RASPBERRY PI-BASED FINGER VEIN RECOGNITION SYSTEM USING PCANet

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By

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LIST OF ABBREVIATIONS

2DPCA	Two-Dimension Principal Component Analysis
2D ² PCA	Two-Directional 2-Dimensional Principal Component Analysis
FV	Finger Vein
FVRS	Finger Vein Recognition System
GNN	General Nearest Neighbors
GUI	Graphical User Interface
IAD	Image Acquisition Device
IR	Infrared
kGNN	k-General Nearest Neighbors
KMMC	K Maximum Margin Criterion
kNN	k-Nearest Neighbors
LBP	Local Binary Pattern
LDP	Local Derivative Pattern
LQP	Local Quinary Pattern
MRO	Mutual Relationship Observation
MSE	Mean Square Error
NIR	Near Infrared
NoIR	No Infrared
PCA	Principal Component Analysis
PCANet	Principal Component Analysis Network
ROI	Region of Interest
RPI	Raspberry pi

- SVM Support Vector Machine
- SWF Similar-Weighted Function

LIST OF SYMBOLS

Λ	Or function
v	And function
E	Element in the set
¢	Not element in the set
Π	Intersection between sets
В	Number of histogram blocks
<i>k</i> ₁	Patch height
<i>k</i> ₂	Patch width
L _i	Number of filters in layer <i>i</i>
m	Histogram height
n	Histogram width
$N^{\delta_x}(x)$	Nearest neighbour of y_i
$N^{\delta_{y_i}}(y_i)$	Nearest neighbour of x
p	Image height
q	Image width
$ra_{x,y}$	Ranking of $d(y, x_i)$ among all x's neighbor
$ra_{y,x}$	Ranking of $d(y, x_i)$ among all y's neighbor
x _{i,mn}	Overlapping patch
$\bar{x}_{i,j}$	Patch means

SISTEM PENGECAMAN URAT JARI BERASASKAN RASPBERRY PI DENGAN MENGGUNAKAN PCANet

ABSTRAK

Sistem Pengecaman Urat Jari (FVRS) merupakan salah satu teknologi biometrik yang dapat mengecam identiti individu berdasarkan corak urat yang unik. Berbanding dengan biometrik lain, ia merupakan cara yang lebih selamat, anti-pemalsuan dan bersih. Oleh itu, ia berjaya digunakan dalam banyak sistem pengesahan masa kini. FVRS asal hanya menyediakan fungsi pengesahan dan bukannya pengenalpastian. Bagi pengenalpastian, proses pemprosesan imej melibatkan process pemprosesan imej, pengekstrakan ciri-ciri dan klasifikasi. Projek ini menggunakan proses pra pemprosesan seperti pengesanan kelebihan, pembetulan orientasi dan pengekstrakan kaisan kepentingan (ROI) yang telah dibangunkan sebelum ini. Objektif utama dalam projek ini adalah melaksanakan teknik pengekstrakan ciri-ciri yang dapat memaksimumkan prestasi FVRS. Rangkaian pembelajaran yang mendalam, iaitu PCANet telah diperkenalkan. PCANet menpunyai tiga komponen pemprosesan asas data, iaitu penapis PCA, penyusunan binari bersepah dan histogram. PCA digunakan bagi pembelajaran bank penapis dari pelbagai lapisan. Penyusunan binari bersepah dan blok histogram merupakan langkah untuk pengindeksan dan penyatuan. Perbandingan antara PCANet dan PCA menunjukkan bahawa PCANet mendapat prestasi yang baik dalam contoh-contoh latihan yang minima, dengan peningkatan sebanyak 21.3% daripada PCA. Faktor-faktor yang mempunyai kesan terhadap prestasi PCANet telah dikaji untuk mengenal pasti batasan PCANet. Bagi klasifikasi, algoritma k- pendekatan jiran (kNN) dengan jarak Euclidean telah digunakan. Algoritma penambahbaikan kNN, iaitu k-pendekatan jiran umum (kGNN) pernah diperkenalkan pada permulaan. Akan tetapi, perbandingan prestasi antara kNN, kGNN dan SVM menunjukkan bahawa kNN lebih sesuai dalam FVRS. Peringkat terakhir dalam projek ini adalah mengabungkan kerja-kerja yang siap sebelum ini supaya dapat digunakan dalam keadaan sebenar. Program ini telah dimuatkan ke Raspberry Pi dengan menggunakan bahasa perisian C++ dan OpenCV. Penilaian prestasi menunjukkan bahawa ketepatan pengenalan FVRS mencapai 92.67%. PCANet telah menunjukkan potensi sebagai garis panduan yang mudah tetapi kompetitif dalam pengecaman urat jari.

RASPBERRY PI-BASED FINGER VEIN RECOGNITION SYSTEM USING PCANet

ABSTRACT

Finger Vein Recognition System (FVRS) is a biometric technology that identifies or verifies an individual identity based on unique vein patterns. Compared with other biometrics, it is more secure, anti-forgery and hygiene. Thus, it successfully utilized in many authentications nowadays. The original FVRS developed only provides verification instead of identification. For identification, the image processing involves process of image pre-processing, feature extraction and classification. The project utilised preprocessing process such as edge detection, orientation correction and Region of Interest (ROI) extraction that have been developed previously. The main objective in this project is to implement a suitable feature extraction technique that can maximize the FVRS performance. A simple deep learning network, namely Principal Component Analysis Network (PCANet) is thus proposed. It composed of three basic data processing components, which are PCA filter, binary hashing and histograms. PCA is employed for learning multistage filter banks. Binary hashing and block histograms are the steps for indexing and pooling. A comparison between PCANet and PCA shows that PCANet is outperform under limited training samples, with an increase of 21.3% than that of PCA. Factors which impact PCANet are studied to identify the limitations of PCANet. For classification, k-Nearest Neighbours (kNN) with Euclidean distance algorithm is implemented. An enhancement version for kNN algorithm, k-General Nearest Neighbours (kGNN) have been proposed at initial stage. However, performance comparison between kNN, kGNN and SVM shows that kNN is more suitable for FVRS implementation. The last stage for this project is to combine previous work done into an embedded system which can be implemented in real finger vein authentication. The program is uploaded in the Raspberry Pi by using C++ language and OpenCV image processing library. The performance evaluation shows that the recognition rate of FVRS achieved 92.67% . Concluded that PCANet serve as a simple but highly competitive baseline in finger vein recognition.

CHAPTER 1

Introduction

1.1 Research Background

Biometric refers details information about someone's body based on the measurement of human physical traits and behaviour traits. Physical traits are fingerprints, palmprints, retina and facial characteristics while voice and gait are example of behaviour traits. Biological traits are used in determining one's identity in most of the contemporary computer science applications nowadays (Kalyani 2017). Numerous advantages of biometric authentication make it take over traditional password or ID card-based authentication.

Traditional authentication is vulnerable to the risk of being exposure or forgotten. Biometric technologies have been extensively researched and developed to meet with the high demand in information security (Parthiban et al. 2014). Biometric traits have less hacking cases as it is more difficult to forge. Finger print and palm print are used in identification and verification over years. Despite being advantageous and easy authentication, it still poses the risk of forgery due to high exposure rate. Unclear fingerprint images due to sweating and injuring degrades system's performance (Sapkale et al. 2016). On the other hand, face recognition becomes tough due to its occurrence such as face-lifts, faces wearing glasses, cap and make up.

Finger Vein Recognition System (FVRS) has successfully attracted the attention of researchers in response to the drawback of several biometrics. Veins are usually the blood vessels which carry blood towards the heart. As like fingerprint and iris patterns, finger vein based blood vessel patterns are unique for each individual (Anand et al. 2013). Vein are located underneath, making it impossible to forge. Thus, it has higher security than other biometrics. Finger vein structure is not easily view in visible light. A special device composed of Near InfraRed Light Emitting Diodes (LED) is utilised for capturing finger vein image (Raghavendra et al. 2015). No direct contact required in this case, making it more hygiene. The outperforms of FVRS enhance the security system to the next level. However, FVRS also affected by many factors such as environmental illumination, ambient temperature, physiological changes and user behaviour. If these factors are not

well established, the performance of FVRS will eventually being affected. Hence, the quality of finger vein image needs to be measure before proceeding to matching (Prasad et al. 2014).

The process of FVRS is composed of four stages where the first stage is image acquisition followed by pre-processing, feature extraction and classification. Robustness in each stage contributes the development of high-quality FVRS. A brief explanation on each stage is shown in Figure 1.1 with descriptions.



Figure 1.1: Overview of FVRS

- Image acquisition: A process in obtaining input image. For obtaining finger vein image, a device that does not affected by the ambient temperature is needed (Prasad et al. 2014). Light absorption in near infrared (NIR) and infrared (IR) wavelengths by different human tissues produce the unique vein image.
- Pre-processing: A process to improve image data by suppressing unwilling distortions or enhancing important features for further processing (Anand et al. 2013). It includes edge detection, orientation correction and ROI extraction.
- iii. Feature extraction: The process of transforming input data into a set of features with relevant information for matching and verification. A core step in determining the accuracy of the system as well as the processing time consumed.
- iv. Classification: A process used to assign corresponding levels with respect to groups with homogeneous characteristics, with the aim of discriminating multiple objects from each other (Anand et al. 2013). Desired result will be obtained if the methods of previous steps are working properly.

Biometric authentication is either verification or identification system. Verification is described as a one-to-one matching system where the system tries to match the biometric presented by the individual against a specific biometric which already exist on file. On the other hand, identification described as one-to-n matching, where n is the total of biometrics in the database. Identification seeks to identify an unknown person or biometric. Most of the authentication systems preferred identification due to its robustness. The final product is expected to be a standalone system which replaced current fingerprint authentication in a more secure and hygiene environment. Thus, a fast and highly accurate automated electronic imaging system comes along with good algorithms in image processing is required.

1.2 Problem Statement

For a finger vein recognition system (FVRS) to be able identify the individual's identity, it is essential to mention about feature extraction and classification phases in image processing. The performance of these two stages have an observable impact on the efficiency of recognition system. Informative features of finger vein (FV) are retrieved from the process of feature extraction where the features are classified based on labelling. Correct labelling indicates the successful of FV recognition.

A suitable feature extraction technique based on the applied input needs to be done with utmost care. Intra-class variability, arising from different lighting conditions, misalignment, non-rigid deformations, occlusion and corruptions have increased the challenge in image classification (Kumar et al. 2014). Various feature extraction techniques have been applied in Finger Vein Recognition System (FVRS), but limitations are still present. Most of existing techniques facing difficulty in thin veins extraction (Kauba et al. 2015), noise blocking (Kauba et al. 2015), inconsistent performance (Tagkalakis et al. 2015), prefixed architecture (Chan et al. 2015) and etc. Hence, returned an unsatisfied result in FVRS.

Besides that, most hand-crafted features cannot simply be adapted to new conditions. New domain knowledge such as learning through deep neural network has been introduced in handling this issue (Wu et al. 2011, Chan et al. 2015, He et al. 2017). It uses convolution architectures which consist of multiple trainable stages stacked on top of each other followed by a supervised classifier. Machine learning takes place to learn filter bank in each stage, but this learning critically depends on expertise in a parameter tuning and various ad hoc tricks. Moreover, deep learning might sensitive to partial occlusion and pose changes (Zhong et al. 2016). Deep learning architecture such as Scatnet does not generalize well when there is a significant illumination changes and corruption (Chan et al. 2015).

Therefore, (Chan et al. 2015) has proposed a simple deep learning baseline for image features extraction. It is named as Principal Component Analysis Network (PCANet) where only three basic data processing components required in the process. It applied the concept of PCA in filtering stage, followed by hashing and histogram. The superior performance of PCANet are used in many recognition systems such as face recognition (Chan et al. 2015). Meanwhile, this technique still not utilised in existing FVRS. The robustness of PCANet technique might replace other feature extraction techniques in improving the performance of FVRS.

The proceeding step after feature extraction is classification. Identification for FVRS is dependent on classifier decision. The original FVRS implemented Support Vector Machine (SVM) method for verification stage (Lim 2017). The drawback of SVM is its time complexity. A better classifier is looking for in FVRS application as robust system is favourable.

Another classifier frequently used is K-Nearest Neighbors (kNN). The performance of kNN has proven having an optimal error rate in Bayes method under certain circumstances (Pan et al. 2017) . Meanwhile, kNN suffers a negative outliers effect as well as accuracy reduction for a small training dataset (Khaniabadi et al. 2014, Pan et al. 2017). The neighbourhood size, k has direct impact to result obtained, optimum k value needs to be determined wisely (Wang et al. 2006, Pan et al. 2017). kNN applies the concept of simple majority vote, ignoring the differences of training samples' qualities and non-equal nearest neighbours which has an impact on classification decision. None of the improvement kNN algorithms able to achieve a balance treatment of all samples.

(Pan et al. 2017) proposed General Nearest Neighbour (GNN) that can overcome the drawbacks of kNN. GNN considered mutual neighbourhood information of both test sample and training samples in classification. It is believed it had better performance than kNN. However, the implementation of kGNN on FVRS has not yet been introduced.

1.3 Research Objectives

The objectives of this project are:

- 1. To enhance the recognition rate of FVRS with the implementation of feature extraction technique.
- 2. To speed up the processing time of FVRS.
- 3. To combine previous work done in an embedded system which can be implemented in real finger vein authentication.

1.4 Scopes of project

The task of this project is to develop FVRS system which able perform recognition based on the individual's finger vein patterns. This is the last stage in FVRS development which is identification. The project targets to improve the recognition rate of FVRS as well as shorten up its processing time with the implementation of feature extraction and classification.

The project is mainly on software development which continuing the original FVRS project. Since image acquisition and image pre-processing have been developed (Lim 2017), the project begins with feature extraction process and end with classification. FV images are taken from people with different ages. For each FV, 6 bmp images with 8-bit depth are captured, there are total 50 datasets collected in the project. The purpose is to examine the performance of FVRS under limited sample size and the recognizing ability when applies to all ages.

ROI images obtained from pre-processing are the input for feature extraction process. Important features of images are extracted before classification. The classification process is then distributing the features into several classes based on the similarities between them. Several steps are involved in determining the proposed algorithms suitable to implement in FVRS. If the proposed algorithm is ineffective to be utilised during progression, a better algorithm is then proposed and implemented in the system.

The program is written in Microsoft Visual Studio with C++ based language and OpenCV library. All parts are then combined for a completing embedded FVRS. For real implementation, Graphic User Interface (GUI) is designed in Raspberry Pi through Qt Creator.

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1.5 Thesis Outline

This thesis is organized as follows:

- a) Chapter 2 will summarize the research information based on the previous work of others on image processing of different fields. It explains new techniques used in image processing section particularly on feature extraction and classification. Algorithms relevant with these techniques will be discussed. Differences among algorithms are mentioned in this chapter.
- b) Chapter 3 will be on discussion of the methods to be applied on this project. The flow charts, block diagrams and operational methods will be explained in this chapter. The working principles of algorithms will describe under this chapter.
- c) Chapter 4 focuses on the analysis of the results of the implementation approach. The discussion will be made based on the recognition rate and robustness of the system
- d) Chapter 5 concludes the project starting from planning stage until implementation stage. The limitations found on this project will also be concluded in this chapter. Future works which might needed for enhancing the project will be suggested under this chapter.

CHAPTER 2

Literature Review

2.1 Introduction

This chapter is about the study of Finger Vein Recognition System (FVRS) and research done previously about image processing techniques in different fields. FVRS system is programmed on microcomputer board along with image acquisition device (IAD) integration. The microcomputer used in this project is Raspberry Pi (RPI). IAD is used for capturing finger vein images with the assist of Near Infrared (NIR) laser line modules as light source. IR sensitive camera, RPI No Infrared (NoIR) camera is employed in the device. The image captured will undergo series of image processing stages.

The development of FVRS normally compose of four stages: image acquisition, preprocessing, feature extraction and classification. The project emphasizes on software development of feature extraction and classification processes. The following section will introduce about FVRS as well as discussing on its advantages over other biometric methods. Various techniques applied on feature extraction and classification previously proposed in FVRS will be discussed in this chapter. The discussion does not focus particularly on FVRS, but also on related techniques which are valuable that might not available in the existing FVRS.

2.2 Finger Vein Recognition System

Veins are blood vessels which carry blood towards the heart (Sapkale et al. 2016). Medical research shows that finger veins of an individual person are varies among different fingers. Its patterns will not vary over the time (Mulyono et al. 2008, Yanagawa et al. 2016). By referring Figure 2.1, finger vein locates beneath the skin's surface. It cannot be viewed under visible light, thus has low exposure rate.



Figure 2.1: Finger Vein of an individual (Lu Yang 2013)

Embedded FVRS is a biometric authentication system which specifies an individual based on finger vein patterns. It consists of three modules: IAD, image processing and identification module. As FV is invisible under visible light, NIR light needed to be setup for image capturing (Chen et al. 2014). IR light sensitive camera captures the image and pass to image processing module. Image pre-processing is carried out to filter unnecessary noise and extracting a good ROI. The next step is extracting important information from the veins known as features. Extracted features are then undergo classification process in identification module where the features class will be figured out. The identity of the person is determined by searching the class of features that has nearest similarity with the test features.

Comparison among various biometric methods based on basic parameters such as anti-forgery, accuracy, speed, resistance and cost are carried out shown in Table 2.1 (Chen et al. 2014). From the table, FVRS obtained the best result in term of security and robustness. As infrared light is penetrating through finger, the FV image captured will face an overexposure or underexposure situation. Meanwhile, it emerges with the balance biometric identification advantages. FVRS obtained a minimum of average grading for its resistance in direct application and cost. FVRS's popularity over others biometric authentication can be generally described as below:

- a) Uniqueness: Finger veins are distinct and unique for each person. Even an individual has distinct veins patterns among each finger (Mulyono et al. 2008, Yanagawa et al. 2016).
- b) Anti-forgery: Finger veins are hidden inside the body and invisible to human naked eyes under visible lighting. Making it impossible to be forge or steal (Sapkale et al. 2016).

- c) **Low exposure rate**: FVRS is a contactless system where veins are untraceable during the authentication process (Sapkale et al. 2016).
- d) Active liveness: Veins are viewable with reflected light due to the peak absorption of near infrared illumination by oxygenated and de-oxygenated haemoglobin in blood (Yang et al. 2013). Only living person able to perform this task. Hence, finger veins can only be taken from a living person (Xie et al. 2015).
- e) Long lasting: FVRS can takes low resolution images, re-enrolment is not needed.
 For adults, finger vein pattern does not vary over the time (Yang et al. 2013, Yanagawa et al. 2016).

Biometrics	Secu	urity	Convenience					
	Anti-	Accuracy	Speed	Speed Resistance Co				
	forgery							
Fingerprint	*	•		*	\checkmark			
Iris	•	~		*	*			
Face	•	*		 	*			
Signature		*		\checkmark				
Vein	\checkmark	\checkmark	\checkmark					
🗶 Insuffici	ent	Norm	al	✓Go	od			

Table 2.1: Comparison of various biometric technologies (Lixiao 2014)

FVRS shows a remarkable development in recent years, after receiving the attention from researches due to its benefits (Sapkale et al. 2016). It is extensively utilised in confidential sectors, automated teller machine, banking sectors, military zone and even hospitality. FVRS prevents unauthorized access as well as protecting individual personal information to the next level (Sugandhi et al. 2014).

2.3 Previously Proposed Feature Extraction Methods for Finger Vein Recognition

In this section, ROI extracted from original image works on feature extraction are reported and discussed. The primary task of FVRS is to assign the input FV image into one of the possible output classes correctly. The process can be divided into two parts which are feature extraction and classification (Kumar et al. 2014). Feature extraction techniques are necessary in image processing to extract a set of features, which maximizes the recognition rate with the least number of elements and to generate a similar feature set for variety of instances of the same symbol (Kumar et al. 2014). Redundant data are eliminated, which minimize the overall data size required for training, increasing efficiency in classification and reducing time consumption. Poor selected features influence the classifier unable to recognise input image model. Selection of good feature extraction technique is critical.

Various types of feature extraction methods have been utilised in finger vein recognition such as Gabor filtering method (Ezhilmaran et al. 2015, Sapkale et al. 2016), repeated line tracking (Ezhilmaran et al. 2015, Kauba et al. 2015), maximum curvature method (Tagkalakis et al. 2015), local binary pattern method (Liu et al. 2016), and dimensionality reduction method (Blahuta et al. 2011, Ezhilmaran et al. 2015).

2.3.1 Gabor Filtering Method

Finger vein extraction always poses a great sensitivity in noise issued by the lowcontrast FV images captured. Gabor filter, linear filter which is powerful in image texture analysis is used to overcome the problem. Gabor filter can be imagined as a sinusoidal plane in certain frequency and orientation which is modulated by a Gaussian envelope (Ezhilmaran et al. 2015).

(Sapkale et al. 2016) proposed a novel method of finger vein enhancement with the implementation of multi-channel Gabor filters. Vein vessel information with variances widths and orientations in images are protruded using Gabor filters. The vein information is represented in different scales and orientations of Gabor filters. All FV information is concatenated to generate an enhanced finger-vein based on reconstruction rule (Ezhilmaran et al. 2015, Sapkale et al. 2016).

The advantages of Gabor filters for finger vein extraction are contributed by its capability of detecting oriented features, directional and frequencies selectivity (Ezhilmaran et al. 2015). However, the widths of veins to be extracted from this conventional are predefined which degrades the accuracy in authentication.

2.3.2 Repeated Line Tracking

Repeated line tracking is a way of tracing line along the vein patterns which starts at various positions (Ezhilmaran et al. 2015, Kauba et al. 2015). In the process, local dark lines are identified, and line tracking is executed by moving along the lines from one pixel to another. The algorithm traced the direction of the vein patterns represented in line based on a cross sectional profile of image in a given direction. The cross section displayed a valley when the point is inside a vein. The step is repeated until the vein pattern can be extracted. The selected pixel (tracking point) increased its value in a corresponding pixel map. The created map at the end have very high intensity boundaries.

Noise may be tracked in this technique, but it does not have much influence because the dark lines of finger vein are more emphasized. Reduction the number of tracking operations and pattern are concerned in minimizing the computational costs (Kauba et al. 2015). The drawback of this method is insufficient in extracting thin veins which proposed in (Tagkalakis et al. 2015). The number of times for the tracking point move across thin veins is probably small.

2.3.3 Maximum Curvature Method

(Tagkalakis et al. 2015) concerned about the failure of repeated line tracking method in thin vein extraction. New proposed solution in tackling previous problem is introduced, namely maximum curvature method. In this method, finger vein appears like a dent with high curvature in the cross-sectional profile. The curvatures of the image profiles are checked and only focus on the centrelines of veins. The centrelines are defined as positions where the curvatures of a cross-sectional profile of a vein image which are locally maxima.

Figure 2.2 illustrated an example of the cross-sectional profile of vein. From the figure, the cross-sectional profile of vein looks like a dent as the vein is darker in comparison with the surrounding area. Figure 2.2 also shown the curvature profile of veins is large without influenced by temporal fluctuation in vein width and brightness. The average mismatched ratio of this method is only 2.83% which is smaller than that of line tracking method (4.63%). On the other hand, the good performance of maximum curvature method is under the assumption that the blood network is properly segmented.



Figure 2.2: Cross-sectional profile of vein (Naoto Miura 2005)

2.3.4 Local Binary Pattern Method

Local binary pattern (LBP) is a texture operator which labelled the pixels of an image by thresholding the neighbourhood of each pixel (Ezhilmaran et al. 2015) and show as binary number. Most of the FVRS used features from a segmented blood vessel network. Finger vein images are not guarantee clear and unable to display irregular shadings and highly saturated region. Optical blurring and scattering problems caused low quality of FV image obtained. Therefore, detection errors might happen in extracting vein patterns which degrading the recognition accuracy (Liu et al. 2016).

Noticed about the performance of recognition is based on detected FV regions, (Liu et al. 2016) proposed a finger vein extraction method using LBP. A unique finger vein code is extracted after aligning the finger vein image based on minutia points. LBP method extract FV codes in the whole finger region and ignoring the accurate detection of vein region. LBP method compared the grey intensity of centre pixel to those of neighbouring pixels in local regions. Therefore, obtaining appropriate irregular shadings and highly saturated region. The false rejection error in the proposed finger vein extraction reduced tremendously.

With its discriminative power and simple computation, LBP texture operator has been widely used in other FVRS developments (Ezhilmaran et al. 2015). Numerous of LBP variants have been introduced in the developments such as Local Line Binary Pattern (LLBP), Local Quinary Pattern (LQP) and Local Derivative Pattern (LDP) (Meng et al. 2012).

Though local patterns provide high accuracy in FVRS, it is essentially a kind of network which is hard to extract. Finger veins contain rich directional information in the network which most of local pattern-based methods does not fully used the hidden directional information in the FV images (Meng et al. 2012).

2.3.5 Dimensionality Reduction Method

A dimensionality reduction method is a technique used for reducing data in highdimensional space. Principal component analysis performs a linear mapping of the data in reducing dimensions. The variance of the data in the low-dimensional representation is maximized. PCA-based techniques introduced in finger vein recognition such as PCA (Blahuta et al. 2011), 2D-PCA (Tateyama et al. 2009)and 2D²-PCA are discussed in the following.

2.3.5.1 <u>PCA</u>

Principal Component Analysis (PCA) or Karhunen-Love transform is a powerful feature representation method in pattern recognition based on statistics and matrix algebra. It is an effective method suitable for simplifying data analysis. In general, it is a transformation of correlated data into uncorrelated data (Blahuta et al. 2011). The observations from PCA analysis are interpreted in term of inter-correlated quantitative dependent variables. The information retrieved is a set of new orthogonal variables called principal components which are eigenvectors of matrix. The mathematical background of PCA is described as follow (Blahuta et al. 2011):

- a) Transform image (matrix) into vector and compute the mean
- b) Compute covariances among vectors and construct covariance matrix
- c) Obtain eigenvalues & eigenvector (eigenspace) from the covariant matrix
- d) Assess an optimal threshold T for choosing the K largest determining components

Meanwhile, the detail description of PCA is more complicated than theoretical concept (Blahuta et al. 2011). (Prasad et al. 2008) have discussed on the limitations of PCA for hyperspectral target recognition. Implementation of PCA on 2D data is not direct. The 2D data needs to unfold into 1D vector as PCA can only proceed with 1D vector data

(Prasad et al. 2008, Rajendran et al. 2014). The dimension of the covariance matrix is huge as unfolded vectors are generally long, making difficulties in calculation. When the training samples are small, the calculated bases cannot represent untrained data accurately.

(Prasad et al. 2008) proposed that PCA is not an optimal projection for a pattern classification aspect such as FV training. The class separation poses possibility of deterioration after PCA transformation. He proposed of using LDA-based transformation for solving small sample size problem instead of PCA. Experimental evidence provided in the paper issues PCA's problem in many applications. Table 2.2 depicts the overall accuracy for three data sets with different techniques. Two mixing ratios (background: target percentage) are used in corrupting target images. Based on the result, PCA technique has lower accuracy compared to LDA, proven his proposed statement is valid.

PCA technique has been applied in embedded FVRS with BP neural network classifier (He et al. 2017). PCA has reduced the large dimensions of finger vein images. Thus, reducing the computation burden and removing noise in discarded dimensions. Meanwhile, tremendous time is consumed in BP NN training. The average accuracy of 81% obtained shown that recognition technique applied is not suitable. Enhancement work such as simplification feature representations needs to carry out to obtained desired performance.

Table 2.2: Overall recognition accuracy for three different data sets at two mixing ratios. Results are reported in % (DF: Multi-classifier Decision Fusion System, SCL: Single-Classifier System, SLDA: Subspace LDA, BA: Band Averaging) (Bruce 2008)

	Mixing Ratio (20:80)						Mixing Ratio (40:60)					
	DSI-P1	DSI-P2	DS2-P1	DS2-P2	DS3-P1	DS3-P3	DSI-P1	DSI-P2	DS2-P1	DS2-P2	DS3-P1	DS3-P3
DF-PCA	76 (4)	83 (3)	72 (4)	72 (4)	73 (4)	72 (4)	76 (4)	83 (3)	71 (4)	71 (4)	71 (4)	71 (4)
DF-	65 (4)	68 (4)	72 (4)	72 (4)	95 (2)	96 (2)	66 (4)	66 (4)	69 (4)	68 (4)	89 (3)	91 (2)
SLDA												
DF-LDA	100	0(0)	100	(0)	100	0 (0)	92	(2)	100	0(0)	96	(2)
SCL-PCA	69	(4)	73	(4)	72	(4)	68	(4)	70	(4)	68	(4)
SCL- SLDA	68	(4)	78	(3)	72	(4)	64	(4)	73	(4)	66	(4)
SCL-BA 62 (4)		97	(1)	97 (1)		59 (4)		95 (2)		86	(3)	

2.3.5.2 <u>2D-PCA</u>

Two-Dimensional Principal Component Analysis (2D-PCA) is a next version of classical PCA based on 2D matrices instead of 1D vectors (Tateyama et al. 2009). With 2D-PCA, original images can be directly transform to covariance matrix. The covariance matrix of 2D-PCA result smaller in size compared to the PCA (Rajendran et al. 2014). This is one of the advantages of 2D-PCA. The other benefits of 2D-PCA are in term of accurate projection directions, less time consuming in eigenvectors calculation. 2D-PCA can achieve better performance than PCA in small training samples (Rajendran et al. 2014).

(You et al. 2015) developed finger vein recognition based 2D-PCA and KMMC. K maximum margin criterion (KMMC) is used to extract nonlinear features of finger vein images. The nonlinear characteristics is due to external factors, i.e. light, temperature, humidity and horizontal misalignment. Even though KMMC returns a better recognition effect but at the same time it has increase complexity in computation and time for training sample. Hence, before taking nonlinear mapping to image data, 2D-PCA algorithm is first applied to minimize image dimension.

Figure 2.3 depicts the proposed feature extraction methods using 2D-PCa followed by KMMC. Suppose *X* is one of the image of training set, 2D-PCA is applied on X in horizontal direction. After 2D-PCA transformation, the classification information, Y is compressed into small number of columns. KMMC algorithm further compressed Y in vertical direction into small number of rows. At the end, the image information is placed to the upper left corner of the image.



Figure 2.3: 2D-PCA followed by KMMC feature extraction method (Lin You 2015)

These combinations successfully reduced training time required while maintaining the recognition rate. The effect on training time with proposed method shown an obvious result when the number of training samples increased.

2.3.5.3 <u>2D²PCA</u>

However, 2D-PCA exists an unresolved problem which is more coefficients are needed to represent an image in contrast with PCA (Zhang et al. 2005). Besides that, 2D-PCA only work for row directions of images (Yang et al. 2012). In handling this issue, (Zhang et al. 2005) proposed another technique called Two-Directional Two-dimensional PCA (2D²-PCA) which captures image information from row and column directions simultaneously. the main advantage of $2D^2$ -PCA over 2D-PCA lies in that the number of coefficients needed in features representation and recognition is much smaller in latter.

The concept of $2D^2$ -PCA working with two directions simultaneously is defined in the following (Zhang et al. 2005):

- (i) Step 1: Working in row direction2D-PCA learns optimal matrix, X from training images where X reflecting information between rows of images.
- (ii) Step 2: Working in column directionThis method is named as alternative 2D-PCA. It learns optimal matrix, Z which reflecting information between columns of images.
- (iii) Step 3: Projecting image matrix A onto X and Z simultaneously

Once obtaining both optimal matrices of X and Z, projecting $m \times n$ size of image A onto X and Z simultaneously to obtain $q \times d$ matrix C. Equation (2.1) shows the formula for obtaining matrix C. Matrix C is named s feature matrix in face recognition.

$$C = Z^T A X \tag{2.1}$$

where Z^T denotes transpose of optimal matrix in column direction, A denotes image matrix and X denotes optimal matrix in row direction.

By referring Equation (2.1), *C* obtained more image information than features obtained using 2D-PCA and alternative 2D-PCA. $2D^2$ -PCA consumed less time than both methods in image processing as the dimension of *C* is small (Yang et al. 2012).

(Yang et al. 2012) noticed about the challenges in finger vein recognition. The challenges are extracting distinguishing features and strong classifier with high accuracy and short recognition time required. For the first task, (Yang et al. 2012) has applied $2D^2$ -PCA feature extraction technique on FVRS. To overcome 2^{nd} problem, classifier based on metric learning is proposed in classification stage. $2D^2$ -PCA extracts informative features in both row and column directions then train a binary classifier based on metric learning. The class imbalance issue is tackled by SMOTE technology by oversampling before classifier is trained. The proposed method has retrieved a very high performance in FVRS with a recognition rate of 99.17%.

The drawback of PCA-based approaches is that unable to distinguish the different roles of variation (Liu 2011). The classification task is challenges with the present of this issue.

2.4 Previously Proposed Classification Methods for Finger Vein Recognition

In this section, the results of previous works on classification to identify FV's identity are reported and discussed. Classification is a process categorise an image based on features of the image. Image classification based on visual content is a challenging task in image processing due to a large amount of intra-class variability (Chan et al. 2015). Numerous classification techniques are utilised in finger vein identification such as Support Vector Machine (SVM) and Nearest Neighbour (NN).

2.4.1 SVM

SVM is a set of supervised learning techniques which categorized as a hyperplane classifier. It has been implemented by (Wu et al. 2011) in vein identification. The hyperplane refers to the decision surface that separates positive training samples from the negatives with largest margin in training process. It measures the maximum distance to the closest point of the training point (support vectors) by applying the concept of structural risk reduction. Figure 2.4 depicted the idea of SVM classifier. The capability of SVM in handling nonlinearly separable data has attracted many researchers utilised it in many applications. (Wu et al. 2011) has conducted experiment on SVM classifier with ANFIS classifier as a comparator. The output from the experiment indicated that SVM is outperform than ANFIS in term of time consumption with equally high identification rate.

Despite of its advantages, SVM is sensitive to noise as reported in (Syazana-Itqan et al. 2016). The mislabelled with SVM degraded the performance of FVRS. Besides that, SVM lack of transparency in result representation. Graphical representation is not an effective method for SVM due to its huge size, whereas no mathematic representation calculating the effectiveness of SVM.



Figure 2.4: A linear separable SVM (Chiung-TsiungLiu 2011)

2.4.2 kNN

k-Nearest Neighbour (kNN) is a non-parametric rule implemented effectively in pattern classification. kNN decision rule is based on majority voting process in its knearest neighbours. The test sample is assigned to the majority class's label. The k-nearest neighbours are depending on the minimum distance between the test sample and training samples (Pan et al. 2017). Given a query point (test sample), y and a set of training samples $T = \{x_j\}_{j=1}^N$ with class labels, $c_1, c_2, ..., n$. N is total number of training samples, x_j is the training sample and n is the number of class (Jaafar et al. 2016). The procedure of kNN classifier is summarised in the following:

i. Calculate distance between query point and training sample. Euclidean distance equation is used as shown in Equation (2.2) (Jaafar et al. 2016).

$$d(y, x_i) = \sqrt{(y - x_i)^T (y - x_i)}$$
(2.2)

where $d(y, x_i)$ =Euclidean distance, y is query point and x_i is training sample.

- ii. Sort the distances calculated in ascending order.
- Number of k training samples which have short distance to the query point are selected based on predefined k-value.

 iv. Query point is classified by assigning class label which has majority vote in k nearest neighbours. Equation (2.3) is the majority voting formula in kNN decision rule, where the weight is expressed in Equation (2.4).

$$y = \arg\max\sum_{x_i^N} wc_m \tag{2.3}$$

$$w = \begin{cases} 1 & c_M = x_j^N \\ 0 & otherwise \end{cases}$$
(2.4)

where w is the weight and c_M is class label by majority voting.

kNN is applied in finger vein database by (Jaafar et al. 2016). The paper discussed on the advantages and limitations of kNN in real utilisation. The advantages of using kNN are simple and fast for small training samples. Moreover, no prior knowledge needed about the structure of training samples. The performance of kNN has been verified having an optimal error rate in Bayes method under the constraint of $k/N \rightarrow 0$ where k represents the neighbourhood size and N represents the training sample size respectively (Cover et al. 1967, Toussaint 1974, Pan et al. 2017). Meanwhile, it still presents some open issues. Firstly, all training images to be saved in memory for distance computation before proceeding to majority voting. This induces complexity in time. Secondly, kNN classification decision is based on majority vote which applying Similar-Weighted Function (SWF). It treated each neighbour has similar contribution to the test sample which is an invalid theory. Moreover, kNN unable perform optimally in limited training samples. The decision rule selects the nearest training samples within the region. Misclassifying test sample situation might occur since the nearest neighbour classes in the region is unreliable. Lastly, the equally-weight treated nearest neighbours regardless distance has increased its sensitiveness to outliers and noise contained in datasets. Thus, cause an accuracy degradation.

2.5 PCANet

After discussing different PCA-based features extraction methods in Section 2.3, PCA network (PCANet) is introduced in this section. PCANet is a simple deep learning network proposed by (Chan et al. 2015) in image classification. The working principle of PCANet is based on three basis data processing components. Three components mentioned are cascaded PCA, binary hashing and block-wise histograms. PCA is employed in multistage filter banks. Binary hashing and block histograms are the steps for indexing and pooling.

Typically, low-level features can be hand-crafted for certain tasks. However, most of the case, hand-crafted features unable adapt in new condition. Deep neural network has become the remedy for the hand-crafted features limitation. It is a process of features learning from data of interest. Abstract representations learned are expected to provide robustness to intra-class variability such as occlusion and corruption. The successful of deep learning in classification is due to the powerful of convolution architectures. One of the example is the convolutional deep neural network (ConvNet) architecture which consist of multiple trainable stages stacked on top of each other followed by supervised classifier. Each stage consists of a convolutional filter bank layer, a nonlinear processing layer and a feature pooling layer. However, the usefulness of the learning is dependent on the accuracy in parameter tuning as well as there exist various of ad hoc tricks. Another prefixed architecture, ScatNet does not generalise well for face recognition due to intravariability. Undesired performance of existing deep learning has motivated researchers on this field in tackling the issues.

The motivation of PCANet is to design an easy and trivial network for training which adapt with different data and tasks. Such simple network could serve as a good baseline in justifying the use of more sophisticated architectures in deep learning networks. The data-adapting convolution filter bank in each stage is PCA filters, the nonlinear layer is set to be the simplest binary quantization (hashing) and block-wise histograms of binary codes are implemented in feature pooling layer.

Figure 2.5 depicts the process of feature extraction from an image in two-stage PCANet. Filtering process is conducted in each stage where more informative features are extracted at the second stage. The features extracted are presented in block-wise histograms form. A clear picture of the process can be referred to Figure 2.6 which shown the block diagram for a two-stage PCA-Net.



Figure 2.5: PCANet illustration on feature extraction from an image through three signal processing components (Tsung-Han Chan 2015)



Figure 2.6: A detailed block diagram of two-stage PCANet (Tsung-Han Chan 2015)

A detailed description of each component in block diagram is explained in following (Chan et al. 2015):

 ${I_i}_N = N$ input training images of size $m \times n$

Patch size (2D filter size) = $k_1 \times k_2$

The number of filter in layer $i = L_i$

i. First stage PCA

 $k_1 \times k_2$ patch is taken around each pixel and overlapping patches of *i*-th image are collected, i.e., $x_{i,1}, x_{i,2}, x_{i,3}, \dots, x_{i,\tilde{m}|\tilde{n}|} \in \mathbb{R}^{k_1 k_2}$. where

$$\tilde{m} = m - \frac{k_1}{2}$$
 and $\tilde{n} = n - \frac{k_2}{2}$ (2.5)

Patch mean is subtracted from each patch and $\overline{X} = [\overline{x}_{i,1}, \overline{x}_{i,2}, ..., \overline{x}_{i,\tilde{m}\tilde{n}}]$ is obtained. The patch means, $\overline{x}_{i,j}$ equation is described in Equation (2.6). **1** is an all-one vector of proper dimension.

$$\bar{x}_{i,j} = x_{i,j} - \frac{\mathbf{1}^T x_{i,j}}{k_1 k_2}$$
(2.6)

By constructing the same matrix for all input images and combined, the output obtained is $X = [\bar{X}_1, \bar{X}_2, ..., \bar{X}_N]$. The output of PCA filters stage are L_i principal eigenvectors of XX^T and expressed in Equation (2.7).

$$W_l^1 = mat_{k_1,k_2}(q_l(XX^T)) \in \mathbb{R}^{k_1 \times k_2}, l = 1, 2, \dots, L_1$$
(2.7)

where W_l^1 is PCA filters, $mat_{k_1,k_2}(v)$ is a function that maps $v \in \mathbb{R}^{k_1k_2}$ to a matrix $W \in \mathbb{R}^{k_1 \times k_2}$ and $q_l(XX^T)$ denotes the *l*-th principal eigenvector of XX^T .

ii. Second stage PCA

This stage repeats the same process as in first stage where the *l*-th filter output of the first stage is convolved with I_i . The equation is described in Equation (2.8). The boundary of I_i is zero-padded before convolving with W_l^1 so that I_i^l has equal matrix size as I_i .

$$I_i^l = I_i * W_l^1, \qquad i = 1, 2, \dots, N$$
(2.8)

Patch mean is then subtracted from each patch and $\overline{Y}_i^l = [\overline{y}_{i,l,1}, \overline{y}_{i,l,2}, ..., \overline{y}_{i,l,\tilde{m}\,\tilde{n}}]$ is obatained. Y^l is further defined as $Y^l = [\overline{Y}_1^l, \overline{Y}_2^l, ..., \overline{Y}_N^l] \in \mathbb{R}^{k_1 k_2 \times N \tilde{m} \tilde{n}}$ for all matrix. All mean-removed patches of *l*-th are collected and concatenated which is described as $Y = [Y^1, Y^2, ..., Y^{L_1}] \in \mathbb{R}^{k_1 k_2 \times L_1 N \tilde{m} \tilde{n}}$. The output from second filter stage is expressed in Equation (2.9).

$$W_l^2 = mat_{k_1,k_2}(q_l(YY^T)) \in \mathbb{R}^{k_1 \times k_2}, l = 1, 2, \dots, L_2$$
(2.9)

 L_2 images of size $m \times n$ will be obtained for each input I_i^l of the second stage and each convolves I_i^l with W_l^2 . Equation (2.10) shows the final output from second PCA stage. Number of images at last PCA stage is expressed in Equation (2.11).

$$O_i^l = \{I_i^l * W_l^2\}_{l=1}^{L_2}$$
(2.10)

No of images at last filtering stage= $L_1 \times L_2 \times ... L_i$ (2.11)

iii. Out Stage (Hashing & Histograms)

The real-valued outputs are then undergone binary quantization, where the binarized outputs are $\{H(I_i^l * W_l^2)\}_{l=1}^{L_2}$, where H(.) is a Heaviside step function. Postive entries have value of 1 and 0 otherwise. The vector of L_2 binary bits is displayed as decimal number, converting L_2 outputs in O_i^l back into single integer-valued "image", T_i^l where the integer range is between 0 and $2^{L_2} - 1$ which is described in Equation (2.12).

$$T_i^l = \sum_{l=1}^{L_2} 2^{l-1} H(I_i^l * W_l^2)$$
(2.12)

 T_i^l is partitioned into B blocks for L_1 images. Histogram with 2^{L_2} binary bits are computed in each block. The last step is concatenate all histograms into one vector, namely $Bhist(T_i^l)$. Hence, features of the input image, f_i is defined in a blockwise histogram which described in Equation (2.13).

$$f_i = [Bhist(T_i^1), Bhist(T_i^2), \dots, Bhist(T_i^{L_1})]^T \in \mathbb{R}^{(2^{L_2})L_1B}$$
(2.13)