First Semester Examination
Academic Session 2020/2021

February 2021

EAS661 – Advanced Structural Mechanics

Duration : 2 hours

Please check that this examination paper consists of **SEVEN (7)** pages of printed material including appendix before you begin the examination.

Instructions : This paper contains **FOUR (4)** questions. Answer **ALL** questions.

Each question **MUST BE** answered on a new page.

...2/-

(1). (a). (i). Derive the governing differential equation for the linear elastic bar problem shown in **Figure 1** where *E*: elastic modulus, *A*: cross-sectional area, *w*: uniformly distributed load per unit length. State clearly where the three basic equations in theory of elasticity have been used.

[6 marks]

(ii). Using the governing differential equation, derive the expression of axial displacement *u* and axial stress σ_x for the linear elastic bar problem. Both ends of the bars are fixed.

[9 marks]

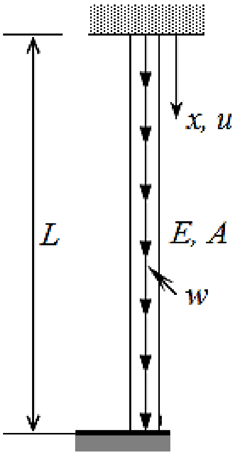

Figure 1

(b). A small thin rectangular plate as shown in **Figure 2** is made of aluminium alloy (E=75 GPa and ν=0.33). Stress components σ_{xx} and σ_{yy} are uniformly distributed as shown in the figure.

(i). determine the magnitude of σ_{yy} so that the dimension b=20 mm remains the same under the load

(ii). determine the percentage of change in dimension a

(iii). write down the corresponding state of stress and state of strain of a point within the rectangular plate

[10 marks]

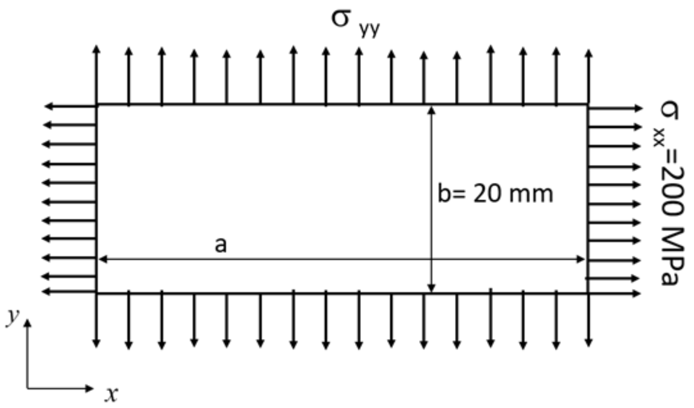

Figure 2

(2). (a). **Figure 3** shows a spring-block assembly. Determine the force in spring 1 using Principle of Minimum Potential Energy. Given that: N = 5 kN, b=100 mm, k_1= 0.5 kN/mm, k_2=2.5 kN/mm.

[7 marks]

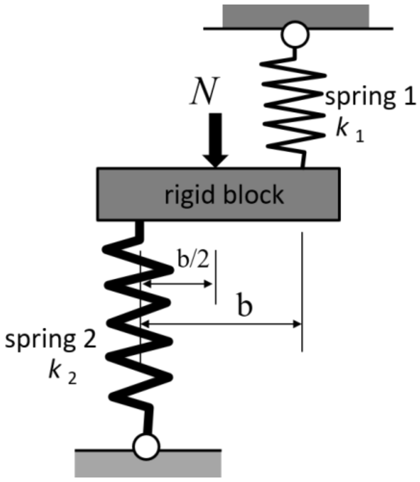

Figure 3

(b). **Figure 4** shows a cantilever beam with length *L* subjected to a uniformly distributed load of intensity *q* and a vertical point load *Q* at point C which is located at a distance of 2/3 *L* from the fixed support A. One linear elastic bar with axial rigidity *EA* and length *0.25L* is attached to the free end B. The following expression for lateral displacement field *v* has been suggested:

$$v = \Delta\left(1 - cos\frac{\pi x}{2L}\right)$$

where Δ is a constant. Show that the above displacement field is kinematically admissible. Determine the displacement at point C by applying the principle of minimum potential energy. Flexural rigidity of the column is *EI*.

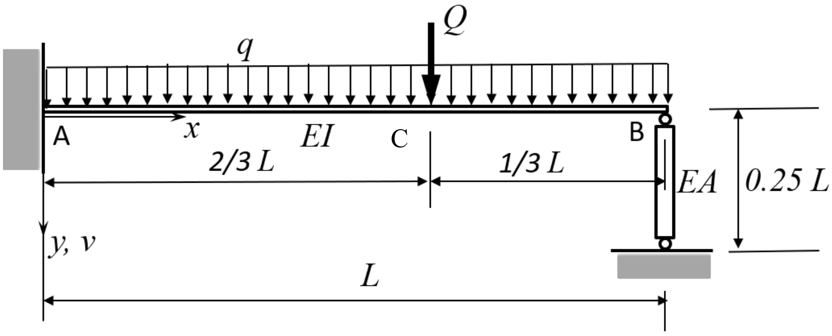

Figure 4

(18 marks)

(3). (a). Write the element stiffness matrices and global matrix for the three bars assembly which is loaded with force P, and constrained at the two ends in terms of E, A and L as shown in **Figure 5**.

[5 marks]

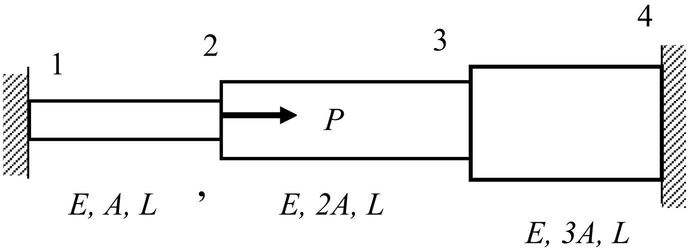

Figure 5

(b). Write the element stiffness matrices and global matrix for the two bars assembly which is loaded with force 10P at node 2 as shown in **Figure 6**. End bars are constrained at end B and free at end A with a gap of Δ at end A. Given the value of P = 60 kN, E = 20 kN/mm^2, L = 200 mm, A= 250 mm^2 and Δ = 1.2 mm, determine:

(i). the displacements at nodes 1, 2 and 3

(ii). the support reaction force at A

[20 marks]

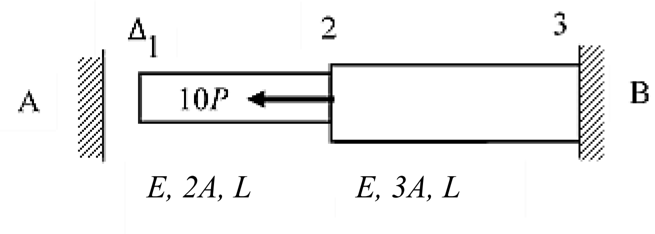

Figure 6

(4). (a). Two plates shown in **Figure 7** and **Figure 8**, shall be analysed as a plane strain problem. Both plates are divided into 9 elements. Each node has been labelled accordingly. Calculate the bandwidth, B = (R+1) NDOF for both plates assuming two degrees of freedom at each node.

[5 marks]

(b). By rearranging the node labeling. Determine the minimum value of R.

[5 marks]

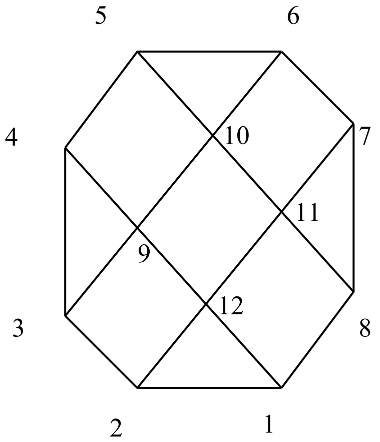

Figure 7

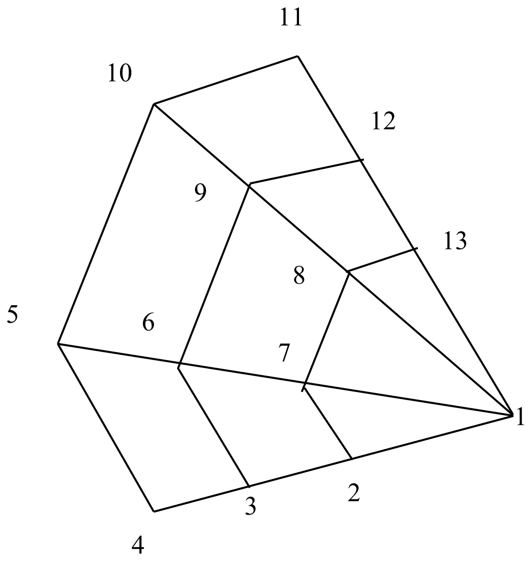

Figure 8

(c). **Figure 9** shows a frame structures labeled as nodes 1, 2 and 3 are subjected to a nodal force of P = 20 kN at node 2 and uniformly distributed load of 6 kN/m. The frame is fixed at node 1 and 3. Given the value of E = 207 GPa, I = 3×10^{-5} m^4 and A = $0.005 m^2$.

(i). Derive the global stiffness matrix for the frame

(ii). Determine the deflection u_2, v_2, θ_2 and u_3, v_3, θ_3 in unit metre and rad, respectively

[15 marks]

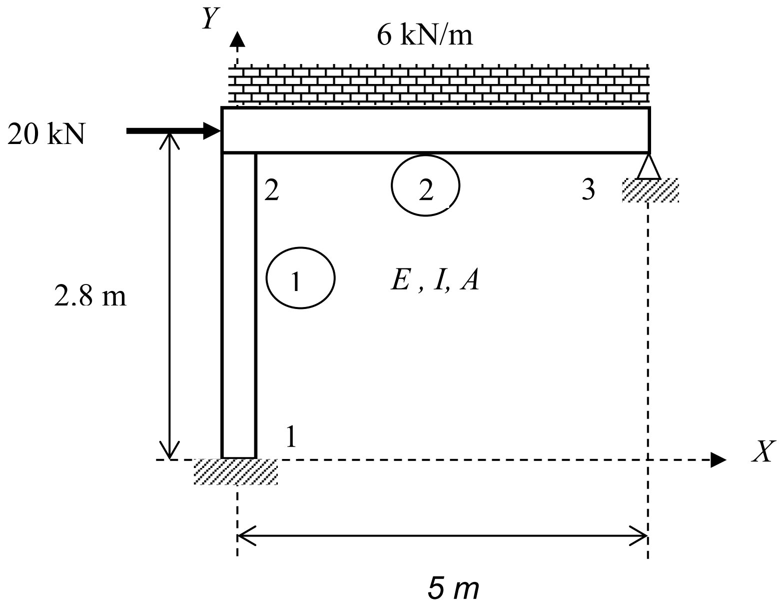

Figure 9

Given the stiffness of the beam and spring elements in dimensional space:

$$k = \frac{EI}{L^3}\begin{array}{c}\begin{array}{cccc} v_i & \theta_i & v_j & \theta_j \end{array}\\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\end{array} \text{ for beam element}$$

$$k = \begin{array}{c}\begin{array}{cc} u_i & u_j \end{array}\\ \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\end{array} \text{ for spring element}$$

-oooOooo-