PERFORMANCE OF NEWTON-RAPHSON METHOD FOR LOAD FLOW ANALYSIS IN ILL-CONDITIONED SYSTEMS

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by

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LIST OF ABBREVIATIONS

IEEE	Institute of Eelctrical and Electronic Engineers
MATLAB	Matrix Laboratory
NR	Newton Raphson
LFSSO	Load Flow Method with Step Size Optimization
CNPF	Continuous Newton Power Flow
NLS-CLM	Non-monotone Line Search with Corrected Lavenberg-Marquardt

ABSTRAK

Kajian aliran kuasa digunakan secara meluas dan penting untuk analisis keadaan mantap pada sistem kuasa. Kekukuhan aliran kuasa adalah penting untuk membantu para jurutera dalam konfigurasi rangkaian yang sukar. Pada hakikatnya, peningkatan permintaan puncak dan pelbagai faktor lain yang telah meningkatkan kebimbangan sistem kuasa kerana ini mungkin menyebabkan sistem jatuh ke sistem yang tidak sihat. Kajian ini membentangkan kajian kaedah Newton-Raphson untuk analisis aliran beban dalam sistem yang tidak sihat. Analisis dilakukan pada IEEE 30-Bus System melalui pengaturcaraan MATLAB. Tumpuan utama kajian ini adalah untuk menganalisis kes-kes yang mempunyai nisbah R/X yang tinggi dan keadaan pemuatan dalam sistem penghantaran dengan melaksanakan kaedah Newton-Raphson pada platform MATLAB. Parameter yang terlibat dalam kajian ini adalah bilangan lelaran yang diperlukan bagi sistem untuk menumpu dan masa pengiraan untuk menjalankan analisis aliran beban di MATLAB. Nilai R/X dan faktor pemuatan dinaikkan secara beransur-ansur untuk melihat kesan pada prestasi Newton-Raphson dalam analisis aliran beban. Penyampaian grafik dimasukkan untuk membandingkan bagaimana kenaikan nisbah R/X dalam baris individu dan sekumpulan baris akan mempengaruhi nombor lelaran sistem untuk menumpu. Sementara itu, untuk faktor pemuatan, bilangan lelaran dan masa pengiraan turut dianalisa.

ABSTRACT

Power flow studies are widely used and essential for steady state analysis on power systems. Robustness of power flow is vital to assist engineers on difficult network configurations. In reality, the increase in peak demand and many more factor has increased the concern of the power system as this might cause the system to fall into ill-conditioned system. This paper presents a review of Newton-Raphson method for load flow analysis in ill-conditioned systems. The analysis was carried out on IEEE 30-Bus System via MATLAB programming. The main focus of this paper is to analyse the ill-conditioned cases of high R/X ratio and loading conditions in a transmission system by implementing the conventional Newton-Raphson method on the MATLAB platform. The parameters that are concerned in this research are the iteration number required for the system to converge and the computation time to run the load flow analysis in MATLAB. The value of R/X ratio and loading factor is increased gradually to observe the effect on the performance of Newton-Raphson in load flow analysis. A graphical presentation is included to compare how the increment of R/X ratio in individual lines and a group of lines would affect the iteration numbers of the system to converge. Meanwhile, for loading factor, the iteration number and computation time also being analysed.

CHAPTER 1

INTRODUCTION

1.1 Background

In a power system, power flows from generating station to the load through different branches of the network. The flow of active and reactive power is known as load flow or power flow. Load flow analysis is an important tool used by power engineers for planning and determining the steady state operation of a power system (Mallik *et al.*, 2012). The main information obtained from the load flow or power flow analysis comprises magnitudes and phase angles of load bus voltages, reactive powers and voltage phase angles at generator buses, real and reactive power flows on transmission lines together with power at the reference bus; other variables being specified (Aroop *et al.*, 2014).

Numerous power flow studies are required to ensure that power is adequately delivered at all time despite normal load fluctuations and undesirable events such as contingencies. Daily fluctuations in the power system operations cause power flow mismatches at busbars. As a result, the busbars voltage magnitude and angle adjust instantly until an equilibrium is reached between the load and the transmitted power. This new equilibrium point can also be obtained from simulation using power flow methods (Lagace, 2012).

Back in past, a numerous numerical method is proposed to solve the non-linear equation in power flow analysis. The flow equation must be solved by iterative techniques using numerical methods. The most commonly used are Gauss-Siedel, Newton-Raphson and Fast Decoupled method. Among these three methods, Newton-Raphson are extensively utilized till today due to its effectiveness in doing the analysis. The reason is because Newton-Raphson have higher convergence rate. Convergence is a term to show how fast a power flow reaches its solution.

It is undoubtedly true that Newton-Raphson presented the best performance in load flow analysis. However, with the industrial developments in the society, the power system kept increasing and the dimension of load flow equation also kept increasing to several thousands. With such increases, any numerical mathematical method cannot converge to a correct solution (Mageshvaran, 2008). In such situation, the problem might arise for ill-conditioned power system such as a system with a high R/X ratio or a system loading that approaches a critical loading.

In this research, the project will be focusing on analysing two ill-conditioned cases which are high R/X ratio and different loading condition by implying the conventional Newton-Raphson method on the MATLAB platform. It is expected that the increasing in R/X ratio and the loading factor would affect the stability of the system to converge.

1.2 Problem Statement

Newton-Raphson is utilized in power flow analysis to determine the voltage magnitudes and phase angles at each bus, hence to compute real and reactive power. This is to ensure the stability of a system at all time and its reliability to meet the power demand. This is to avoid any unforeseen condition in future.

Implementing Newton-Raphson in power flow analysis is efficient and practical as it converges fast within a few iteration numbers. However, in some cases, the method might be diverged in some ill-condition cases. Conventional power flow methods are known to have difficulties in analysing ill-conditioned systems. This is due to the nature of Jacobian matrix as ill-conditioned systems cause the condition number of the Jacobian to be extremely high leading to round off error accumulations during the course iterative solution and giving rise to oscillations or divergence of load flow solutions (Bijwe and Kelapure, 2003).

This paper will present the implementation of the conventional Newton-Raphson method for load flow analysis specifically in ill-conditioned system. The analysis was performed to analyse the convergence properties of the conventional method in ill-conditioned system. The investigation will highlight two ill-conditioned cases which are high R/X ratio and different loading. This project aims to focus on two parameters which are the iteration number required for a system to converge and the computation time to run the load flow analysis in MATLAB.

1.3 Research Objectives

The objectives of this project are as followed:

- To review and understand the performance of conventional Newton-Raphson in load flow analysis
- To analyse Newton-Raphson method in ill-condition cases of different R/X ratio.
- To analyse Newton-Raphson in ill-condition cases of different loading.

1.4 Project Scope

In this project, the concerns will be on analysing the ill-conditioned cases by implementing the conventional Newton-Raphson in analysing the power flow in generating system. The highlighted ill-conditioned cases in this research are the high R/X ratio and loading condition. The test will be conducted based on two parameters which are the iteration number required for the system to converge and the computational time to run the load flow analysis.

The implemented conventional method of Newton-Raphson will be carried out using MATLAB. In addition, the algorithms will be implemented on IEEE standard bus system which is IEEE 30-Bus System. The obtained result is analysed to observe the effect of high R/X ratio and different loading on the stability and the convergence of a system.

1.4 Thesis Outline

Chapter 1 briefly explains the background of power flow study and the limitations of Newton-Raphson in ill-conditioned system. Hence, the problem statement and the objectives of the project is identified. Then, the scope of project is presented.

Chapter 2 presents the literature review that are related to this project. The reviews are basically on the power flow studies, Newton-Raphson method and the ill-conditioned cases.

Chapter 3 describes the methodology involves in this research. The formation of power flow equations and the Newton-Raphson algorithm is explained. Then, the steps involving the two highlighted issues to observe the stability of the system are explained in detail.

Chapter 4 discuss the results and the findings from the test as described in Chapter 3. The iteration number and computation time are illustrated in a graphical representation.

Chapter 5 concludes the outcomes and objectives achieved. Besides, suggestion of future works and development also included.

CHAPTER 2

LITERATURE REVIEW

2.1 Load Flow Studies

A load flow study is the analysis of an electrical network carried out by an electrical engineer. The purpose is to understand how power flows around the electrical network. Carrying out a load flow study assists the engineer in designing electrical systems which work correctly, have sufficient power supplied by the power grid, where equipment is correctly sized, reactive power compensation is correctly placed and transformer taps are optimised (McFadyen, 2014).

Power flow studies provide a systematic mathematical approach to determine the various bus voltages, phase angles, active and reactive power flows through different branches, generators, transformer settings and load under steady state conditions. The power system is modelled by an electric circuit which consists of generators, transmission network and distribution network (Kothari and Nagarath, 2007). In addition to power flow, a load flow study is often used to investigate other parameters such as current flow, system power factors, losses, and equipment loading (McFadyen, 2014).

Power flow studies are performed at various points in the network for different operating conditions subject to the constraints on generator capacities and specified net interchange between operating systems and several other restraints. Power flow or load flow solution is essential for continuous evaluation of the performance of the power systems so that suitable control measures can be taken in case of necessity. In practice it will be required to carry out numerous power flow solutions under a variety of conditions (Akhani, 2015).

The Figure 2.1 shows a typical diagram that is often used in load flow analysis to investigate and determine the parameters involved.

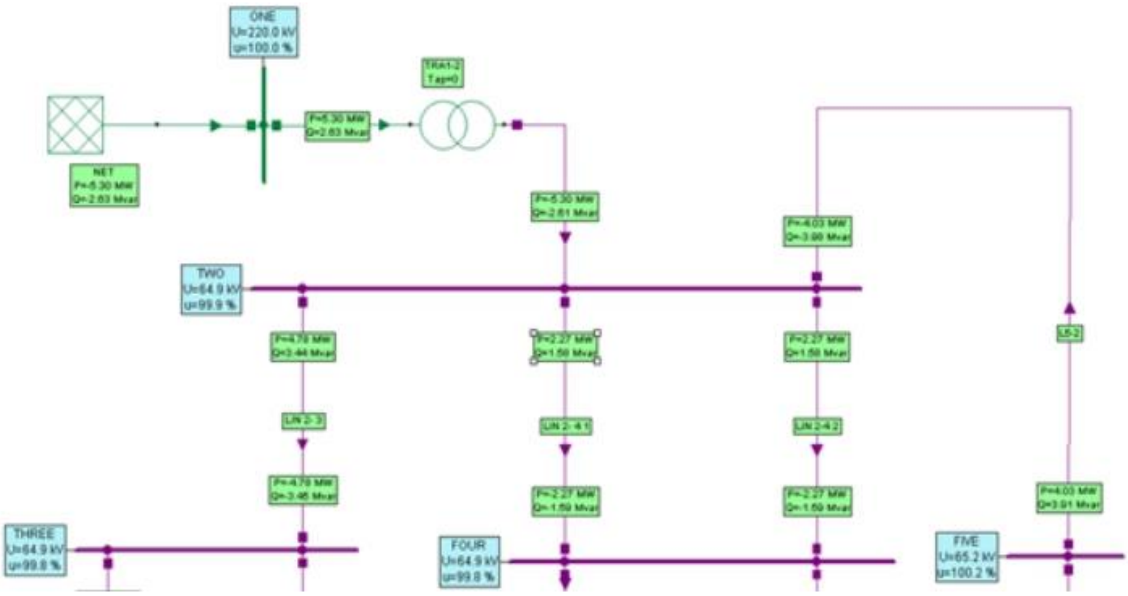

Figure 2.1: Load Flow Study (McFadyen, 2014)

The image in Figure 2.1 shows a typical load flow study. This diagram illustrates part of the network and shows the calculated the flow of real power (P) and reactive power (Q), with the arrows indicating the direction of flow. In addition, busbar voltages are shown.

The power flow calculation of distribution network is an important part of distribution analysis, it is a vital basis of analysing the rationality, reliability and economy of distribution network’s planning and running mode (Yang *et al.*, 2015).

For larger power distribution systems, a formal ‘load flow study’ is carried out; typically using software, with the results presented in a report (McFadyen, 2014). With the advent of the modern digital computers possessing large storage and high speed the mode of power flow studies have changed from analog to digital simulation. Many algorithms are developed for digital power flow solutions. The methods basically distinguish between themselves in the rate of convergence, storage requirement and time of computation. The loads are generally represented by constant power (Akhani, 2015).

To carry out the studies, engineers builds a network of nodes interconnected by admitttances; where each system node has four key parameters:

1. Active power (P)
2. Reactive power (Q)
3. Voltage magnitude (V)
4. Voltage phase angle (δ)

In defining nodes in a software model, the engineer typically considers three types:

1. **Load Bus [P-Q bus]** – a bus where the real and reactive power are specified. The voltage (magnitude and phase angle) is calculated by the study.
2. **Generator Bus [P-V bus]** – a bus which the voltage and real power generation is known. The reactive power and phase angle of the voltage are calculated by the study.
3. **Slack Bus (Swing bus)** – where the voltage magnitude and phase are assumed known. The active and reactive power are calculated by the study.

There are five major factors for evaluating the load flow analysis method, namely speed, storage, reliability, versatility which refers to the ability to handle conventional and uncommon features or suitability for incorporation into more complicated processes and lastly simplicity meaning ease of writing coding and enhancing the algorithm and the computer program based on it (Stott, 1974).

Computer software programming is specifically designed and written to perform and execute the load flow analysis algorithm to obtain the solution which typically includes voltage magnitudes and angles at PQ buses, voltage angles at PV buses, reactive power generations and loads at PV buses, power flows and MVA loadings at both ends of each transmission line and transformer and the total system losses (Kamel *et al.*, 2013).

2.2 Newton-Raphson Method

This method was named after Isaac Newton and Joseph Raphson. The origin and formulation of Newton-Raphson method was dated back to late 1960s (Aroop *et al.*, 2014). To date, Newton-Raphson is the most used method for solving load flow analysis due to its robustness and effectiveness on large power system and able to solve problem that could not be solved using other conventional method.

Among all the power flow methods, the Gauss-Seidel method remains serviceable but suffers from slow convergence characteristics. The incentive to accelerate the convergence led to the Newton-Raphson method which consists of solving a linearization of the active and reactive power equations around an operating point. The Newton-Raphson method is an iterative algorithm for simultaneously solving a set of nonlinear equations and exhibits a quadratic convergence when the initial operating point is close to the solution (Lagace *et al.*, 2008).

Newton-Raphson method is concisely considered as the state of the art power-flow technique and widely accepted and referred in industry applications as it improves the converging behaviour of Gauss-Siedel method which has poor convergence characteristics and high iteration numbers. However, the main disadvantage of the Newton-Raphson method is the computational complication which is the necessity for factorizing and updating the Jacobian matrix during the iterative solution process. (Kamel *et al.*, 2013).

The Newton-Raphson approach is the most preferred load flow method because of its various advantages. It has powerful convergence characteristics compared to alternative processes and considerably low computing times are achieved when the sparse network equations are solved by the technique of sparsity-programmed ordered elimination (Wadhwa, 2009).

In modern times, the power system network expands more complicated because such alternative resources type generator (like wind energy, solar energy, etc.) with large scale battery connects to the power grid which causes the power system more fluctuating. In that situation, the traditional NR method may not solve those cases, due to low converge rate and long computation time (Shao Puguang, 2015).

In 2010, a new load based on Newton-Raphson method was presented. The matrices used in the method are the constant matrices of conductance and susceptance. In 2011, a new iterative solution technique for power flow analysis to reduce the computation complexity and time of the conventional solution techniques was introduced. In addition, the decoupled iterative schemes with constant Jacobian matrices were also widely used and developed as the method was reliable to certain extent and rapid in convergence without updating the Jacobian matrix and thus simplifying the numerical computing process (De Moura and De Moura, 2013).

2.3 Ill-conditioned Case

Linear systems theory defines an ill-conditioned system as the one for which its condition number is sufficiently large. This large condition number could be a result of (Gutierrez and Badrinana, 2011):

i. the proximity of the load level to the system's maximum load ability or maximum loading point(MLP), also known as the saddle-node bifurcation point.

ii. choosing an initial state that conducts the standard calculation method to a trajectory that diverges and the condition number increases. This behavior occurs although the system loading is within the feasible region and an operating point does exist.

Fast-Decoupled method is also one of the preferred methods in power flow calculation. However, in the distribution network, because of the high ratio of R/X leading to the result that the principal diagonal are not dominant, it is rather difficult for Fast-Decoupled method to converge (Tan *et al.*, 2013).

There are many other factors causing Fast-Decoupled method to converge slowly such as heavy loading at some buses which results in low voltages at these buses. In this case, the convergence deteriorates because the Fast-Decoupled formulation is derived assuming all bus voltages are near the nominal value. Many attempts and modifications have been made to obtain better convergence of Fast-Decoupled method while most of the methods concerns on high R/X ratio problems. For instance, tuning the power flow calculation according to the R/X ratio and the bus voltage in the power systems obtains better convergence (Lu and Li, 2000).

The power system with near singular or singular Jacobian matrix is named bad-conditioned or ill-conditioned. Some reasons may lead to change the condition of the power system to ill-condition. Some of these reason are position of the swing bus, installation of some equipment such as flexible AC transmission system (FACTS) and high ratio of R/X in radial networks (Pourbagher and Derakhshandeh, 2017).

The distribution systems usually fall into the category of ill-conditioned power systems for generic Newton-Raphson like methods with its special features, such as (Bijwe and Kelapure, 2003):

i. Radial or weakly meshed topologies
ii. High R/X ratio of the distribution lines
iii. Unbalanced operation
iv. Loading conditions
v. Dispersed generation
vi. Non-linear load models

2.4 MATLAB Simulation

Matrix Laboratory (MATLAB) is a matrix-based computation system designed for scientific and engineering application. It was developed by Cleve Moler in 1970s.

MATLAB has some advantages contributing to its large popularity (Abdel and Guenther, 2011):

i. Has very fast speed of calculation which gives instantaneous feedback
ii. Easy to use since data can be easily entered
iii. High level-command for two-dimensional and three-dimensional data visualization and graphic

For this research, all simulations are done in MATLAB platform. Its features that being able to implement matrix are

For this research, all simulations are done in MATLAB platform. The features that it has make it as the most appropriate software to implement the load flow study algorithms.

2.5 Summary

From the review, it is known that Newton-Raphson has the strongest features to conduct power flow studies due to its powerful convergence characteristic. Newton-Raphson method has very powerful converging properties over a wide range. However, due to the complexity of network, the large-scale battery in power system and other factors that causing the power system to fluctuate had merely affect the performance of this traditional method. At this rate, NR unable to solve the case and start to diverge. There are many factors that contribute to ill-conditioned system. The most concern factor is due to the high ratio of R/X and the critical loading. Hence, in this paper, the analysis would focus on these two factor and the NR method is chosen to carry out the analysis since it has the best performance.

CHAPTER 3

METHODOLOGY

3.1 Overview

This chapter discuss the fundamental of load flow analysis using numerical formulation and equations. The algorithm involved; Newton-Raphson(NR) is derived and explained in this chapter.

The algorithm is applied to a standard bus system to be tested and analyse on reliability and stability of the system. The test would be focusing on two factors which are the high R/X ratio and loading conditions. The performance of NR is observed and analysed based on the number of iterations and computing time graphically.

The Newton-Raphson method is implemented on bus system which is IEEE 30-Bus System. The computer software that is used to perform the algorithm involved are carried out in MATLAB.

3.2 Theory

This section explains the theory of load flow analysis including the bus admittance matrix formation and power flow equations using numerical formulations and equations.

3.2.1 Bus Admittance Matrix

The conventional numerical methods that are used in this paper is based on the node-voltage method (Saadat, 2010). This method is the most suitable and commonly used for power system analysis.

To perform the nodal analysis, the impedance, Z are converted to admittance, Y, which is the inverse of impedance, Z. Consider two buses, bus i and bus j with the resistance of R_{ij} and inductive reactance of X_{ij}, the line admittance y_{ij} is expressed in terms of the line impedance Z_{ij} as follows:

$$Z_{ij} = R_{ij} + jX_{ij} \tag{3.1}$$

$$y_{ij} = \frac{1}{Z_{ij}} = \frac{1}{R_{ij} + jX_{ij}} \tag{3.2}$$

Figure 3.1 shows a simple admittance diagram of a 4-bus system .

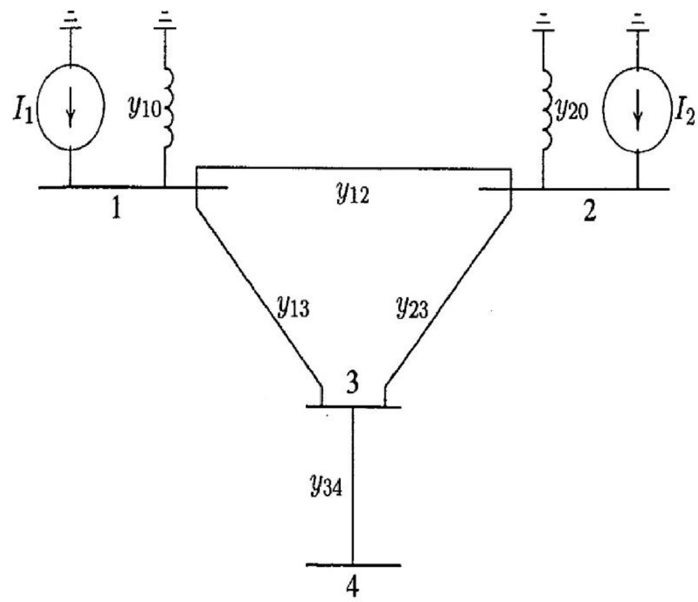

Figure 3.1: 4-bus system admittance diagram (Saadat,2010)

Applying Kirchoff's Current Law (KCL) on Figure 3.1:

$$I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3) \tag{3.3}$$

$$I_2 = y_{20}V_2 + y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3) \tag{3.4}$$

$$0 = y_{23}(V_3 - V_2) + y_{13}(V_3 - V_1) + y_{34}(V_3 - V_4) \tag{3.5}$$

$$0 = y_{34}(V_4 - V_3) \tag{3.6}$$

Rearranging Equations (3.3) to (3.6):

$$I_1 = (y_{10} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3 \tag{3.7}$$

$$I_2 = -y_{12}V_1 + (y_{20} + y_{12} + y_{23})V_2 - y_{23}V_3 \tag{3.8}$$

$$0 = -y_{13}V_1 - y_{23}V_2 + (y_{13} + y_{23} + y_{34})V_3 - y_{34}V_4 \tag{3.9}$$

$$0 = -y_{34}V_3 + y_{34}V_4 \tag{3.10}$$

From Equations (3.7) to (3.10), introducing the admittance Y,

$$Y_{11} = y_{10} + y_{12} + y_{13} \tag{3.11}$$

$$Y_{22} = y_{20} + y_{12} + y_{23} \tag{3.12}$$

$$Y_{33} = y_{13} + y_{23} + y_{34} \tag{3.13}$$

$$Y_{44} = y_{34} \tag{3.14}$$

$$Y_{12} = Y_{21} = -y_{12} \tag{3.15}$$

$$Y_{13} = Y_{31} = -y_{13} \tag{3.16}$$

$$Y_{23} = Y_{32} = -y_{23} \tag{3.17}$$

$$Y_{34} = Y_{43} = -y_{34} \tag{3.18}$$

Thus, the general node equation reduces to :

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4 \tag{3.19}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 \tag{3.20}$$

$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4 \tag{3.21}$$

$$I_4 = Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4 \tag{3.22}$$

Extending to n-bus system, the node voltage equation in matrix form is:

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1i} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2i} & \cdots & Y_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ Y_{i1} & Y_{i2} & \cdots & Y_{ii} & \cdots & Y_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{ni} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} \tag{3.23}$$

or

$$I_{bus} = Y_{bus}V_{bus} \tag{3.24}$$

The Y_{bus} is known as the bus admittance matrix. In general, the diagonal element of each node is the sum of admittance connected to it, which known as the self-admittance or driving point admittance;

$$Y_{ii} = \sum_{j=0}^{n} y_{ij} \qquad j \neq i \tag{3.25}$$

The off-diagonal element is equal to the negative of the admittance between the nodes, which known as the mutual admittance or transfer admittance;

$$Y_{ij} = Y_{ji} = -y_{ij} \tag{3.26}$$

Thus, the bus admittance matrix is written as:

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & - & Y_{1n} \\ Y_{21} & Y_{22} & - & Y_{2n} \\ - & - & - & - \\ Y_{n1} & Y_{n2} & - & y_{nn} \end{bmatrix} \tag{3.27}$$

3.2.2 Power Flow Equation

Consider a typical bus of a power system network as shown in Figure 3.2 where all the transmission lines are represented by their corresponding π-model.

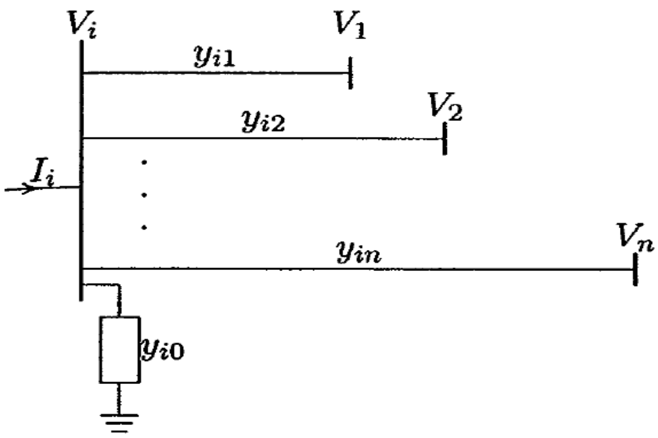

Figure 3.2: Typical bus of power system (Saadat, 2010)

Applying KCL to bus i, the current I_i can be calculated as:

$$I_i = y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \dots + y_{in}(V_i - V_n)$$
$$= (y_{i0} + y_{i1} + y_{i2} + \dots + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \dots - y_{in}V_n \quad (3.28)$$

or

$$I_i = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j \quad (3.29)$$

The real and reactive power at bus i can be expressed as:

$$P_i + jQ_i = V_i I_i^* \quad (3.30)$$

or

$$I_i = \frac{P_i - jQ_i}{V_i^*} \quad (3.31)$$

Substituting (3.31) into (3.29):

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j \qquad j \neq i \quad (3.32)$$

From Equation (3.32), the relation show that the power flow equation is in nonlinear algebraic equation and should be solved by iterative techniques.

3.2.3 Newton-Raphson Method

From Equation (3.28), rewrite in polar form:

$$I_i = \sum_{j=1}^{n} |Y_{ij}||V_j|\angle\theta_{ij} + \delta_j \tag{3.33}$$

By substituting Equation (3.33) into complex power equation, Equation (3.30):

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{3.34}$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \tag{3.35}$$

Using Taylor's Series expansion, Equation (3.34) and Equation (3.35) is expanded about the initial estimate and neglecting all higher order terms.

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \hline \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \frac{\partial P_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_2}{\partial \delta_n}^{(k)} & \frac{\partial P_2}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial P_2}{\partial |V_n|}^{(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_n}{\partial \delta_n}^{(k)} & \frac{\partial P_n}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial P_n}{\partial |V_n|}^{(k)} \\ \hline \frac{\partial Q_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_2}{\partial \delta_n}^{(k)} & \frac{\partial Q_2}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial Q_2}{\partial |V_n|}^{(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_n}{\partial \delta_n}^{(k)} & \frac{\partial Q_n}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial Q_n}{\partial |V_n|}^{(k)} \end{array}\right] \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \hline \Delta |V_2^{(k)}| \\ \vdots \\ \Delta |V_n^{(k)}| \end{bmatrix} \tag{3.36}$$

In Equation (3.36), bus 1 is assumed to be slack bus. This equation is also known as Jacobian matrix. The Jacobian matrix gives relationship between small changes in voltage angle and voltage magnitude with the small changes in real and reactive power. It can be written as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \tag{3.37}$$

For J_1,

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \tag{3.38}$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{3.39}$$

For J_2 ,

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}| \cos\theta_{ii} + \sum_{j \neq i} |V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \tag{3.40}$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{3.41}$$

For J_3 ,

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \tag{3.42}$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{3.43}$$

For J_4 ,

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}| \sin\theta_{ii} - \sum_{j \neq i} |V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \tag{3.44}$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{3.45}$$

The new estimate bus voltages is obtained:

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)}$$
$$|V_i^{(k+1)}| = |V_i^{(k)}| + \Delta|V_i^{(k)}| \tag{3.45}$$

The process is continued until the residuals are less than specified accuracy.

3.3 High R/X Ratio

As mentioned in Chapter 3.1, the formulation of Newton-Raphson is implemented on IEEE 30-Bus System via MATLAB programming to generate the load flow solution.

Generally, the first step is by finding and considering the range of R/X ratio for the test system to be analysed in MATLAB programming. Throughout this project, the range of the R/X ratio will start from 1 to 10. Hence, the value of new R for each line is calculated parallel with the value of R/X ratio.

The result obtained is tabulated and illustrated in a graph. For a system that failed to converge, it will be denoted as NC in the table.

3.3.1 Individual Line

In this subtopic, the line data in IEEE 30-Bus System is modified suitably to the increment of R/X ratio. The R/X ratio for each individual line is increased within a range from 1 to 10. The test is carried out by modifying the line data of the bus test system for one line at a time. Hence, each line is modified for 10 times since the R/X ratio is increased up to 10 factors.