First Semester Examination
Academic Session 2020/2021

February 2021

EAS663 – Dynamics and Stability of Structures

Duration : 2 hours

Please check that this examination paper consists of **ELEVEN (11)** pages of printed material including appendix before you begin the examination.

Instructions : This paper contains **SIX (6)** questions. Answer **FOUR (4)** questions.

Each question **MUST BE** answered on a new page.

(1). A 6.0 m long simply supported beam-column, which is prevented from swaying and from deflecting out of the plane of bending, is required to support a factored design axial load of 1000 kN and factored design end moments of 300 and 150 kNm which bend the beam-column in single curvature about the major axis. Design a suitable UB or UC beam-column of S275 steel. (Refer **Appendix A** for section properties of UB or UC)

[25 marks]

(2). A single degree of freedom system is excited by a dynamic force as shown in **Figure 1**. Assume the girder is rigid whereas the columns are flexible to the lateral deformation but rigid in vertical direction. Using E = 30 GPa and I = 15(10^6) mm^4, 5 % damping and neglect the mass of columns, determine

(i) The natural period of building
(ii) The steady state amplitude of vibration, and
(iii) The maximum shear force and bending moment in the column

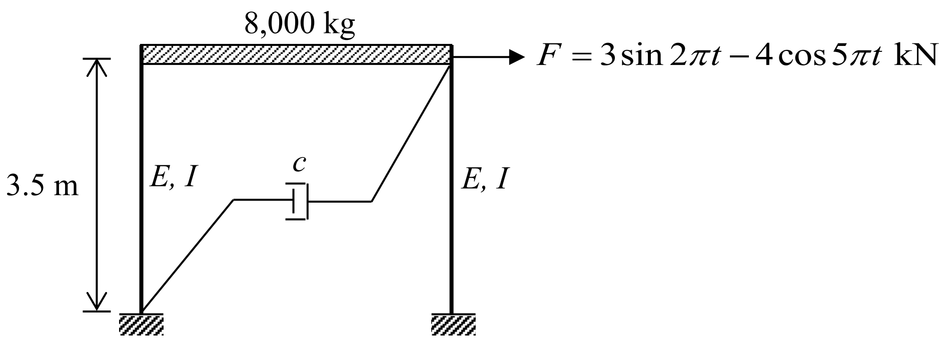


Figure 1

[25 marks]

(3). **Figure 2** shows a damped multi-degree of freedom system with the mass m_i, damping coefficient c_i and stiffness k_i at i-level. Use various masses such as m_1, m_i and m_N as examples,

(i) Formulate the equations of motion for this N-storey building excited by a dynamic load. State all assumptions made.

(ii) Explain the way to determine the natural frequencies and vibration mode shapes.

(iii) Outline the process to solve the equations of motion. Show clearly in your answer how to uncouple the equations of motion.

(iv) Explain how to solve this problem if the dynamic excitation is an earthquake ground motion with the ground acceleration of $\ddot{u}_g(t)$.

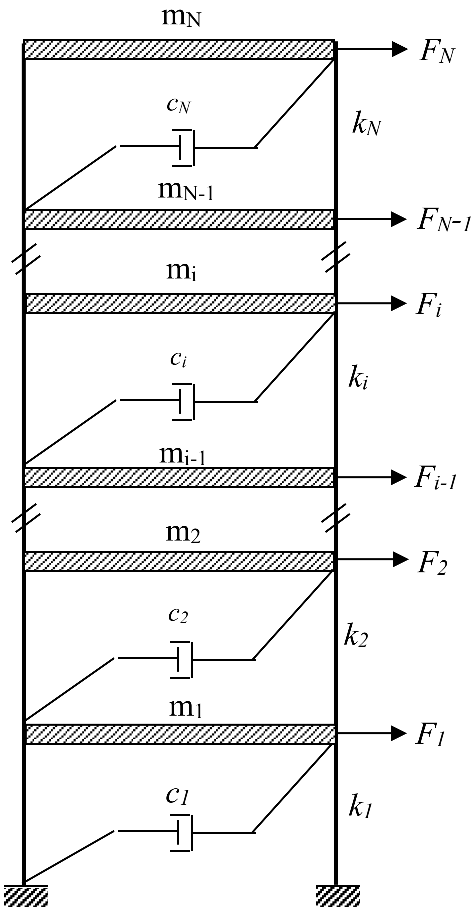


Figure 2

[25 marks]

(4). (a). Using suitable sketches, explain the meaning of neutral equilibrium. Obtain the critical load of the column shown in **Figure 3** using method of neutral equilibrium. The lower end of column is pinned and the upper end is prevented from rotating but free to translate laterally. Given flexural rigidity of the column is *EI*.

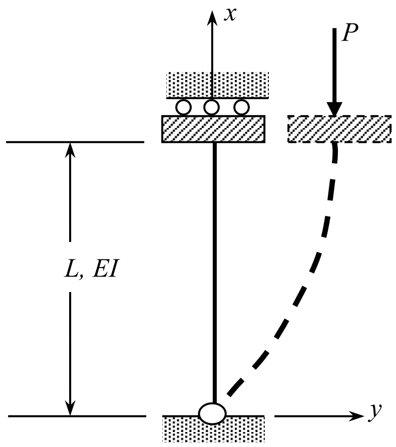


Figure 3

[10 marks]

(b). The total deflection *y* from vertical for the initially bent imperfect column shown in **Figure 4** is given as follows:

$y = a/(1-\alpha)\ sin\frac{\pi x}{L}$

where a is the amplitude of the initial deformation at mid-height of the column and $\alpha = P/P_E$.

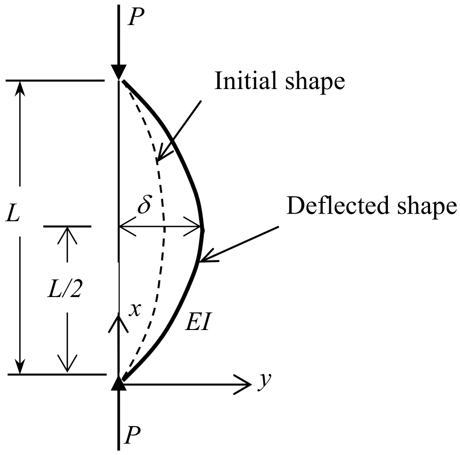


Figure 4

Using the above equation for y, derive the expression for the mid-height deflection δ. Next, explain the difference in behavior between imperfect and perfect column using the derived expression for δ. Use suitable graph in your explanation.

[15 marks]

(5). (a). Obtain the critical load of the cantilever column shown in **Figure 5** by using Rayleigh-Ritz method for the following assumed deflection of the cantilever column:

$$y = Ax^2 + Bx^3$$

where *A, B* : constants. Provide one possible reason for the difference of the obtained solution in comparison with the exact solution of $P_{cr,\ exact} = 2.467\ EI/L^2$.

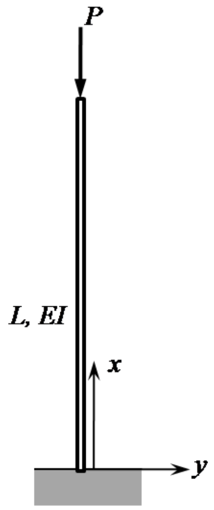


Figure 5

[15 marks]

(b). The mid-span deflection δ of the simply supported beam-column subjected to transverse load Q and axial force P as shown in **Figure 6** is given by the following equation:

$$\delta = \delta_0 \times 1 / (1 - P/P_{cr})$$

where $\delta_0 = QL^3/48EI$ which corresponds to the mid-span deflection in the absence of P and P_{cr}: critical load for the simply supported beam-column. Using the above equation of δ, explain the behavior of beam-column. Next, using suitable Q vs δ plots for the case of $P/P_{cr}=0$, $P/P_{cr}=0.25$, $P/P_{cr}=0.5$, explain the effect of the presence of constant axial force on bending stiffness of beam-column.

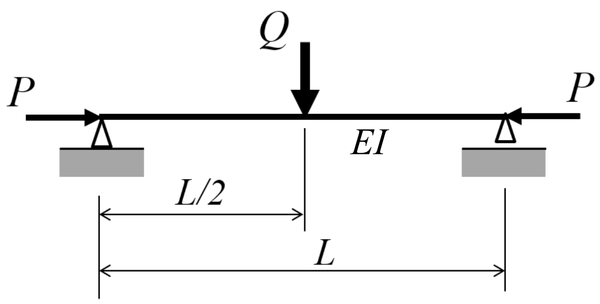


Figure 6

[10 marks]

(6). (a). Using matrix method, determine the critical load of column BC in the frame shown in **Figure 7**. Both A and C are fixed supports. Refer Appendix B for the stiffness matrix of a beam-column member.

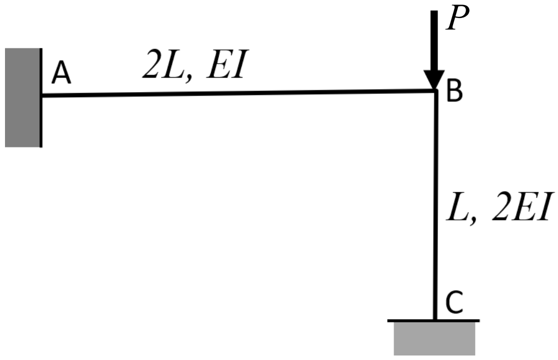


Figure 7

[15 marks]

(b). **Figure 8 (a)** and **(b)** show a braced and un-braced frame, respectively. Sketch the corresponding buckling mode for the case of braced and unbraced frames, respectively. Subsequently, show that the range of critical load P_{cr} for the unbraced frame shown in **Figure 8(b)** is as follows.

$0.25\, P_E < P_{cr} < P_E$, where P_E : Euler buckling load.

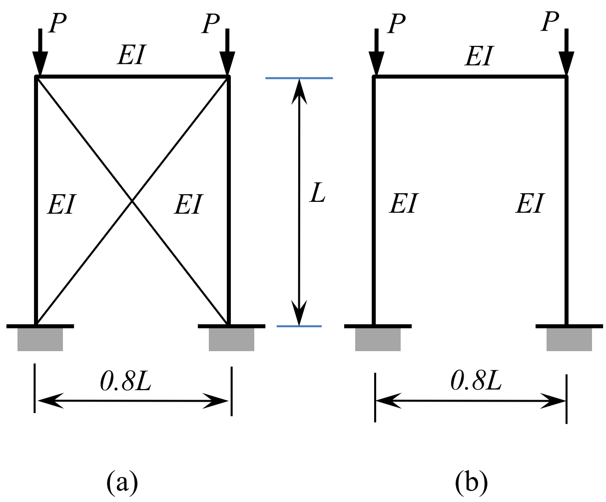


Figure 8

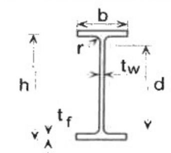
[10 marks]

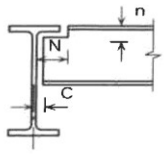
APPENDIX A

BS EN 1993-1-1:2005
BS 4-1:2005

UNIVERSAL BEAMS

Advance UKB





Dimensions

Section Designation	Mass per Metre	Depth of Section	Width of Section	Thickness		Root Radius	Depth between Fillets	Ratios for Local Buckling		Dimensions for Detailing			Surface Area	
				Web	Flange			Flange	Web	End Clearance	Notch		Per Metre	Per Tonne
		h	b	t_w	t_f	r	d	c_f / t_f	c_w / t_w	C	N	n		
	kg/m	mm	mm	mm	mm	mm	mm			mm	mm	mm	m^2	m^2
1016x305x487 +	486.7	1036.3	308.5	30.0	54.1	30.0	868.1	2.02	28.9	17	150	86	3.20	6.58
1016x305x437 +	437.0	1026.1	305.4	26.9	49.0	30.0	868.1	2.23	32.3	15	150	80	3.17	7.25
1016x305x393 +	392.7	1015.9	303.0	24.4	43.9	30.0	868.1	2.49	35.6	14	150	74	3.14	8.00
1016x305x349 +	349.4	1008.1	302.0	21.1	40.0	30.0	868.1	2.76	41.1	13	152	70	3.13	8.96
1016x305x314 +	314.3	999.9	300.0	19.1	35.9	30.0	868.1	3.08	45.5	12	152	66	3.11	9.89
1016x305x272 +	272.3	990.1	300.0	16.5	31.0	30.0	868.1	3.60	52.6	10	152	62	3.10	11.4
1016x305x249 +	248.7	980.1	300.0	16.5	26.0	30.0	868.1	4.30	52.6	10	152	56	3.08	12.4
1016x305x222 +	222.0	970.3	300.0	16.0	21.1	30.0	868.1	5.31	54.3	10	152	52	3.06	13.8
914x419x388	388.0	921.0	420.5	21.4	36.6	24.1	799.6	4.79	37.4	13	210	62	3.44	8.87
914x419x343	343.3	911.8	418.5	19.4	32.0	24.1	799.6	5.48	41.2	12	210	58	3.42	9.96
914x305x289	289.1	926.6	307.7	19.5	32.0	19.1	824.4	3.91	42.3	12	156	52	3.01	10.4
914x305x253	253.4	918.4	305.5	17.3	27.9	19.1	824.4	4.48	47.7	11	156	48	2.99	11.8
914x305x224	224.2	910.4	304.1	15.9	23.9	19.1	824.4	5.23	51.8	10	156	44	2.97	13.2
914x305x201	200.9	903.0	303.3	15.1	20.2	19.1	824.4	6.19	54.6	10	156	40	2.96	14.7
838x292x226	226.5	850.9	293.8	16.1	26.8	17.8	761.7	4.52	47.3	10	150	46	2.81	12.4
838x292x194	193.8	840.7	292.4	14.7	21.7	17.8	761.7	5.58	51.8	9	150	40	2.79	14.4
838x292x176	175.9	834.9	291.7	14.0	18.8	17.8	761.7	6.44	54.4	9	150	38	2.78	15.8
762x267x197	196.8	769.8	268.0	15.6	25.4	16.5	686.0	4.32	44.0	10	138	42	2.55	13.0
762x267x173	173.0	762.2	266.7	14.3	21.6	16.5	686.0	5.08	48.0	9	138	40	2.53	14.6
762x267x147	146.9	754.0	265.2	12.8	17.5	16.5	686.0	6.27	53.6	8	138	34	2.51	17.1
762x267x134	133.9	750.0	264.4	12.0	15.5	16.5	686.0	7.08	57.2	8	138	32	2.51	18.7
686x254x170	170.2	692.9	255.8	14.5	23.7	15.2	615.1	4.45	42.4	9	132	40	2.35	13.8
686x254x152	152.4	687.5	254.5	13.2	21.0	15.2	615.1	5.02	46.6	9	132	38	2.34	15.4
686x254x140	140.1	683.5	253.7	12.4	19.0	15.2	615.1	5.55	49.6	8	132	36	2.33	16.6
686x254x125	125.2	677.9	253.0	11.7	16.2	15.2	615.1	6.51	52.6	8	132	32	2.32	18.5
610x305x238	238.1	635.8	311.4	18.4	31.4	16.5	540.0	4.14	29.3	11	158	48	2.45	10.3
610x305x179	179.0	620.2	307.1	14.1	23.6	16.5	540.0	5.51	38.3	9	158	42	2.41	13.5
610x305x149	149.2	612.4	304.8	11.8	19.7	16.5	540.0	6.60	45.8	8	158	38	2.39	16.0
610x229x140	139.9	617.2	230.2	13.1	22.1	12.7	547.6	4.34	41.8	9	120	36	2.11	15.1
610x229x125	125.1	612.2	229.0	11.9	19.6	12.7	547.6	4.89	46.0	8	120	34	2.09	16.7
610x229x113	113.0	607.6	228.2	11.1	17.3	12.7	547.6	5.54	49.3	8	120	30	2.08	18.4
610x229x101	101.2	602.6	227.6	10.5	14.8	12.7	547.6	6.48	52.2	7	120	28	2.07	20.5
610x178x100 +	100.3	607.4	179.2	11.3	17.2	12.7	547.6	4.14	48.5	8	94	30	1.89	18.8
610x178x92 +	92.2	603.0	178.8	10.9	15.0	12.7	547.6	4.75	50.2	7	94	28	1.88	20.4
610x178x82 +	81.8	598.6	177.9	10.0	12.8	12.7	547.6	5.57	54.8	7	94	26	1.87	22.9
533x312x273 +	273.3	577.1	320.2	21.1	37.6	12.7	476.5	3.64	22.6	13	160	52	2.37	8.67
533x312x219 +	218.8	560.3	317.4	18.3	29.2	12.7	476.5	4.69	26.0	11	160	42	2.33	10.7
533x312x182 +	181.5	550.7	314.5	15.2	24.4	12.7	476.5	5.61	31.3	10	160	38	2.31	12.7
533x312x151 +	150.6	542.5	312.0	12.7	20.3	12.7	476.5	6.75	37.5	8	160	34	2.29	15.2

Advance and UKB are trademarks of Corus. A fuller description of the relationship between Universal Beams (UB) and the Advance range of sections manufactured by Corus is given in section 12.

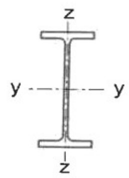
+ These sections are in addition to the range of BS 4 sections.

FOR EXPLANATION OF TABLES SEE NOTE 2

BS EN 1993-1-1:2005
BS 4-1:2005

UNIVERSAL BEAMS

Advance UKB



Properties

Section Designation	Second Moment of Area		Radius of Gyration		Elastic Modulus		Plastic Modulus		Buckling Parameter	Torsional Index	Warping Constant	Torsional Constant	Area of Section
	Axis y-y	Axis z-z	Axis y-y	Axis z-z	Axis y-y	Axis z-z	Axis y-y	Axis z-z	U	X	I_w	I_T	A
	cm^4	cm^4	cm	cm	cm^3	cm^3	cm^3	cm^3			dm^6	cm^4	cm^2
1016x305x487 +	1022000	26700	40.6	6.57	19700	1730	23200	2800	0.867	21.1	64.4	4300	620
1016x305x437 +	910000	23400	40.4	6.49	17700	1540	20800	2470	0.868	23.1	56.0	3190	557
1016x305x393 +	808000	20500	40.2	6.40	15900	1350	18500	2170	0.868	25.5	48.4	2330	500
1016x305x349 +	723000	18500	40.3	6.44	14300	1220	16600	1940	0.872	27.9	43.3	1720	445
1016x305x314 +	644000	16200	40.1	6.37	12900	1080	14800	1710	0.872	30.7	37.7	1260	400
1016x305x272 +	554000	14000	40.0	6.35	11200	934	12800	1470	0.872	35.0	32.2	835	347
1016x305x249 +	481000	11800	39.0	6.09	9820	784	11300	1240	0.861	39.9	26.8	582	317
1016x305x222 +	408000	9550	38.0	5.81	8410	636	9810	1020	0.850	45.7	21.5	390	283
914x419x388	720000	45400	38.2	9.59	15600	2160	17700	3340	0.885	26.7	88.9	1730	494
914x419x343	626000	39200	37.8	9.46	13700	1870	15500	2890	0.883	30.1	75.8	1190	437
914x305x289	504000	15600	37.0	6.51	10900	1010	12600	1600	0.867	31.9	31.2	926	368
914x305x253	436000	13300	36.8	6.42	9500	871	10900	1370	0.865	36.2	26.4	626	323
914x305x224	376000	11200	36.3	6.27	8270	739	9530	1160	0.860	41.3	22.1	422	286
914x305x201	325000	9420	35.7	6.07	7200	621	8350	982	0.853	46.9	18.4	291	256
838x292x226	340000	11400	34.3	6.27	7980	773	9160	1210	0.869	35.0	19.3	514	289
838x292x194	279000	9070	33.6	6.06	6640	620	7640	974	0.862	41.6	15.2	306	247
838x292x176	246000	7800	33.1	5.90	5890	535	6810	842	0.856	46.5	13.0	221	224
762x267x197	240000	8170	30.9	5.71	6230	610	7170	958	0.869	33.1	11.3	404	251
762x267x173	205000	6850	30.5	5.58	5390	514	6200	807	0.865	38.0	9.39	267	220
762x267x147	169000	5460	30.0	5.40	4470	411	5160	647	0.858	45.2	7.40	159	187
762x267x134	151000	4790	29.7	5.30	4020	362	4640	570	0.853	49.8	6.46	119	171
686x254x170	170000	6630	28.0	5.53	4920	518	5630	811	0.872	31.8	7.42	308	217
686x254x152	150000	5780	27.8	5.46	4370	455	5000	710	0.871	35.4	6.42	220	194
686x254x140	136000	5180	27.6	5.39	3990	409	4560	638	0.870	38.6	5.72	169	178
686x254x125	118000	4380	27.2	5.24	3480	346	3990	542	0.863	43.8	4.80	116	159
610x305x238	209000	15800	26.3	7.23	6590	1020	7490	1570	0.886	21.3	14.5	785	303
610x305x179	153000	11400	25.9	7.07	4930	743	5550	1140	0.885	27.7	10.2	340	228
610x305x149	126000	9310	25.7	7.00	4110	611	4590	937	0.886	32.7	8.17	200	190
610x229x140	112000	4510	25.0	5.03	3620	391	4140	611	0.875	30.6	3.99	216	178
610x229x125	98600	3930	24.9	4.97	3220	343	3680	535	0.875	34.0	3.45	154	159
610x229x113	87300	3430	24.6	4.88	2870	301	3280	469	0.870	38.0	2.99	111	144
610x229x101	75800	2910	24.2	4.75	2520	256	2880	400	0.863	43.0	2.52	77.0	129
610x178x100 +	72500	1660	23.8	3.60	2390	185	2790	296	0.854	38.7	1.44	95.0	128
610x178x92 +	64600	1440	23.4	3.50	2140	161	2510	258	0.850	42.7	1.24	71.0	117
610x178x82 +	55900	1210	23.2	3.40	1870	136	2190	218	0.843	48.5	1.04	48.8	104
533x312x273 +	199000	20600	23.9	7.69	6890	1290	7870	1990	0.891	15.9	15.0	1290	348
533x312x219 +	151000	15600	23.3	7.48	5400	982	6120	1510	0.884	19.8	11.0	642	279
533x312x182 +	123000	12700	23.1	7.40	4480	806	5040	1240	0.886	23.4	8.77	373	231
533x312x151 +	101000	10300	22.9	7.32	3710	659	4150	1010	0.885	27.8	7.01	216	192

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+ These sections are in addition to the range of BS 4 sections.

FOR EXPLANATION OF TABLES SEE NOTE 3

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APPENDIX B

Stiffness matrix for a beam-column member

$$[k] = EI\begin{bmatrix} \frac{12}{L^3} & -\frac{6}{L^2} & -\frac{12}{L^3} & -\frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{4}{L} & \frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & \frac{6}{L^2} & \frac{12}{L^3} & \frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{2}{L} & \frac{6}{L^2} & \frac{4}{L} \end{bmatrix} - P\begin{bmatrix} \frac{6}{5L} & -\frac{1}{10} & -\frac{6}{5L} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{2L}{15} & \frac{1}{10} & -\frac{1}{30} \\ -\frac{6}{5L} & \frac{1}{10} & \frac{6}{5L} & \frac{1}{10} \\ -\frac{1}{10} & -\frac{1}{30} & \frac{1}{10} & \frac{2L}{15} \end{bmatrix}$$

where EI:flexural rigidity of member; L:length of member; P:axial force of member

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