generated when there is a temperature difference between the fluid with surface. The temperature profile is uniform with $T = T_\infty$ at the end left plate surface. However, when fluid elements meet the surface plate, heat transfer process occurs until thermal equilibrium is reached. At the same time, energy conversion occurs with those elements with adjacent elements. This condition further forms the gradient of temperature and this area is known as thermal boundary layer. The thermal boundary layer thickness, δ_T is defined as value y with the ratio $\frac{T_w - T}{T_w - T_\infty} = 0.99$. T_w is the surface temperature, T is the temperature of the fluid within the boundary layer and T_∞ is the free stream fluid temperature (Long and Sayma, 2009). Increasing the distance x from the left edge of the plate causes the transfer effect the heat permeates into the free flow, thereby causing increased thickness of the layers thermal boundary layer. Similarly to the temperature, C_w and n_w are the surface concentration and microorganism, respectively, while C_∞ and n_∞ are the ambient fluid concentration and microorganism, respectively (Cengel and Cimbala, 2014)

There are three types of boundary layer which are laminar (layered), turbulent (disordered) and transitional boundary layers. These boundary layers are dependent on the value of the Reynolds number. In this thesis, we considered laminar boundary layer for all the problems in Chapter 4 until Chapter 8.

In the laminar boundary layer, any mass or momentum exchange occurs between adjacent layers only at the microscopic scale that cannot be seen with the eye. Since the fluid may therefore be considered as moving in layers, this kind of flow is now called laminar flow. Laminar boundary layers are found only when the Reynolds numbers are small. When Reynolds numbers is low, the boundary layer is laminar and the streamwise velocity changes uniformly as one move away from the wall (Massey and Smith, 2006).

2.8 Governing Equations

The general governing equations consist of the following continuity, momentum, energy, mass and microorganism which is can be written in vector form: (Hill and Pedley, 2005; Potter et al., 2012):

Continuity equation: $$\nabla \cdot \bar{\mathbf{V}} = 0. \tag{2.16}$$

Momentum equation: $$\rho \bar{\mathbf{V}} \cdot \nabla \bar{\mathbf{V}} = -\nabla p + \mu\left(\nabla^2 \bar{\mathbf{V}}\right). \tag{2.17}$$

Energy equation: $$\bar{\mathbf{V}} \cdot \nabla \mathbf{T} = \alpha\left(\nabla^2 \mathbf{T}\right) + \tau\left[D_B \nabla \mathbf{T} \cdot \nabla \mathbf{C} + \left(\frac{D_T}{T_\infty}\right) \nabla \mathbf{T} \cdot \nabla \mathbf{T}\right]. \tag{2.18}$$

Concentration equation: $$\bar{\mathbf{V}} \cdot \nabla \mathbf{C} = D_B\left(\nabla^2 \mathbf{C}\right) + \left[\left(\frac{D_T}{T_\infty}\right) \nabla^2 \mathbf{T}\right]. \tag{2.19}$$

Microorganism equation: $$\bar{\mathbf{V}} \cdot \nabla \mathbf{n} + \hat{v}\left[\nabla \cdot \left(\mathbf{n} \nabla \mathbf{C}\right)\right] = D_n\left(\nabla^2 \mathbf{n}\right). \tag{2.20}$$

where $\hat{v} = \left(\dfrac{bW_c}{\Delta C}\right)$, $\nabla = \dfrac{\partial}{\partial x}\vec{i} + \dfrac{\partial}{\partial y}\vec{j} + \dfrac{\partial}{\partial z}\vec{k}$, $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$,

Next, the governing equations can be written in the three dimensional cartesian coordinates form: (This mathematical model refers in Chapter 4 - 3D case).

Continuity equation

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0. \tag{2.21}$$