IMPROVING ACCURACY IN AUTOMATIC MODULATION CLASSIFICATION OF DIGITAL MODULATED SIGNALS USING DESIGN OF EXPERIMENT METHOD

CHAN WUI HUNG

UNIVERSITI SAINS MALAYSIA

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by

CHAN WUI HUNG

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LIST OF ABBREVIATIONS

ADTS	Asynchronous Delay tap Sampling
ADTP	Asynchronous Delay tap Plot
AE	Auto-Encoder
AF	Ambiguity Function
AltBOC	Alternative Binary Offset Carrier
AMC	Automatic Modulation Classification
AML	Approximate Maximum Likehood
ANC	Auto-Encoder employing Non-negativity Constraint
ASK	Amplitude-Shift Keying
BOC	Binary Offset Carrier
BPNN	Backpropagation Neural Network
BPSK	Binary Phase-Shift Keying
CBOC	Composite Binary Offset Carrier
CNN	Convolution Neural Network
CPFSK	Continuous-Phase Frequency-Shift Keying
DBN	Deep Belief Network
DoE	Design of Experiment
DL	Deep Learning

DNN	Deep Neural Network
DWDM	Dense Wavelength Division Multiplexing
FB	Feature-Based
FSK	Frequency-Shift Keying
FT	Fourier Transform
GFSK	Gaussian Frequency-Shift Keying
GMSK	Gaussian Minimum-Shift Keying
HH-AMC	Hierarchical Hybrid Automatic Modulation Classification
HOM	High Order Moments
ITD	Instantaneous Time Domain
KNN	K-Nearest Neighbour
LB	Likelihood-Based
LFM	Linear Frequency Modulated
LSVM	Linear Support Vector Machine
MIMO	Multiple Input Multiple Output
MSK	Minimum-Shift Keying
OFDM	Orthogonal Frequency-Division Multiplexing
OTA	Over-The-Air
PSK	Phase-Shift Keying
QAM	Quadrature Amplitude Modulation

QPSK	Quadratic Phase-Shift Keying
SAE	Sparse Auto-Encoder
SAE-AF	Sparse Auto-Encoder based on Ambiguity Function
SDR	Software Defined Radio
SNR	Signal To Noise Ratio
SVM	Support Vector Machine
UDNN	Unsorted Deep Neural Network
WT	Wavelet Transform
4-QAM	4-Quadrature Amplitude Modulation
8-PSK	8-Phase-Shift Keying
16-QAM	16- Quadrature Amplitude Modulation
64-QAM	64- Quadrature Amplitude Modulation

PENINGKATAN KETEPATAN DALAM KLASIFIKASI MODULASI AUTOMATIK UNTUK ISYARAT DIGITAL TERMODULAT DENGAN MENGGUNAKAN KAEDAH REKA BENTUK EKSPERIMEN

ABSTRAK

Klasifikasi modulasi automatik (AMC) merupakan sistem untuk mengklasifikasi format modulasi isyarat yang diterima. Ia merupakan system di antara penerima dan demodulator. Sistem ini penting kerana proses klasifikasi isyarat yang diterima mestilah boleh diharap supaya informasi yang diterima adalah tepat. Oleh itu, terdapat banyak penyelidikan telah dijalankan untuk mencari alternatif yang boleh meningkatkan ketepatan klasifikasi system AMC. Dalam projek ini, teknik persampelan tunda tak segerak (ADTS) telah dicadangkan dalam klasifikasi modulasi. Dengan menggunakan teknik ADTS, plot tunda kelewatan tak segerak (ADTP) yang unik dan berbeza telah dijana untuk setiap isyarat digital termodulat QPSK, 16-QAM dan 64-QAM. Data-data ini dibina semula untuk menjadi input kepada menyokong pengelas mesin vektor (SVM) yang terbina dalam MATLAB. Kaedah reka bentuk eksperimen (DoE) digunakan untuk meningkatkan ketepatan sistem AMC. Dalam DoE, reka bentuk factorial 2^2 telah digunakan. Dua faktor yang dipilih ialah keterlambatan ketik dan tempoh persampilan yang digunakan dalam ADTS. Hasil klasifikasi mnunjukkan bahawa ketepatan pengelas adalah sebanyak 95.1%. Melalui DoE, ketepatan pengelas menggunakan nilai optimum ialah 97.6%. Keadaan ini menunjukkan peningkatan akurasi dalam system AMC apabila DoE digunakan. Sebagai kesimpulan, teknik-teknik yang dicadangkan mampu meningkatkan ketepatan system AMC.

IMPROVING ACCURACY IN AUTOMATIC MODULATION CLASSIFICATION (AMC) OF DIGITAL MODULATED SIGNALS USING DESIGN OF EXPERIMENT (DOE) METHOD

ABSTRACT

An automatic modulation classification (AMC) is a system is used to classify the modulation format of a received signal. It is a system placed in between the receiver and the demodulator. The AMC is crucial as the classification of received signal must be reliable to ensure the received information is correct. Therefore, a lot of studies had been conducted to look for the alternative for the improvement of classification accuracy of the AMC system. In this project, asynchronous delay tap sampling (ADTS) is proposed as a technique in modulation classification. From the ADTS, unique and distinct asynchronous delay tap plot (ADTP) is generated for each of the QPSK, 16-QAM and 64-QAM digital modulated signal. These data are then reconstructed to become the input of a built-in support vector machine (SVM) classifier in MATLAB. Design of experiment (DoE) method is applied to improve the accuracy of the AMC system. In DoE, 2^2 factorial design method is applied. The two selected factors are the delay tap and the sampling period used in ADTS. The results of the classification showed that the accuracy of the classifier is 95.1%. Through DoE, the accuracy of the classifier using the optimum values is 97.6%. This shows an improvement in the accuracy of the AMC system by using the DoE method. In conclusion, the proposed techniques are fully capable of improving the accuracy of the AMC system.

CHAPTER 1

INTRODUCTION

1.1 Research Background

Automatic modulation classification (AMC) is a process between signal detection and demodulation [1]. It is used to identify the type of modulation of a signal and its parameter. The examples of these parameters are carrier frequency and symbol rate [2]. AMC is widely used in military and civilian to securely transmit necessary signals while blocking the unwanted signals. In military field, AMC is used to recover transmitted signals, to jam communications between adversary units and to protect friendly communications from opponents. While in civilian scene, AMC is used to select a modulation technique from a set of predefined modulations based on the system specification and channel conditions [3].

A modulation classifier can be split into two steps: signal pre-processing and classification algorithm [4]. Signal pre-processing estimate the parameters of the received signal. These parameters include signal power, signal to noise ratio (SNR), time of arrival, pulse width or carrier frequency. Classification algorithm determines the accuracy requirement in the signal pre-processing [1].

There are two types of AMC algorithms, which are the likelihood-based (LB) and feature-based (FB) methods. In LB method, the likelihood function of the received signal is compared with a threshold signal to come out with a decision [5]. In feature-based method, the features include instantaneous time domain (ITD) parameters, Fourier transform (FT), wavelet transform (WT), higher order moments (HOM), etc [6]. The decision making is based on the observe value of these features.

Recent years, a lot of works have been done to improve the performance of AMC system in term of accuracy of the classification. Deep neural networks (DNN) [7], sparse auto-encoders based on ambiguity function (SAE-AF) [8] and a few other works were carried out to improve the performance of AMC system [9] [10] [11] [12]. A more detail review can be found in Chapter 2: Literature Review.

Motivated by the success in using asynchronous delay tap sampling (ADTS) in [13] [14] and [15] the ADTS method is being used in this project for modulation classification of the received digital modulated signals. Even though this technique is being used to determine joint modulation format/bit rate classification and signal-to-noise (SNR) ratio, no research have been done to vary the parameters being used in ADTS for accuracy improvement yet.

DoE is a statistical method for any process or system to describe the changes to the output when the input variables are changed [16]. Factors affecting the performance of a process or system can be studied based on the response in the output. Through the outcomes of the experiment, a model can be developed for improvement or other decision-making.

In the past years, there were researchers who applied DoE method in optimization of their system or design [17] [18] [19] [20]. It is proven that DoE is a powerful approach for improvement and optimization. Thus, in this project, DoE method is used to improve the accuracy in automatic modulation classification (AMC) of digital modulated signals.

1.2 Problem Statements

In the previous works [7] – [12], the AMC systems demonstrated have high complexity and thus increase the computational efforts. Other alternative with lower complexity are to be explored for the ease of computational effort. Recent researches have shown that ADTS offer low-complexity and flexibility in implementation and have shown promising results in their research [13] - [15]. The applicability of ADTS are to be confirmed in this project.

In [15], the application of ADTS in the determination of joint modulation format/bit rate and SNRs had demonstrated good recognition accuracy. However, the paper did not study the effect of varying the delay tap time between each sample pair and the sampling time for the ADTS. Hence, it remained an unknown if the changes in these parameters would improve the classification accuracy of the system. The optimum values for the parameters are yet to be determined.

1.3 Objectives

In order to develop an AMC system with the utilisation of ADTS and by improving it through DoE approach, the following three objectives are the main concern of this project:

(i) To investigate the applicability of ADTS in AMC system.

(ii) To study the utilization of DoE in improving the accuracy of AMC system.

(iii) To evaluate the performance of the AMC system after the application of DoE.

CHAPTER 2

LITERATURE REVIEW

2.1 Chapter Introduction

This chapter reviews the works and researched that had been done on AMC system and ADTS. In Section 2.2, the results of the study on AMC system carried out by various researchers are discussed. In Section 2.3, a brief concept on the ADTS is explained and the results of the research on this topic are also discussed.

2.2 Preliminary Study on Automatic Modulation Classification

Afan Ali and Fang Yangyu [7] proposed an AMC method based on a deep autoencoders-based network with non-negativity weight constraint. This proposal applied both deep neural network (DNN) and autoencoder employing non-negativity constraint (ANC). There are 3 layers of DNN based on autoencoders which are fully connected. The autoencoders are trained to learn a sparse, part-based representation of the input data. This is achieved by breaking the input data into parts and reconstruct the original signal when all the parts are added together. After that, the DNN is trained with the stacked autoencoders and a softmax classification layer while constraining the weights of the overall network. There five modulation classes chosen in this paper, i.e, BPSK, 8-PSK (phase-shift keying), 4-QAM (quadratic-amplitude modulation), 16-QAM and 64-QAM. All the simulations were performed in MATLAB environment. The authors then compared the result obtained with a three-layer sparse autoencoder (SAE). The classification accuracy of this proposed method showed a better performance than the literatures reviewed by the authors. Table 2.1 shows the comparison of the results obtained for the two models. Based on Table 2.1, as the sample length used increases, the accuracy also increases.

In another paper, Afan and Yangyu [21] aimed to reduce the complexity of the existing AMC system in the literatures reviewed by the authors. They proposed an unsorted deep neural network (UDNN) classifier. This classifier contains a single layer of *k*-sparse autoencoder. It is trained to learn compact representation of input data and is used to extract distinct features from the input data. The results obtained showed that the proposed UDNN has good classification capability and reduced complexity compare to the linear support vector machine (LSVM), approximate maximum likelihood (AML) and backpropagation neural network (BPNN) classifiers.

[9] consider feature-based (FB) AMC method in the paper. The features of the digitally modulated signals are extracted automatically with deep learning (DL) method, which is fine-tuning stacked sparse auto-encoders based on ambiguity function (SAE-AF). In this method, three components are included: auto-encoder (AE), deep belief network (DBN) and convolution neural network (CNN). The first step of the proposed method is to convert raw signals into ambiguity function (AF) images. Then, the DL model is used to extract the features of the images. Finally, the extracted features are fed into a softmax classifier for identification purpose. In the experiment, digital modulations including ASK, PSK, QAM, FSK, MSK, LFM and OFDM were used as data source. The experiment was carried out using MATLAB. The results obtained using the proposed method are compared with the results obtained using support vector machine (SVM) method. There is a significant improvement using the SAE-AF developed by the authors.

In addition, [10] designed a decision tree based novel hierarchical hybrid automatic modulation classifier (HH-AMC) which combines both likelihood-based (LB) and features-based (FB) methods. The authors first conducted experiments on LB and FB methods. Based on the results obtained, the suitable AMC technique is chosen at each node of the decision tree. Over-the-air (OTA) performance evaluation is conducted using software defined radios (SDR). The modulation scheme used were BPSK, QPSK, 8-PSK, 16-QAM, 32-QAM, CPFSK, GFSK and GMSK. This experiment showed a high value of probability of correct classification (>0.99) for selected modulation formats except for 32-QAM. However, the method selected has high complexity and thus increase the computational time. Table 2.2 shows the result obtained.

[11] proposed a high-efficiency classification system using genetic backpropagation neural network (BPNN). The system consists of two parts: feature extraction and classification algorithm. In feature extraction, the signal is first processed by Hilbert transform to obtained the instantaneous parameters such as amplitude, phase and frequency. Then, these parameters become the input of the BPNN for classification purpose. The BPNN is designed using generic algorithm (GA) for optimization. The signals chosen were ASK, PSK, FSK, BOC, CBOC and AltBOC. The classification has shown an accuracy of 94.5% under SNR of 3dB, which showed a good performance compare to previous work. The authors also compared the results obtained with other systems, which are tabulated in Table 2.3.

In [12], they aimed to improve the performance of the feature based AMC classification for applications in spectrum monitoring. The modulation classification uses higher order cumulants and antenna array. In higher order cumulants, the combination of multiple cumulants in different orders form a multi-cumulant vector for classification. This method utilizes the information from various cumulants to improve the efficiency.

[22] designed and compared 4 models of feature based AMC: classification tree, *K*-nearest neighbour (KNN), artificial neural network (ANN) and support vector machine (SVM). All the 4 models were evaluated under multiple-input-multiple-output (MIMO) channels in terms of accuracy and computational complexity. In terms of accuracy, SVM and ANN classifiers have better accuracy among the 4 models. Bur overall all the 4 classifiers have close performances. In terms of complexity, KNN and SVM have the highest order of computational complexity.

2.3 Asynchronous Delay tap Sampling (ADTS)

Asynchronous delay tap sampling is applied due to the fact that it DoEs not require clock synchronization with the signal [13]. It can be done in a lower rate than the data rate of received signal [23].

In this technique, the signal is sampled in pair. There is a time delay between the pair of sampled signals x_i and y_i, which is known as delay tap, Δt. A pair of signals is sampled again after a sampling period, T_s. Figure 2.1 shows the realisation of the ADTS. The pair of signals is used to create a histogram called asynchronous delay tap plot (ADTP). This delay tap plot contains information of the signal and is necessary in modulation classification [23].

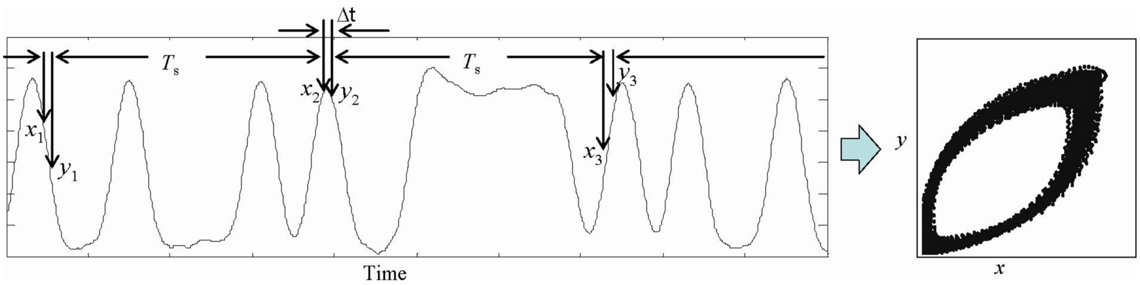

Figure 2.1: ADTS and ADTP [24]

[13] used this method to simultaneously classify modulation format/bit rate. The authors utilised the deep machine learning architectures, which in this case is deep neural network (DNN) to extract the features in ADTPs. A large amount of ADTPs were generated to function as the training data set for the DNN. The results showed good classification accuracies and reduced complexity as well as cost-effective. However, the authors did not try to vary the delay tap and the sampling period to see if the classification accuracy will show any improvement.

In [25], they used ADTS to monitor the chromatic dispersion (CD) in dense wavelength division multiplexing (DWDM) system. ADTS is used to sample optical signal followed by the generation of ADTP. The generated delay tap plot or histogram contains the information of the waveform distortion due to impairment during transmission. This proposed method successfully monitor the CD through the graph of amplitude ratio of delay tap sampling against the residual CD.

2.4 Chapter Summary

Various classifiers are used as the AMC system in the literatures reviewed i.e. DNN, KNN, SVM and ANN. These classifiers have shown good performances, with average classification accuracy of 90% and above. Most of the digital modulated signals included in the reviewed literatures are commonly known signals, for example BPSK, QPSK, 4-QAM, 16-QAM and 64-QAM signals. ADTS technique is being applied in simultaneous classification of modulation format/bit rate as well as in the monitoring of impairments in optical communication network. The implementation of this technique is because it DoEs not require synchronization and can be done in a lower rate.

CHAPTER 3

METHODOLOGY

3.1 Chapter Introduction

In this chapter, the methods used in this project are explained. In Section 3.2, the flow of this project is presented in the form of flow chart together with brief explanations. Section 3.3 shows the digital modulated signals generated using MATLAB and are turned into useful data. In Section 3.4, ADTS technique is applied on the data from Section 3.3. In Section 3.5, data reconstruction is carried out followed by classification of the data i.e. signals using a built-in classifier in MATLAB. Finally, in Section 3.6, the application of DoE method is explained.

3.2 Project Flow Chart

Figure 3.1 shows the flow chart for the overall project flow.

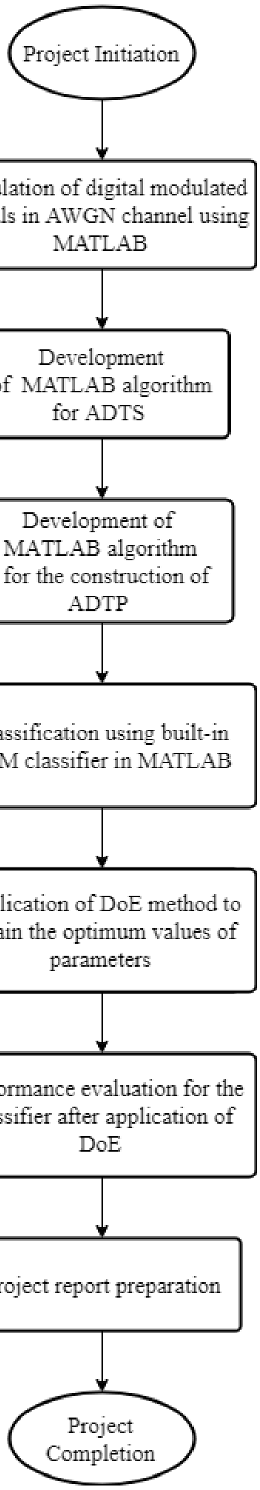

Figure 3.1: Flow chart of project flow

First of all, QPSK, 4-QAM and 64-QAM signals are generated in MATLAB environment. Then, the MATLAB algorithms for ADTS and ADTP are developed. Afterwards, the output data is reconstructed and classified using a built-in classifier, which is the SVM classifier in MATLAB. The results are tabulated as confusion matrix. DoE method is then applied for accuracy improvement. The results are again tabulated for performance evaluation. Finally, preparation for the report is carried out.

3.3 Generation of Digital Modulated Signal Using MATLAB

MATLAB (matrix laboratory) is used for the generation of the digital modulated signals. The selected modulation format is QPSK, 16-QAM and 64-QAM signals. These signals are modelled in AWGN channel with SNR of 5dB. The generated signals are in the form of vector matrix or column matrix as shown in Figure 3.2.

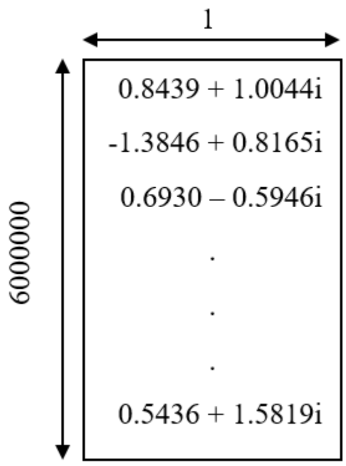

Figure 3.2: Generated QPSK signal

Figure 3.2 shows the vector matrix which has a dimension of 6000000 x 1. The values in the vector matrix are in the form of complex numbers. These complex numbers are converted into magnitude before the next step of the project. To convert the complex number into magnitude, Eq. (3.1) is utilised.

$$X = a + bi \tag{3.1}$$

where X = magnitude

a = real part of the complex number

b = imaginary part of the complex number

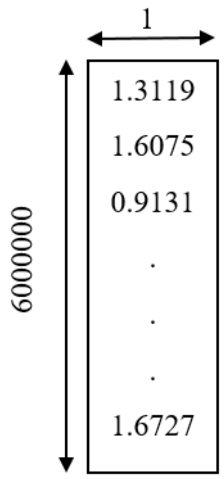

Figure 3.3: Vector matrix in the form of magnitude

Figure 3.3 shows the vector matrix in the form of magnitude after conversion from the complex number. This is done for all the signals. There are 1000 sample signals generated for QPSK, 16-QAM and 64-QAM signals respectively, Thus, there are a total of 3000 sample signals combined.

3.4 Asynchronous Delay Tap Sampling (ADTS)

The signal generated in section 3.3 will undergo ADTS. The signal magnitudes are sampled asynchronously in pairs to obtain a pair of values (x_i , y_i) as shown in Figure 3.4.

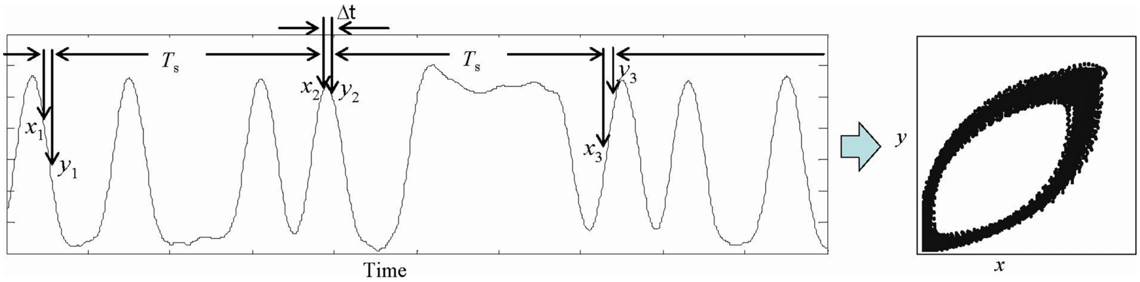

Figure 3.4 ADTS and ADTP

The delay between the samples in each pair is Δt, while the sampling period is T_s. After obtaining the sample pairs, a 2D histogram or also known as asynchronous delay tap plot (ADTP) is plotted as shown in Figure 3.5.

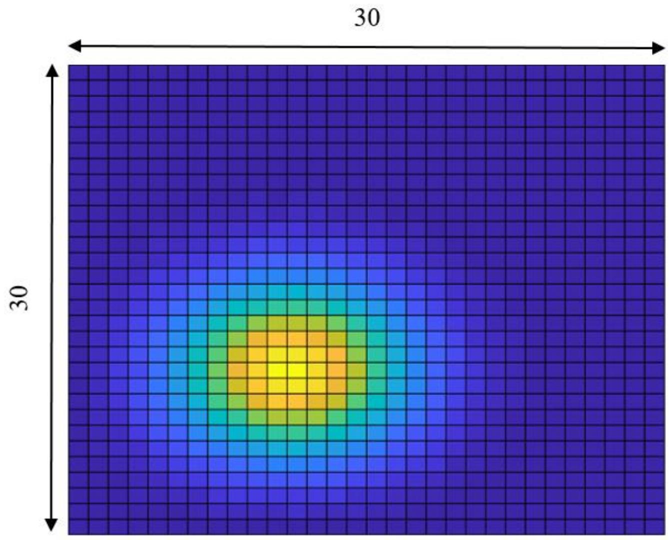

Figure 3.5: ADTP of QPSK signal

The ADTP in Figure 3.5 is 30 x 30 in dimension. In other words, the number of bins chosen for the ADTP is 30 x 30. Each bin contains the number of elements fall into that particular bin. This number is indicated by the colour of the bin. By referring to Figure 3.5, the colour bar shows the number of element in the bin.

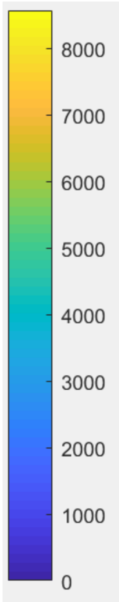

Figure 3.6: Colour bar for the ADTP

Figure 3.6 shows the colour bar used to indicate the number of element in each bin of the ADTP. The yellow colour shows the highest number while blue colour shows the number of zero.

3.5 Modulation Classification

The ADTPs generated in Section 3.4 are used in this section. These ADTPs are first reconstructed before the classification of the modulation format can proceed. Section 3.5.1 explains the reconstruction of the mass amount of data. In Section 3.5.2, the working principal of the support vector machine (SVM) classifier, which is a built-in classifier in MATLAB is explained.

3.5.1 Data Reconstruction

The ADTP generated is represented mathematically in matrix form before it is used for classification in Section 3.5.2. The matrix representation of the ADTP is as shown in Figure 3.7.

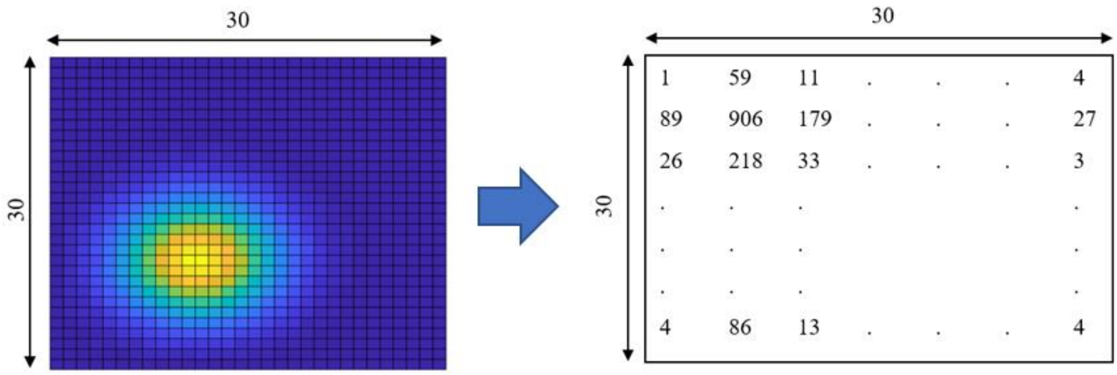

Figure 3.7: Matrix representation of ADTP.

As mentioned, the number of bins used for the generation of ADTP are 30 x 30 in dimension. There is a total of 3000 ADTPs generated and the same amount were converted into matrix form. Thus, there is a total of 3000 matrices. Each of these matrices is then converted into 1 x 900 row matrix as shown in Figure 3.8.

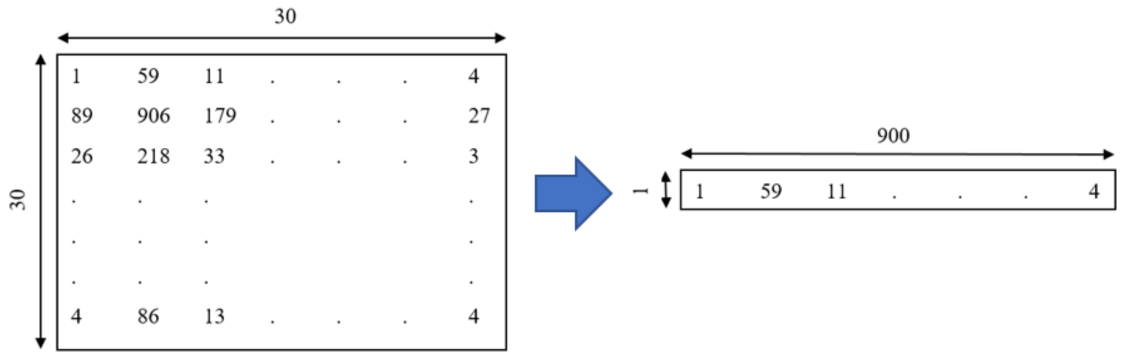

Figure 3.8: Conversion of 30 x 30 matrix into 1 x 900 row matrix

After the conversion, all the 3000 row matrices are combined to form a 3000 x 900 2D matrix. The well-arranged 2D matrix is then randomly permutated. Random permutation is done to ensure that the data with different modulation formats are evenly distributed over the entire matrix.

3.5.2 Classification

Classification is carried out to classify the modulation formats of the signals as well as to determine the accuracy of the classifier. As mentioned, the classifier used as the AMC system is the SVM classifier, a built-in classifier in MATLAB. The resultant matrix from Section 3.5.1 is the input data for the SVM classifier. First, the data is divided in a ratio of 7:3, where 70% or 2100 samples of the data are the training data set, while the remaining 30% or 900 samples of the data are the testing data set.

The classifier is trained using the 2100 samples from training data set. After that, the 900 samples from the testing data set are used to determine the accuracy of the trained model. There are 300 samples for each of the QPSK, 16-QAM and 64-QAM signals respectively. The resultant classification accuracy is presented similar to Table 3.1.

Table 3.1: Classification accuracy of the model

		Predicted Class		
		QPSK	**16-QAM**	**64-QAM**
True Class	**QPSK**	*a*	*b*	*c*
	16-QAM	*l*	*m*	*n*
	64-QAM	*x*	*y*	*z*

From Table 3.1, *a*, *m* and *z* are the number of signals correctly classified for QPSK, 16-QAM and 64-QAM respectively. *b* and *c*, *l* and *n* as well as *x* and *y* are the number of signals wrongly classified for QPSK, 16-QAM and 64-QAM respectively. To calculate for the percentage of the correctly classified signals for QPSK, Eq. (3.2) is used.

$$QPSK\_Percentage = \frac{a}{300} \times 100 \tag{3.2}$$

To calculate for the percentage of the correctly classified signals for 16-QAM and 64-QAM, the value *a* is substituted with *m* and *z* respectively. The accuracy of the model or classifier is calculated using Eq. (3.3).

$$Accuracy = \frac{a + m + z}{900} \times 100 \tag{3.3}$$

3.6 Design of Experiment

The DoE method is adopted from [16]. The delay tap, Δt and the sampling period, T_s are taken as the factors that affect the accuracy of the modulation classification. These two values are varied to determine the optimum values that can improve the accuracy of the modulation classification. Since, this experiment involves only 2 factors, hence, a 2^2 factorial design is carried out. In 2^2 factorial design, each of the factors is ran at two levels: low and high levels. Thus, there are four possible combinations for the experiment as shown in Table 3.2.

Table 3.2: Combinations of the factor for DoE

Delay tap, Δt	Sampling period, T_s	Accuracy I	Accuracy II	Accuracy III	Total
-	-	95.6%	97.8%	94.3%	287.7%
-	+	96.7%	98.1%	95.9%	290.7%
+	-	94.2%	96.9%	93.5%	284.6%
+	+	97.5%	98.3%	95.4%	291.2%

From Table 3.2, the low and high levels of the delay tap and the sampling period are denoted by “-” and “+” respectively. The obtained results for each combination are compared to determine which combination give the best outcome or the highest accuracy. All the values in Table 3.2 are examples to better explain Figure 3.9.

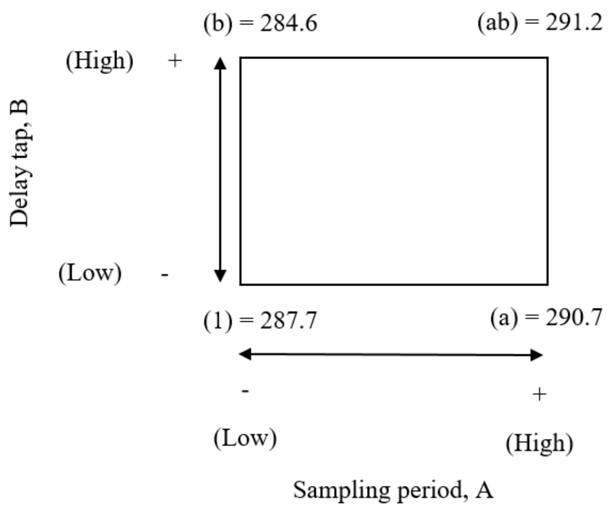

Figure 3.9: Treatment combinations in 2^2 factorial design

Figure 3.9 shows the treatment combinations of the design. In this example, the effect of the sampling period is referred as “A” while the effect of the delay tap is referred as “B”. “AB” refers to the sampling period and delay tap interaction. The high level of any factor in the treatment combination is denoted by the corresponding lowercase letter, while the absence of the other lowercase letter means that the factor is in low level. Hence, *a* represents A at high level and B at low level, *b* represents A at low level and B at high level. When both A and B are at high level, *ab* is used. On the other hand, when both A and B are at low level, (1) is used instead.

In Eq. (3.4), (3.5) and (3.6), n is the number of repeated experiment, where in this case n = 3. The average main effect of A is found from the effect of A at the low level of B, using the Eq. (3.4).

$$A = \frac{1}{2n}\{[ab - b] + [a - (1)]\} \tag{3.4}$$

The average main effect of B is found from the effect of B at the low level of A, using the Eq. (3.5).