BENCHMARK ANALYSIS OF HEAT CONDUCTION PROBLEMS WITH ADAPTIVE FEA CODE POLYDE

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NOMENCLATURE

Abbreviation

FEM	Finite Element Method	DOF	Degree of freedom
PDE	Partial Differential Equation	NDOF	Number of degree of freedom

Symbols

r_o	Outer radius (m)	R_{th}	Thermal resistance (K/W)
r_i	Inner radius (m)	T	Temperature (K)
N_L	Number of long fins	T_{max}	Maximum Temperature (K)
N_M	Number of middle fins	T_{min}	Minimum temperature (K)
L_L	Length of long fins (m)	T_∞	Ambient air temperature (K)
L_M	Middle fins length (m)		
H	Fins height (m)		
t	Thickness (m)		
k	Thermal conductivity (W/mK)		
$\dot{q}$	Heat flux (W/m^2)		
A	Surface area (m^2)		
h	Heat convection coefficient (W/m^2K)		
h_{avg}	Average heat convection coefficient (W/m^{2}K)		

ABSTRAK

FEM ubah suai merupakan topik yang penting dalam bidang penyelidikan. Memandangkan pelbagai jenis pelaksanaan kod penyelidikan telah dihasilkan, hal ini penting untuk menguji algoritma dalam masalah penanda aras supaya penilaian prestasi dapat dilakukan dengan cara yang standard. Kerja ini mengkaji prestasi kod FEM ubah suai PolyDE dalam masalah pemindahan haba mantap tiga dimensi. Simulasi PolyDE telah dijalankan dengan strategi adaptiviti yang berbeza menggunakan polinomial order elemen yang berbeza pada domain geometri permukaan melengkung dan permukaan rata. Strategi adaptiviti yang dikaji ialah h-FEM dan hp-FEM. H-FEM melakukan "mesh refinement" di rantau yang memerlukan mesh halus manakala hp-FEM melibatkan gabungan "mesh refinement" dan penukaran polinomial order elemen untuk meningkatkan ketepatan penyelesaian. Dalam had bilangan darjah kebebasan yang diuji, telah didapati bahawa ralat hp-FEM menyuai lengkung quadratik manakala ralat h-FEM menyuai garisan linear dalam plot logaritma-logaritma berkenaan dengan bilangan darjah kebebasan dan masa CPU (saat). Kebaikan penyuaian dibukti dengan nilai pekali penentuan, R^2 yang melebihi 0.9 untuk semua penyuaian garisan linear dan lengkung quadratik. Kesimpulannya, prestasi hp-FEM dalam kadar penumpuan ralat lebih baik berbanding dengan h-FEM. Perbezaan polynomial order elemen tidak mendatangkan kesan yang ketara dalam kadar penumpuan ralat. PolyDE mempunyai isu dengan pengurusan ingatan. Contohnya, bilangan darjah kebebasan yang diperoleh daripada simulasi hp-FEM dengan polynomial order permulaan satu pada geometri permukaan lengkung terhad pada 28468. Prestasi PolyDE pada geometri permukaan lengkung adalah memuaskan, dengan nilai kadar penumpuan ralat yang tinggi berbanding dengan geometri permukaan rata. Keputusan simulasi sink haba jejarian yang dihasilkan semula di PolyDE adalak konsisten dengan [1]. Kesimpulannya, PolyDE berkaliber untuk aplikasi dalam analisis sink haba LED sebenar. Kajian kes telah ditetapkan untuk menanda aras prestasi PolyDE.

ABSTRACT

Adaptive FEM is a topic of interest in research. As different implementations of research code have been developed, it is crucial to test the algorithms on benchmark problems so assessment of the performance can be done in a standard way. This work studies the performance of adaptive FEM code PolyDE on 3-dimensional steady-state heat transfer problem. PolyDE simulations were run for different adaptivity strategies with elements of different polynomial orders on curved surface and flat surface geometry domain. The adaptivity strategies studied were h-FEM and hp-FEM. H-FEM involves refining the mesh at the region where finer elements are required while hp-FEM involves combination of refining the mesh and changing the polynomial orders of the elements to improve the accuracy of the solution. Within the limitation of the number of degree of freedom tested, it was found that error for hp-FEM fits quadratic curves while the error for h-FEM fits linear line in logarithmic-logarithmic plot with respect to number of degree of freedom (NDOF) and CPU time (sec). The goodness of fits was proven with coefficient of determination, R^2, which shows value above 0.9 for all the fitted linear lines and quadratic curves. Hence, it was concluded that hp-FEM performs better than h-FEM in faster error convergence. Difference in polynomial order of elements has no significant effect on error convergence rates. PolyDE has issue with memory management. For example, NDOF obtained is limited at 28468 for simulation with hp-FEM starting polynomial order 1 on the curved surface geometry. PolyDE's performance on curved surface domain geometry is acceptable, with the simulation results for curve surface geometry showing higher error convergence rate than the flat surface geometry. The radial heat sink simulation results replicated in PolyDE is in agreement with [1]. It is concluded that PolyDE is reliable for application in actual LED heat sink analysis. The case studies have been established for benchmarking the performance of PolyDE.

Benchmark Analysis of Heat Conduction Problems with Adaptive FEA Code PolyDE

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Keywords: Heat sink, h-FEM, hp-FEM, curved surface geometry, PolyDE

1. INTRODUCTION

The finite element method (FEM) is a numerical method for solving engineering and mathematical physics problem. FEM subdivides the geometry domain into 'finite elements' and computes the approximate solution for the weak form of the PDE using, among other methods, variational methods by minimizing the error norm [2]. FEM is important and practical because most of the real-world engineering problems involve complicated geometries, loadings and material properties in which there is no analytical solution. The application of FEM was originally in field of structural engineering, such as the published work by Hrennikoff [3] in 1941 and McHenry [4] in 1943 [5]. Wilson and Nickel solved heat conduction problem in 1966, marking the beginning of FEM application in non-structural field [6].

Adaptive FEM is a version of FEM which incorporates automatic mesh refinement in the algorithm for faster error convergence. The adaptive FEM was introduced by Babuska and Theinbolt in the late 1970s [7, 8]. The refinement methods include h-refinement, p-refinement, and hp-refinement. H-refinement involves refining the mesh at the region where finer elements are required such as at high singularity region; p-refinement is such that the mesh element polynomial order is changed without changing the mesh element size; while hp-refinement combines h-refinement and p-refinement to optimize the advantage of both [2].

Examples of adaptive finite element libraries available as open source include Alberta [9], DealII [10], FEniCS [11], FETK [12], Hermes [13], libMesh [14], Phaml [15], and 2dhp90 [16]. It is not straightforward to assess the efficiency of an adaptive FEM algorithm. To characterize the efficiencies of different algorithms, a common approach is by solving benchmark problems. Examples of benchmark problems are the suit of 12 benchmark problems for adaptive FEA collected by Dr. William Mitchell (NIST) [17]. The work by Zhonghua Ma et al [18] solved the benchmark problems in [17] using HERMES [13]. The results shows that hp-FEM converge the fastest with respect to number of degree of freedom (NDOF) and CPU time.

In this project, the goal is to establish case studies for benchmarking the performance of adaptive FEM code like PolyDE. The first objective is to construct a heat sink with a circular base and cylindrical fins, hereinafter known as cylindrical heat sink, and a heat sink with rectangular base and rectangular fins, hereinafter known as rectangular heat sink for simulation in PolyDE. Error convergences are compared between different polynomials orders of element and also compared between h-FEM and hp-FEM. A concern on FEM is the distortion on curved surface geometry, therefore performance of PolyDE on curved surface and flat surface geometry domain are compared [19]. The next objective is to assess the reliability of results with PolyDE. Numerical simulation of radial heat sink from the work by S.H Yu et al [1] is replicated in PolyDE for results comparison and analysis. The last objective is to simulate a radial heat sink to model the actual application of heat sink for 50mm diameter 2W LED module.

2. LITERATURE REVIEW

The adaptive FEM methods are h-refinement, p-refinement, and hp-refinement. The hp-refinement is known to be the most optimal method. It is an active area in research, with different algorithms being researched on. In the work by Zienkiewicz et al. in 1989, the procedure is by implementing h-refinement with lower order elements such as linear or quadratic element, to obtain certain percentage accuracy specified. It is then followed by p-refinement which further reduces the error and improves the accuracy [2, 20].

A concern on FEM is the distortion on curved surface geometry domain. The curved surface geometry is approximated by edges of the collections of elements, such as piecewise straight lines or flat surface if linear elements are used; the original surface will not be fully recovered [19]. This affects the solution accuracy greatly in fields where solution is sensitive to the geometry such as structural analysis. Therefore, the recovery of curved surface is of importance.

One of the widely used methods is the plane to surface transformation method used by O.C. Zinekiewicz and D. V. Phillips [21]. The mesh is first generated in a two-dimensional parent domain before mapped onto the curved surface. T.S Lau and S.H. Lo proposed a scheme for automatic generation of unstructured triangular meshes of arbitrary density distribution over curved surface [22]. The elements are generated directly on the curved surface using the advancing front technique.

One of the objectives in this project is to study the performance of PolyDE on curved surface. Simulation is run for rectangular heat sink as a control. The simulation results on cylindrical heat sink is compared to the rectangular heat sink. If the error convergence rate of cylindrical heat sink simulations results is similar or higher than the rectangular heat sink, PolyDE is considered to perform well on curved surface domain geometry.

3. METHODOLOGY

For 3D steady-state heat conduction with no material heat generation, the governing equation is:

$$\frac{\partial^2 \mathrm{T}}{\partial^2 \mathrm{x}} + \frac{\partial^2 \mathrm{T}}{\partial^2 \mathrm{y}} + \frac{\partial^2 \mathrm{T}}{\partial^2 \mathrm{z}} = 0$$

The settings for PolyDE are written in “FEMsettings.txt”. The type of linear solver used is unsymmetric multifrontal (UMF) method solver. The UMF method is common as finite element solver, which also available in software like ANSYS and ABAQUS. Table 3.1 shows the description of the main settings.

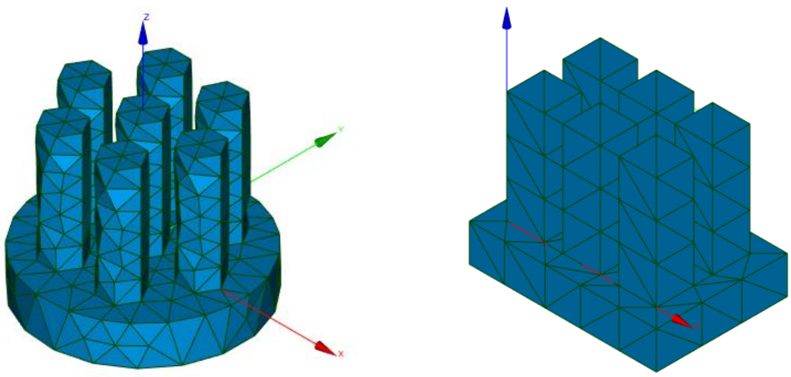
Table 3.1: Main settings in PolyDE

Parameter	Description
ADAPTION_TYPE	To define types of adaption strategy
ADAPT_STEPS	To specify number of adapt steps
HP_ALGORITH	To specify the percentage of elements to be refined, for example, TOP5 specify that top 5% elements with the highest error are to be refined.
LINSOLVERTYPE	To define the type of linear solver, such as UMF
POLYORDER	To define the starting polynomial order

The meshes are built from Salome using tetrahedral elements. The simulation is run in PolyDE. Temperature plots are obtained from Paraview.

3.1 Computational Performance of PolyDE

A cylindrical heat sink with 1761 starting elements and rectangular heat sink with 431 starting elements are constructed in Salome. Figure 3.1 shows cylindrical heat sink and rectangular heat sink geometry domain.



(a) (b)

Figure 3.1: Mesh prepared in Salome for (a) cylindrical heat sink and (b) rectangular heat sink

Performance of PolyDE on different adaption methods is studied. The simulations are run with h-adaption polynomial order of 1, 2, 3, hereinafter known as h1, h2, and h3-adaption, and with hp-adaption starting polynomial order of 1 and 2, hereinafter known as hp1 and hp2-adaption. The simulations are run for maximum number of adaption steps obtained before the program crashes. The relative error in energy norm for the solution, hereinafter known as error, obtained from a posteriori error estimations technique built-in in PolyDE is plotted against NDOF and CPU Time (seconds) [23]. The error convergence rates are compared between different polynomials orders of elements, and also compared between h-adaption and hp-adaption.

Performance of PolyDE on curved surface domain is studied. Simulation is run for rectangular heat sink as a control. The simulation results on cylindrical heat sink is compared to the rectangular heat sink to analyse performance of PolyDE on curved surface domain.

The material assigned to the heat sinks is steel with thermal conductivity, $k=50.2$ W/mK . The heat sinks bases are assigned with Dirichlet boundary condition $T=373K$ and the rest of the surface are assigned with Robin boundary conditions with heat convection coefficient, h=10W/m^2K and ambient air temperature, T_∞ = 293K. Figure 3.2 shows the boundary conditions for cylindrical heat sink and rectangular heat sink.

```
!-------------------------------------------!
! boundary conditions.txt                   !
!       - Boundary Conditions for Polyde    !
!-------------------------------------------!
 200 pvalue= 2930. , 0. qvalue= -10., 0. , Robin
 ! h=10, Tair=293K, p=h*Tair, q=-h

 0 pvalue= 373.   , 0. qvalue=0., 0. ,     T100C
```

Figure 3.2: Boundary conditions for cylindrical heat sink and rectangular heat sink

Robin boundary condition for convection is often used in practice. The use of Robin boundary condition usually cause difficulty to linear solver since the system matrix is non-symmetric. This alters the efficiency of solutions.

3.2 Assessment of Reliability of Results with PolyDE

To compare PolyDE simulation with the results from the work of S.H. Yu et al [1], PolyDE simulation is done for aluminum radial heat sink with middle fins length L_M of 0.005m, 0.0025m, and 0.0045m are constructed. Only a single set of fins is simulated because of the symmetrical characteristics in geometry. The simulation is run with h2-adaption. The parameters and dimensions are shown in Table 3.2 and Figure 3.3.

Table 3.2: Parameters for radial heat sink simulation [1]

Parameters	
Number of long fins, N_L	20
Number of middle fins, N_M	20
Outer radius, r_o *(m)*	0.075
Inner radius, r_i *(m)*	0.010
Length of long fins, L_L *(m)*	0.055
Length of middle fins, L_M *(m)*	0.005, 0.0025, 0.0045
Fins height, *H (m)*	0.0213
Thickness, *t (m)*	0.002
Thermal conductivity, *k (W/mK)*	202.4
Heat flux input, $\dot{q}$ *(W/m^2)*	700
Ambient air temperature, T_∞ *(K)*	303.15
Heat convection coefficient, *h (W/m^2K)*	5.50, 4.85, 3.90

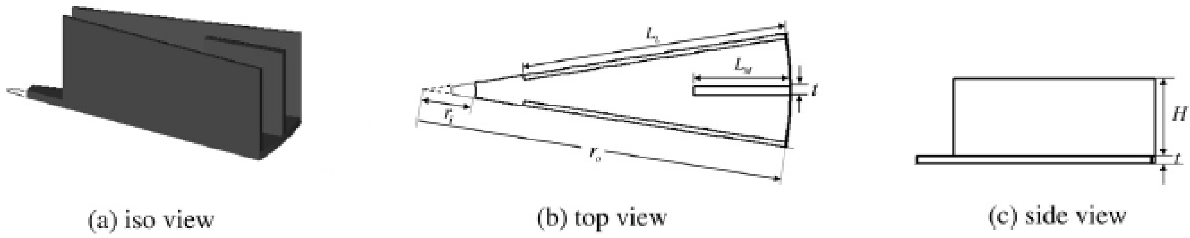


Figure 3.3: Dimensions of radial heat sink [1]

The boundary conditions are listed as:

(a) Heat sink base: constant heat flux, $\dot{q} = -k\frac{\partial T_s}{\partial n}|_{heat\ sink\ base}$ =*700W/*m^2

(b) Periodic interface: symmetric condition, $\frac{\partial T_s}{\partial n}|_{sectional\ wall} = 0$

(c) Inner curved surface, $\frac{\partial T_s}{\partial n}|_{sectional\ wall} = 0$

(d) Remaining surface: robin boundary condition, $hT_s + k\frac{\partial T_s}{\partial n}|_{wall} = hT_\infty$

The study in [1] uses computational fluid dynamics simulation where the average heat convection coefficient, $\mathrm{h_{avg}}$ is derived from the simulation results. In PolyDE, heat convection coefficient is assigned to the surfaces with the same h_{avg} value obtained from Figure 8 of [1]. Figure 3.4 shows the boundary conditions for radial heat sink with middle fins length of 0.005m.

```
!-----------------------------------------!
! boundary conditions.txt                 !
!       - Boundary Conditions for Polyde  !
!-----------------------------------------!
 200 pvalue= 1667.33 , 0. qvalue= -5.50, 0. , Robin
 ! h=5.50, Tair=303.15K, p=h*Tair, q=-h

 200 pvalue= 700. , 0. qvalue= 0., 0. ,       Neumann
 ! q/A=700
```

Figure 3.4 Boundary conditions for radial heat sink with middle fins length, L_M of 0.005m,

3.3 Computation of Radial Heat Sink for 2W LED Module

To model the actual heat sink application for 50mm diameter 2W LED module, the radial heat sink for middle fins length, L_M of 0.025m is downscaled by factor of 3, such that the diameter is 0.050m. The downscaled radial heat sink is simulated with heat input of 2W. The parameters and dimensions are shown in Table 3.3.

Table 3.3: Parameters for downscaled radial heat sink simulation

Parameters	
Number of long fins, N_L	20
Number of middle fins, N_M	20
Outer radius, r_o *(m)*	0.0250
Inner radius, r_i *(m)*	0.0033
Length of long fins, L_L *(m)*	0.0183
Length of middle fins, L_M *(m)*	0.0083
Fins height, *H (m)*	0.0071
Thickness, *t (m)*	0.00067
Thermal conductivity, *k (W/mK)*	202.4
Heat flux input, $\dot{q}$ (W/m^2)	1037.03
Ambient air temperature, T_∞ *(K)*	303.15
Heat convection coefficient, *h* (W/m^2K)	4.85

All boundary conditions are maintained the same as the original model, with exception of heat flux input. Figure 3.5 shows the boundary conditions for downscaled radial heat sink.

```
!--------------------------------------------!
! boundary conditions.txt                    !
!       - Boundary Conditions for Polyde     !
!--------------------------------------------!
 200 pvalue= 1470.28 , 0. qvalue= -4.85, 0. , Robin
 ! h=4.85, Tamb=303.15K, p=huamb, q=-h

 200 pvalue= 1037.03 , 0. qvalue= 0., 0. ,    Neumann
 ! q/A = 2/0.00192859 = 1037.03
```

Figure 3.5 Boundary conditions for downscaled radial heat sink

4. RESULTS AND DISCUSSION

For computational performance analysis of PolyDE, simulation is done on cylindrical and rectangular heat sink to study the error convergence rates. For the assessment of reliability of results with PolyDE, the thermal resistance of radial heat sink, *Rth* from PolyDE and the work by S.H. Yu et al [1] are compared. Radial heat sink for 2W LED module is simulated to model actual LED application. Table 4.1 shows the abbreviation used for different adaption strategy.

Table 4.1: Description for abbreviation used for different adaption strategy

Adaption Strategy	Description
h1	h-adaption with polynomial order 1
h2	h-adaption with polynomial order 2
h3	h-adaption with polynomial order 3
hp1	hp-adaption with starting polynomial order 1
hp2	hp-adaption with starting polynomial order 2

4.1 Computational Performance of PolyDE

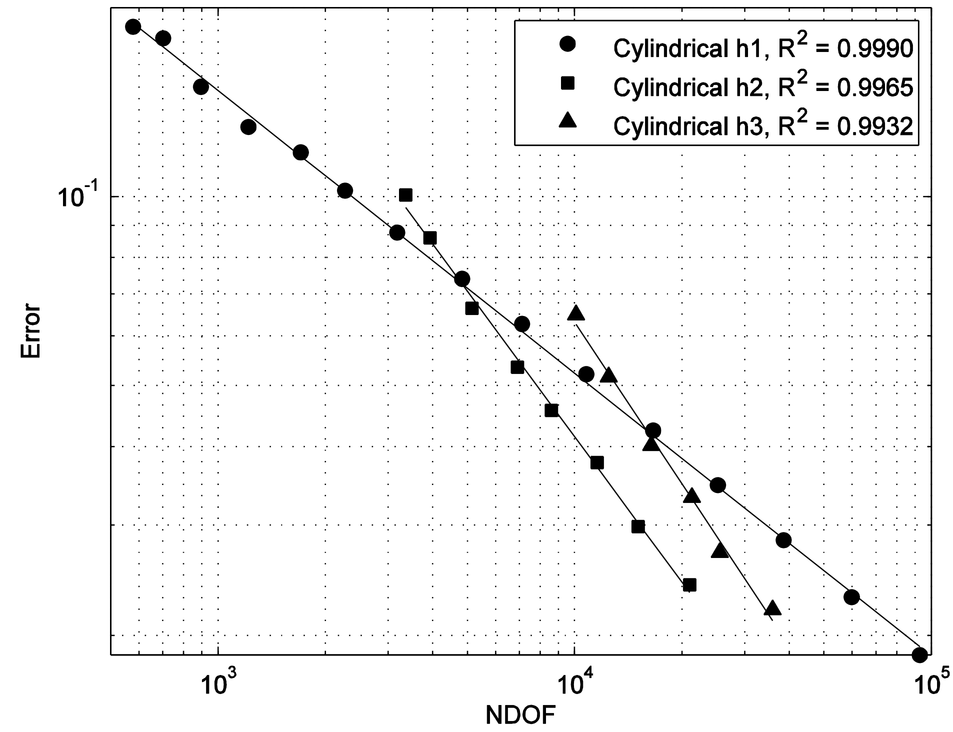
For h-adaption, $log_{10}Error$ is expected to decrease almost linearly with $log_{10}NDOF$ and $log_{10}CPU\ Time$, linear regression lines are fitted for the logarithmic-logarithmic plots. The gradient of the fitted lines are shown in Table 4.2 and Table 4.3. For hp-adaption, second order polynomial curves are fitted. Table 4.4 and Table 4.5 show the fitted curves equation while Table 4.6 and Table 4.7 show the gradient expression for the fitted curves.

Figure 4.1 shows that, for cylindrical heat sink, the error convergence rates with respect to NDOF of h-adaptions in descending order are h3, h2 and h1-adaption. The error convergence rates with respect to CPU Time in descending order are h2, h3, and h1-adaption.

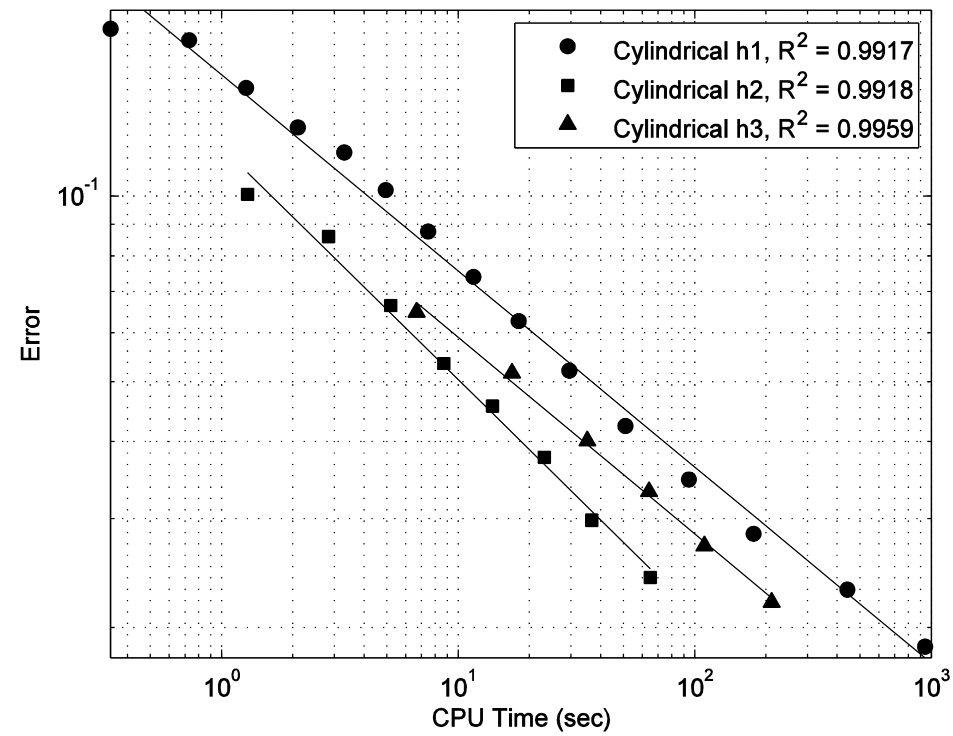
Figure 4.2 shows that for cylindrical heat sink, the error convergence rate with respect to NDOF for hp2-adaption is almost linear which is not expected. It is likely due to insufficient data set since only 4 data points are acquired for hp2-adaption. This is because PolyDE running out of memory at higher levels of adaption steps with hp2-adaption. The error convergence rates with respect to CPU Time for hp1 and hp2-adaption shows almost similar curvature for cylindrical heat sink, with gradient expression “$-0.0760\text{x} - 0.2351$” and "$-0.0760\text{x} - 0.1557$" respectively.

Figure 4.3 shows that for rectangular heat sink, the error convergence rate with respect to NDOF is the highest for h2, followed by h3, and then h1-adaption while the error convergence rates with respect to CPU Time is the highest for h2, followed h1, and then h3-adaption.

From Figure 4.4, for rectangular heat sink, the error convergence rate with respect to NDOF for hp2 is more curved than hp1-adaption. The gradient expression for hp1 and hp2-adaption are “$-0.3244\text{x} + 0.6836$” and “$-0.5108\text{x} - 1.3567$” respectively. The error convergence rate with respect to CPU Time for hp2 is also more curved than hp1-adaption.

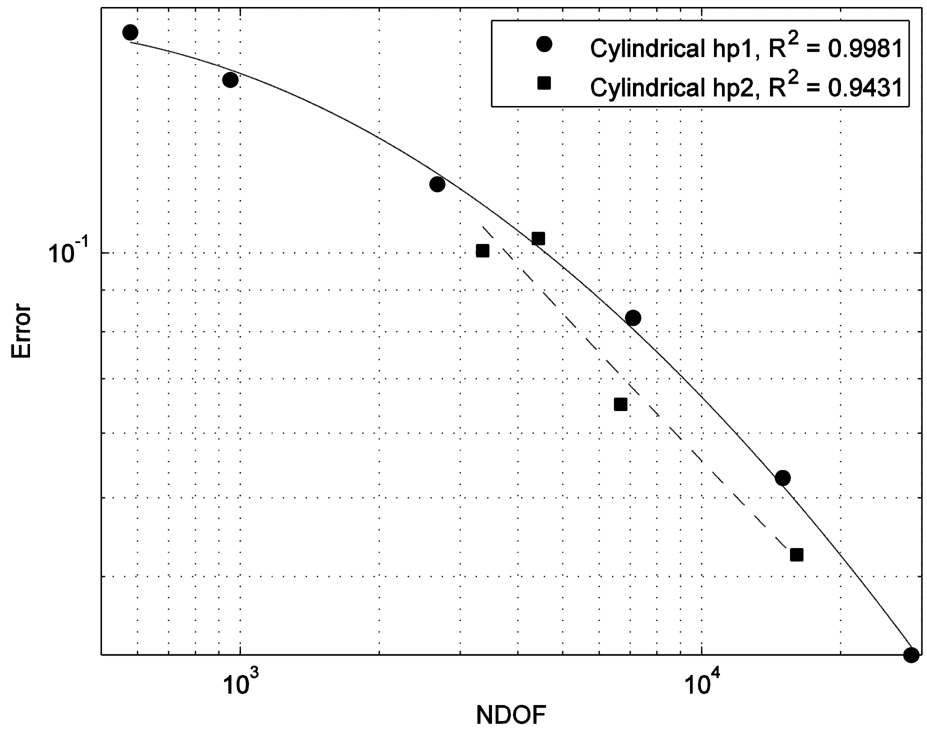


(a)



(b)

Figure 4.1: Error convergence rate regression for cylindrical heat sink for h1, hp2, and h3-adaption with respect to (a) NDOF and (b) CPU Time (sec)



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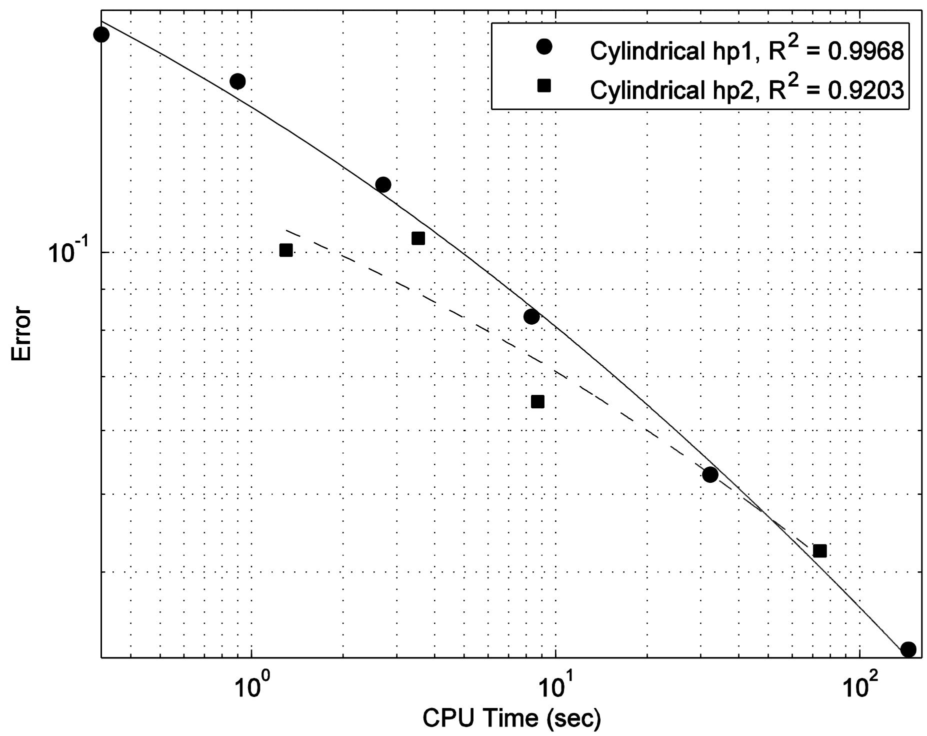
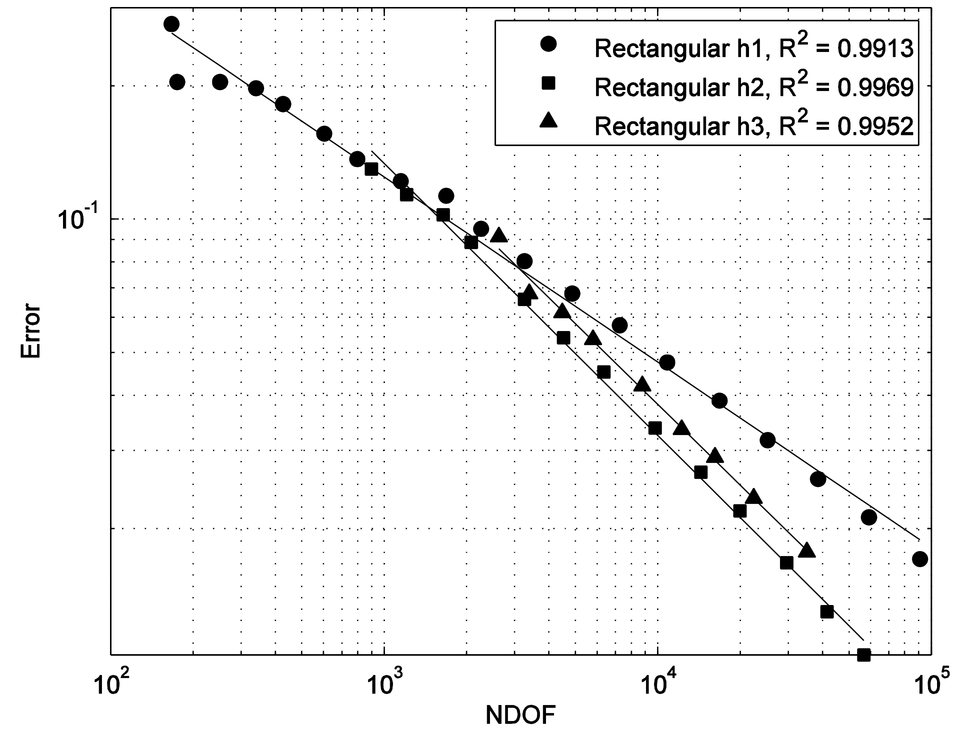
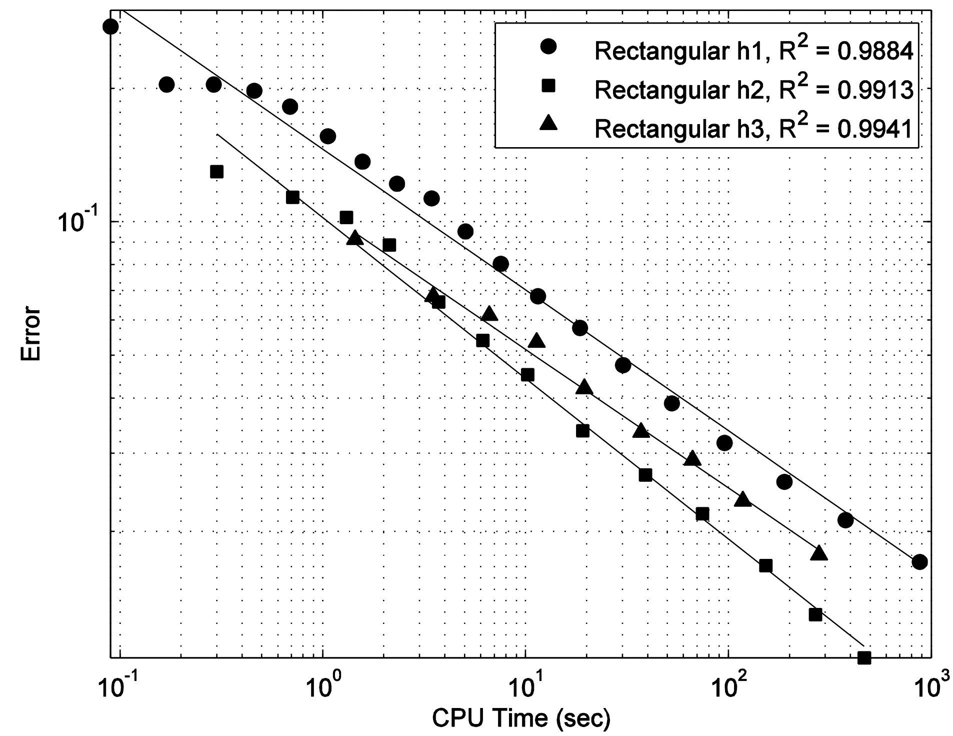


(b)

Figure 4.2: Error convergence rate regression for cylindrical heat sink for hp1 and hp2-adaption with respect to (a) NDOF and (b) CPU Time (sec)



(a)



(b)

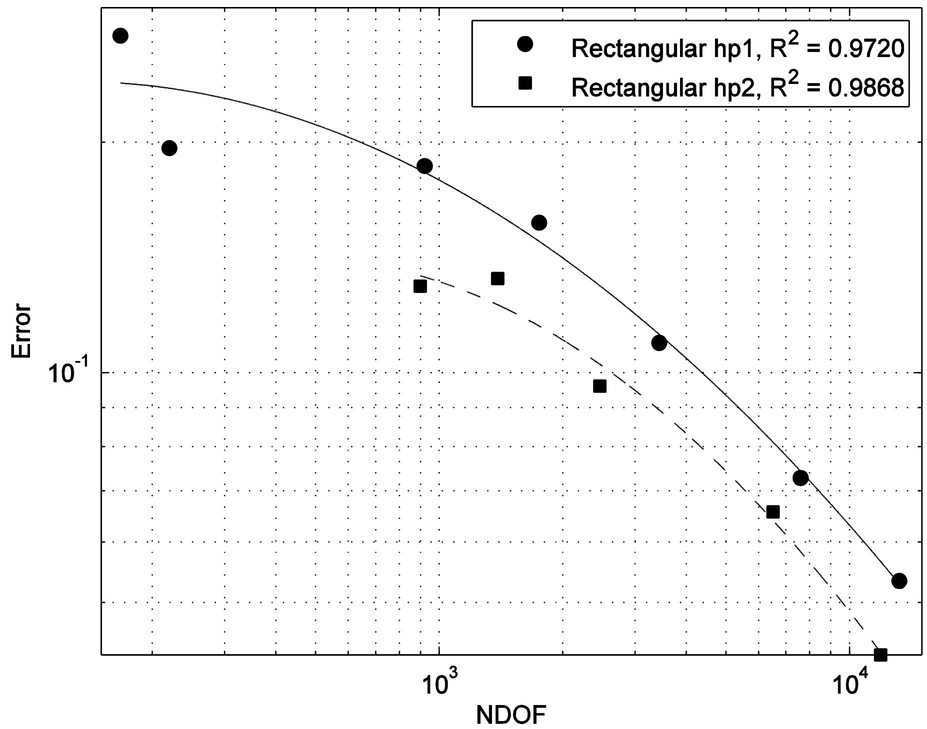
Figure 4.3: Error convergence rate regression for rectangular heat sink for h1, h2, and h3-adaption with respect to (a) NDOF and (b) CPU Time (sec)

The gradient expression for hp1 and hp2-adaption are “$-0.0682x - 0.2217$" and “$-0.1628x - 0.1229$" respectively.

Comparing element of different polynomial order, the error convergence rates with respect to both NDOF and CPU Time do not show consistent observations. The increase in element polynomial order reduces the error but increases the NDOF. Therefore, the change in polynomial order of the starting mesh does not affect the error convergence rate significantly.

Comparing h-adaption and hp-adaption, hp-adaption converges faster than h-adaption. From Figure 4.1 and Figure 4.3, errors for h-adaption fits linear lines while from Figure 4.2 and Figure 4.4, errors for hp-adaption fits quadratic curves, hence the gradient for hp-adaption is increasing logarithmically with NDOF. The goodness of fits for the fitted lines and curves are proved with high coefficient of determination, R^2 values, which are all above 0.9, as shown in the figures. Hp-adaption has better error convergence as it optimize the advantages of h-adaption, the element size refinement and p-adaption, the element polynomial order refinement [2].

From Table 4.2, Table 4.3, Table 4.6 and Table 4.7, cylindrical heat sink shows higher error convergence gradient than rectangular heat sink for h1, h2, h3, and hp1-adaption, with exception of hp2-adaption. As mentioned previously, the fitted curve for cylindrical heat sink hp2- adaption is not able to fully capture the trend to show meaningful information. Higher error convergence rates in cylindrical heat sink shows that PolyDE performs well on curved surface geometry domain. Figure 4.5 shows the error convergence rate for h2-adaption, with cylindrical heat sink having higher error convergence rate than rectangular heat sink.



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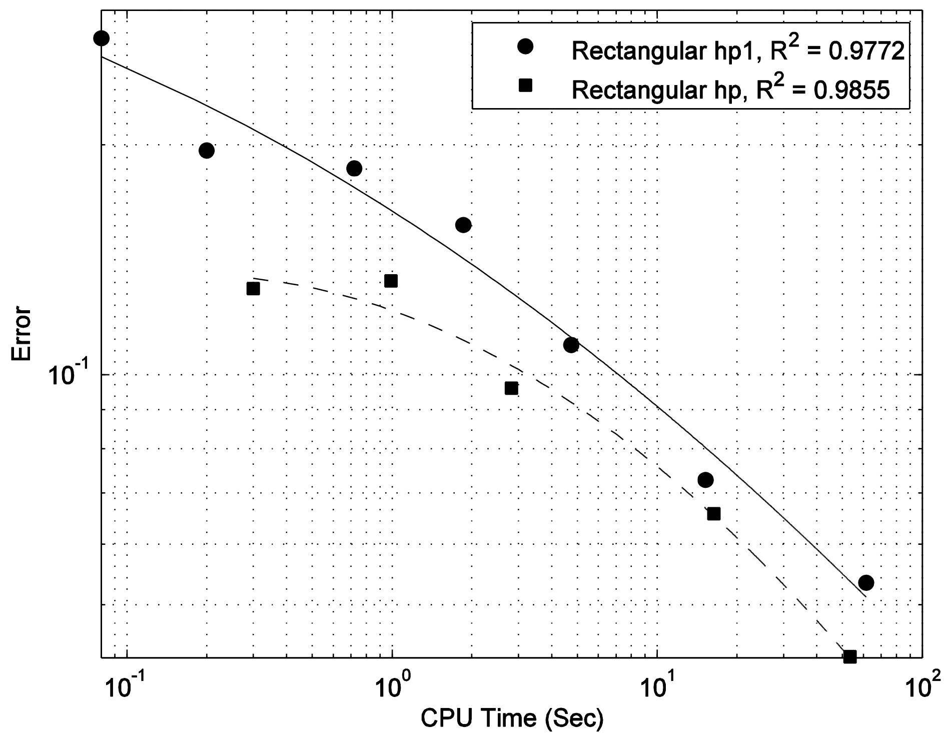
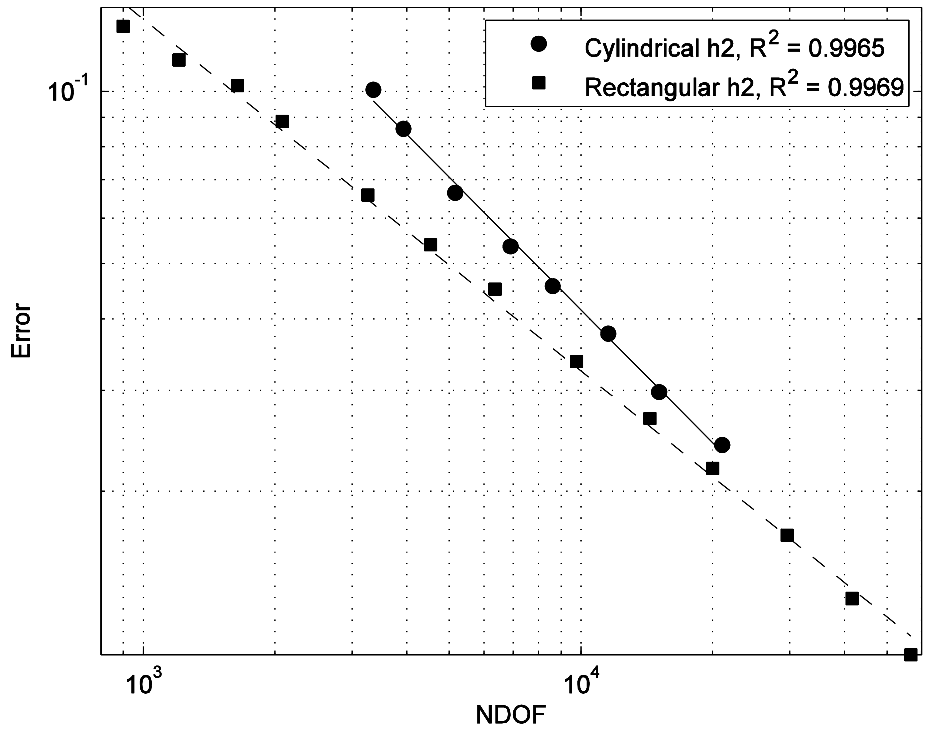


(b)

Figure 4.4: Error convergence rate regression for rectangular heat sink for hp1 and hp2-adaption with respect to (a) NDOF and (b) CPU Time (sec)



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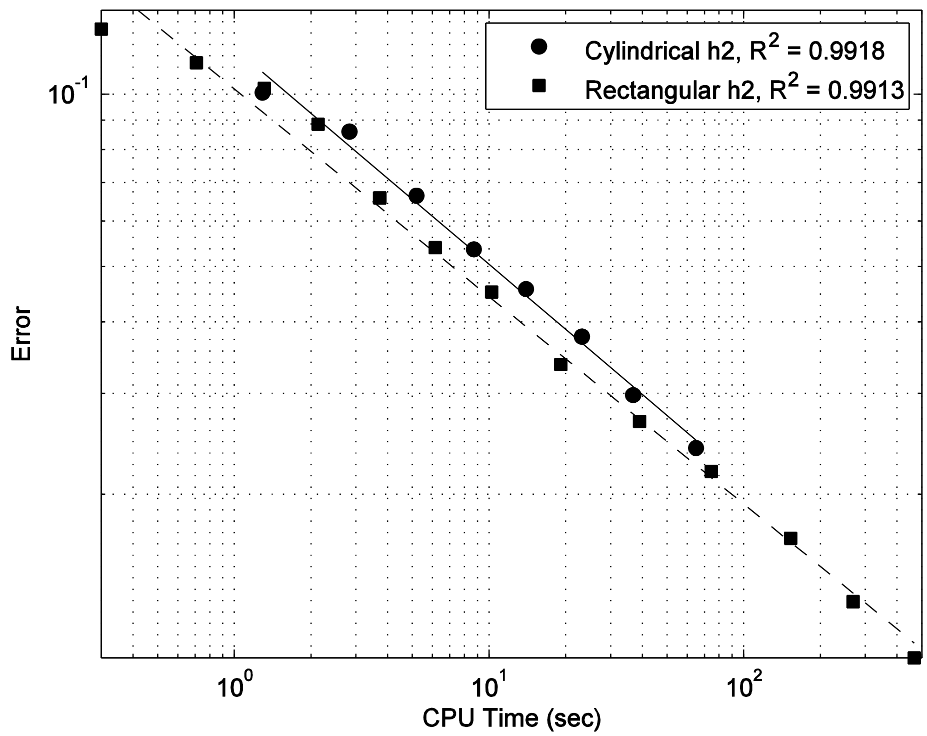


(b)

Figure 4.5: Error convergence rate regression for h2-adaption for cylindrical heat sink and rectangular heat sink with respect to (a) NDOF and (b) CPU Time (sec)

Table 4.2: Regression gradient value for $log_{10}Error\ \ Vs\ \ log_{10}NDOF$ for h1, h2, h3-adaption

Adaption	Gradient		
	h1	h2	h3
Cylindrical Heat Sink	-0.4504	-0.7705	-0.8563
Rectangular Heat Sink	-0.4182	-0.6144	-0.6050

Table 4.3: Regression gradient value *for* $log_{10}Error\ \ Vs\ \ log_{10}CPU\ Time$ for h1, h2, h3-adaption

Adaption	Gradient		
	h1	h2	h3
Cylindrical Heat Sink	-0.3179	-0.3773	-0.3188
Rectangular Heat Sink	-0.3179	-0.3624	-0.3127

Table 4.4: Regression equation for $log_{10}Error\ Vs\ log_{10}NDOF$ for hp1 and hp2-adaption

Adaption	Regression equation	
	hp1	hp2
Cylindrical Heat Sink	$y = -0.1912x^2 + 0.9412x - 1.8822$	$y = 0.0283x^2 - 0.8189x + 1.5673$
Rectangular Heat Sink	$y = -0.1622x^2 + 0.6836x - 1.3397$	$y = -0.2554x^2 + 1.3567x - 2.6521$

Table 4.5: Regression equation for $log_{10}Error\ Vs\ log_{10}CPU\ Time$ for hp1 and hp2-adaption

Adaption	Regression equation	
	hp1	hp2
Cylindrical Heat Sink	$y = -0.0380x^2 - 0.2351x - 0.8196$	$y = -0.0380x^2 - 0.1557x - 0.9546$
Rectangular Heat Sink	$y = -0.0341x^2 - 0.2217x - 0.7858$	$y = -0.0814x^2 - 0.1229x - 0.9160$

Table 4.6: Regression gradient for $log_{10}Error\ Vs\ log_{10}NDOF$ for hp1 and hp2-adaption

Adaption	Regression gradient	
	hp1	hp2
Cylindrical Heat Sink	$-0.3824x + 0.9412$	$0.0566x - 0.8189$
Rectangular Heat Sink	$-0.3244x + 0.6836$	$-0.5108x - 1.3567$

Table 4.7: Regression gradient for $log_{10}Error\ Vs\ log_{10}CPU\ Time$ for hp1 and hp2-adaption

Adaption	Regression gradient	
	hp1	hp2
Cylindrical Heat Sink	$-0.0760x - 0.2351$	$-0.0760x - 0.1557$
Rectangular Heat Sink	$-0.0682x - 0.2217$	$-0.1628x - 0.1229$

PolyDE Memory Usage

The convergence results discussed earlier strongly suggest that PolyDE's memory usage management has yet to be optimized. The UMF solver used is known for its high memory consumption. Table 4.8 shows the maximum NDOF and number of elements obtained for different adaption strategies. The maximum NDOF and number of elements obtained are limited, especially for hp2-adaption. Improvement in memory usage management helps to provide more data set for error convergence study.

Table 4.8: Maximum NDOF and number of elements obtained

Adaption	Cylindrical Heat Sink		Rectangular Heat Sink	
	Maximum NDOF obtained	Maximum No. of Elements Obtained	Maximum NDOF obtained	Maximum No. of Elements Obtained
h1	92858	471420	90903	457701
h2	21018	13333	56752	37458
h3	35898	6992	35078	7039
hp1	28468	7139	13236	2244
hp2	16070	3450	11910	1476

Polynomial Order Control Algorithm

For hp-adaption in PolyDE, the polynomial orders of elements are automatically reduced if needed, which is a rare feature in adaptive FEM code [2]. Table 4.9 shows that hp-adaption with element of starting polynomial order 2, having some of the elements reduced to polynomial order 1 after adaption. This feature is beneficial as it helps to reduce unnecessary NDOF and computation time.

Table 4.9: Rectangular heat sink hp2-adaption data log

Adaption Step	NDOF	No. Matrix Entries	Polynomial order			CPU time (sec)	Error
			Min	Max	Mean		
0	899	19841				0.30	0.129652927250940
1	1387	33629	1	3	2	0.99	0.132686538963065
2	2463	80709	1	4	2	2.82	0.095981717331999
3	6511	325623	1	5	2	16.38	0.065739068192339
4	11910	724666	1	6	3	53.39	0.042703370340418

Mesh refinement is such that meshes are refined at region where there is high parameter variation while the mesh is maintained coarser at region with less parameter variation. Such property is known as h-adaptive, and is implemented in h-adaption and hp-adaption. Figure 4.6 shows the mesh density of the heat sink is uniform at every region before adaption. Figure 4.7 and Figure 4.8 show that after h and hp-adaption, the meshes near the heat sink base are finer than the fins regions. There is higher temperature variation near the heat sink base and lesser temperature variation at the fins.

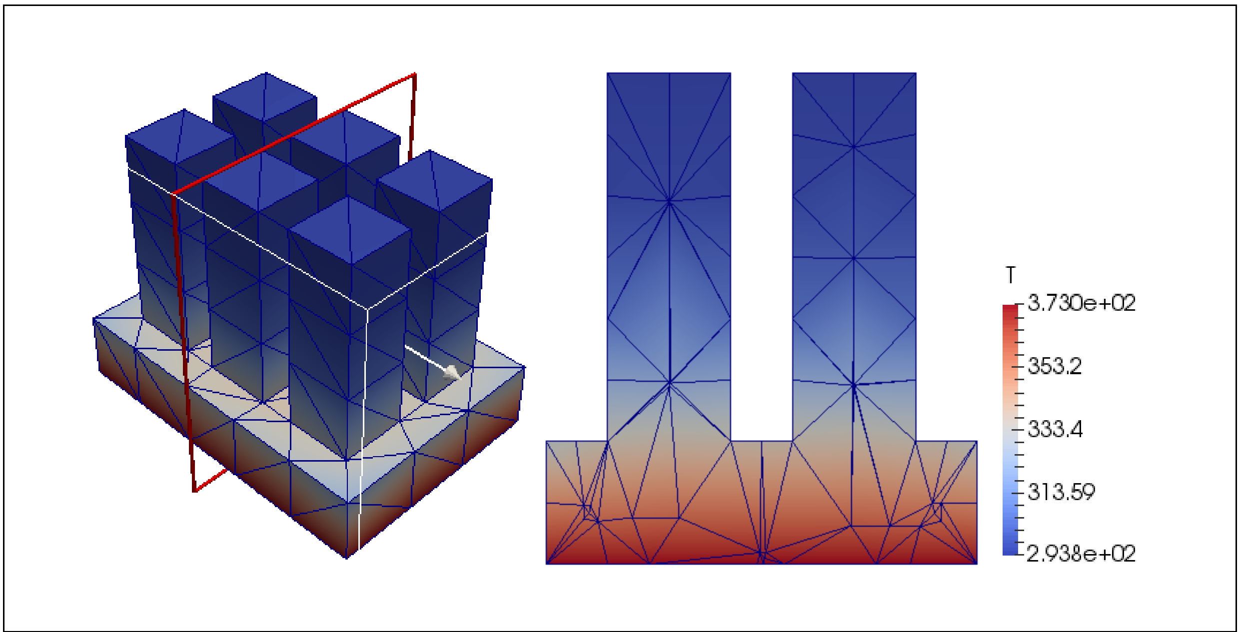


Figure 4.6: Rectangular heat sink mesh before adaption

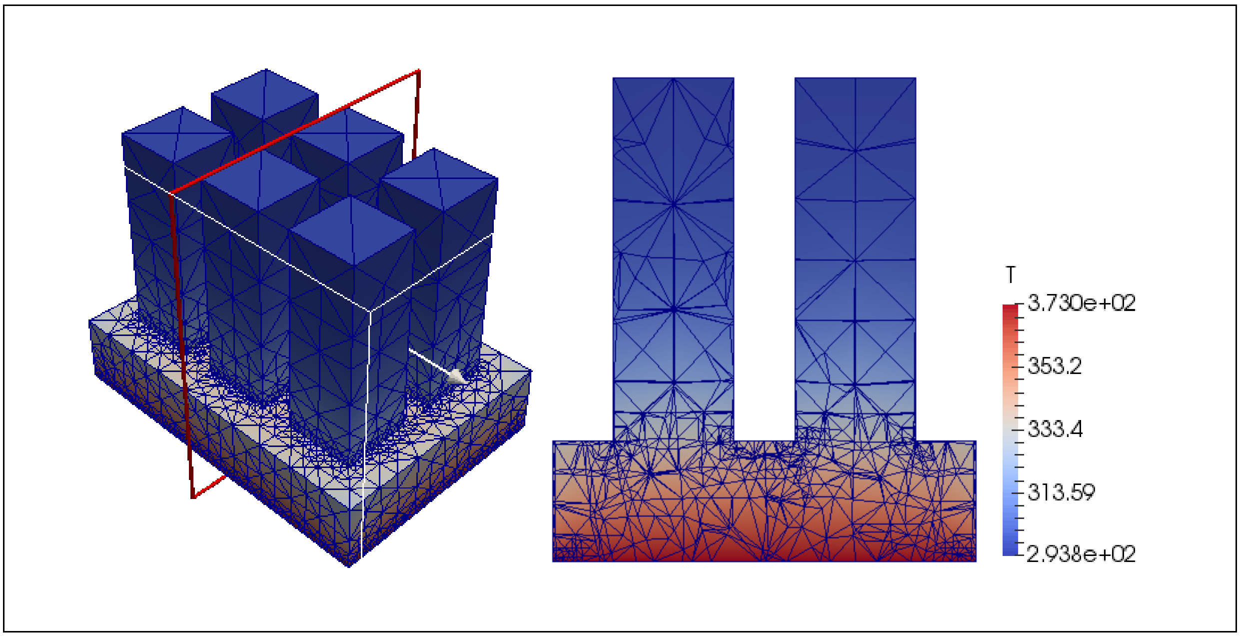


Figure 4.7: Rectangular heat sink mesh after h2-adaption with 56752 NDOF

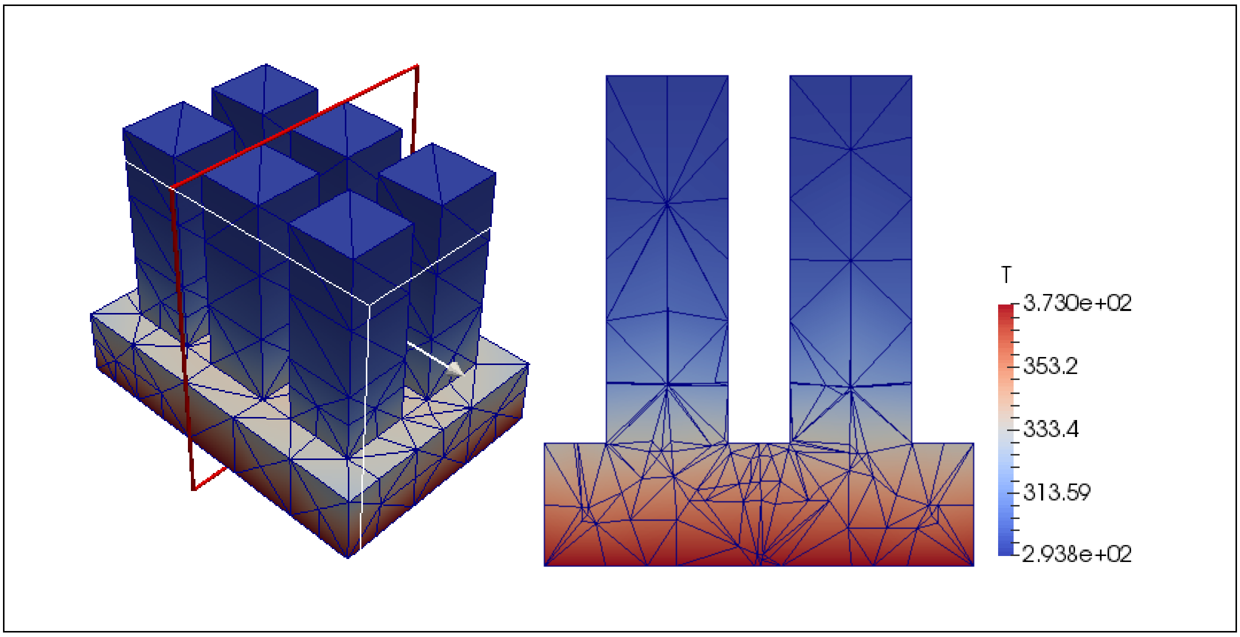


Figure 4.8: Rectangular heat sink mesh after hp2-adaption with 13236 NDOF

4.2 Assessment of Reliability of Results with PolyDE

Contrary to this work, the study in [1] uses computational fluid dynamics simulation, where the average heat convection coefficient, h_{avg} is derived from the simulation results. In

PolyDE, heat convection coefficient is assigned to the surfaces with the same h_{avg} value obtained from Figure 8 of [1]. Therefore, exact results are not to be expected from PolyDE.

Figure 4.9 shows the plot of thermal resistance, R_{th} against middle fin length, L_M from PolyDE simulation and from [1]. As discussed in [1], as the middle fins length increase, the thermal resistance decreases until the middle fins length reaches an optimum length. Upon exceeding the optimum length, the thermal resistance increases as the middle fins length increases. From the interpolation of 3 data points in PolyDE simulation, thermal resistance, R_{th} decreases as middle fins length, L_M increases from 0.005 to 0.025m, and R_{th} increases as L_M increases from 0.025 to 0.045m. The data trend shown in PolyDE simulation is in agreement with [1], such that there is an optimum middle fins length where the thermal resistance, R_{th} is minimum. The percentage deviation of R_{th} of PolyDE simulation from [1] is less than 2%, as shown in Table 4.10. It is concluded that the PolyDE simulation model is acceptable, suggesting that it is reliable for application in actual LED heat sink analysis.

The thermal resistance is given by

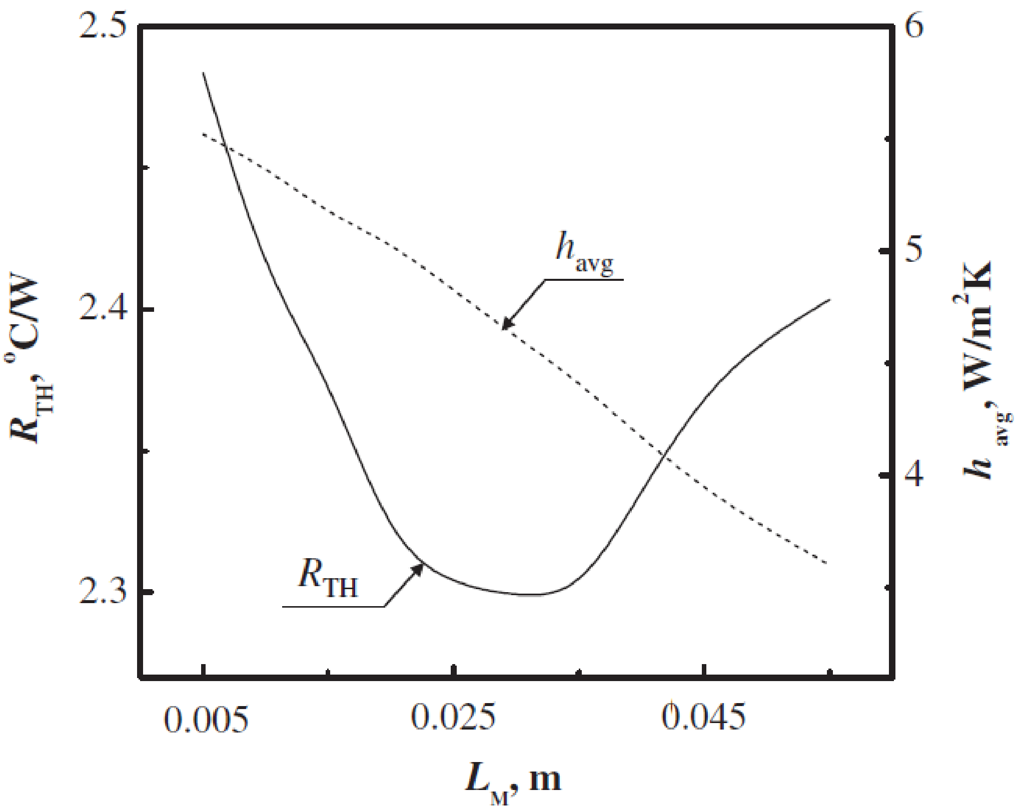
$$R_{th} = \frac{T_{max} - T_{\infty}}{\dot{q}A} \qquad (1)$$

Where $\dot{q}$ is the heat flux input to the heat sink base and A is the surface area of the heat sink base.

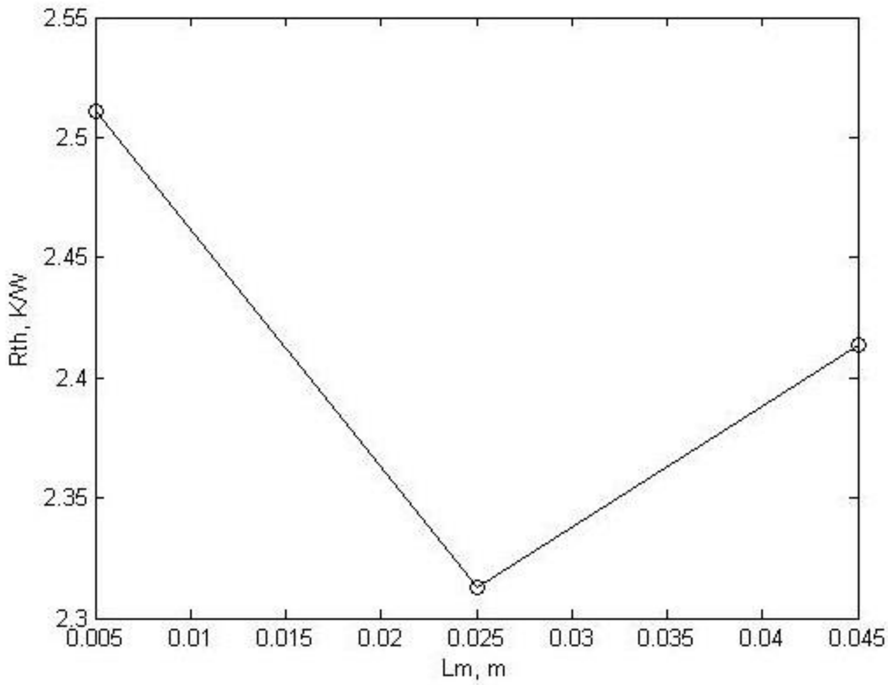
Table 4.10: The effect of middle fins length, L_M on thermal resistance, R_{th} of radial heat sink

L_M (m)	h_{avg} (W/m^2K)	T_{max} (K)	T_{min} (K)	R_{th} PolyDE (K/W)	R_{th} [1] (K/W)	Percentage deviation (%)
0.005	5.50	333.660	333.313	2.51109	2.48	1.25
0.025	4.85	331.252	330.992	2.31290	2.31	0.13
0.045	3.90	332.477	332.210	2.41372	2.37	1.84

Temperature plot for radial heat sink with middle fins length, L_M of 0.005, 0.0025, and 0.005m are shown in Figure 4.10 to Figure 4.12.



(a)



(b)

Figure 4.9: Plot of thermal resistance, R_{th} against middle fin length, L_M from (a) results in [1] and from (b) PolyDE simulation

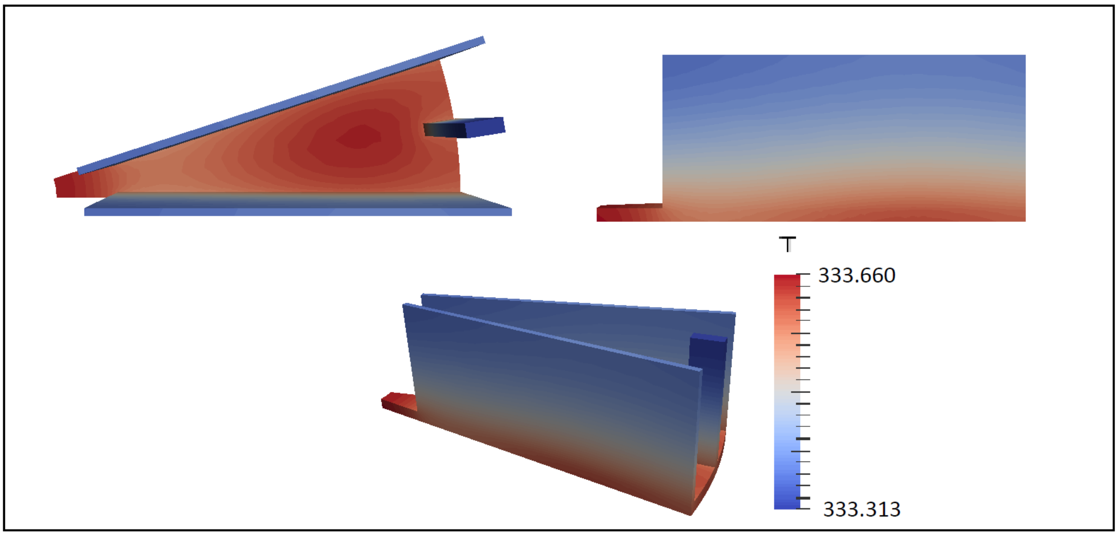


Figure 4.10: Temperature plot for L_M = 0.005m, T_{max} = 333.660 K, T_{min} = 333.313 K

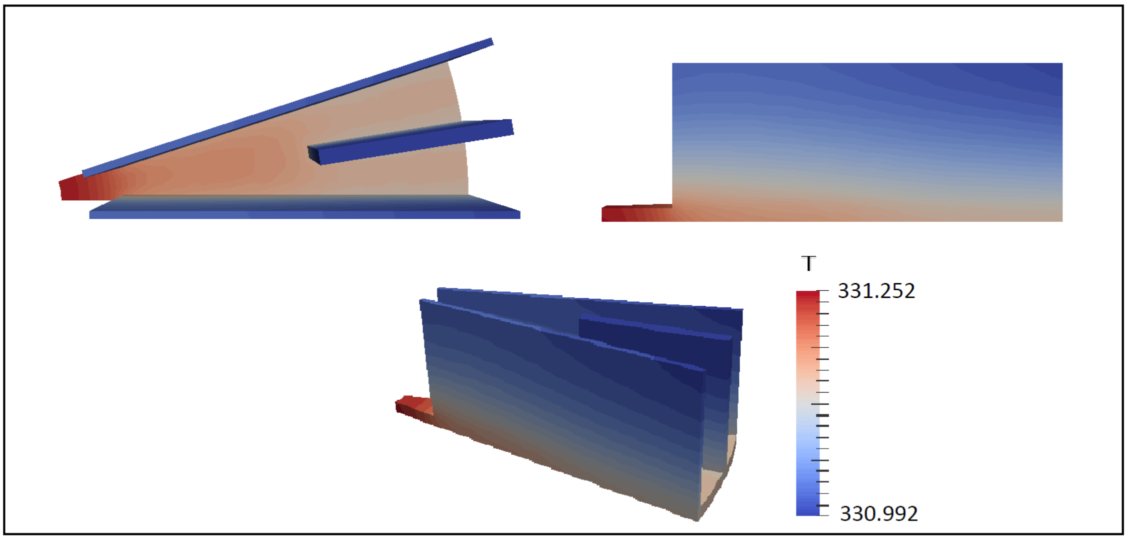


Figure 4.11: Temperature plot for L_M = 0.025m, T_{max} = 331.252 K, T_{min} = 330.992 K

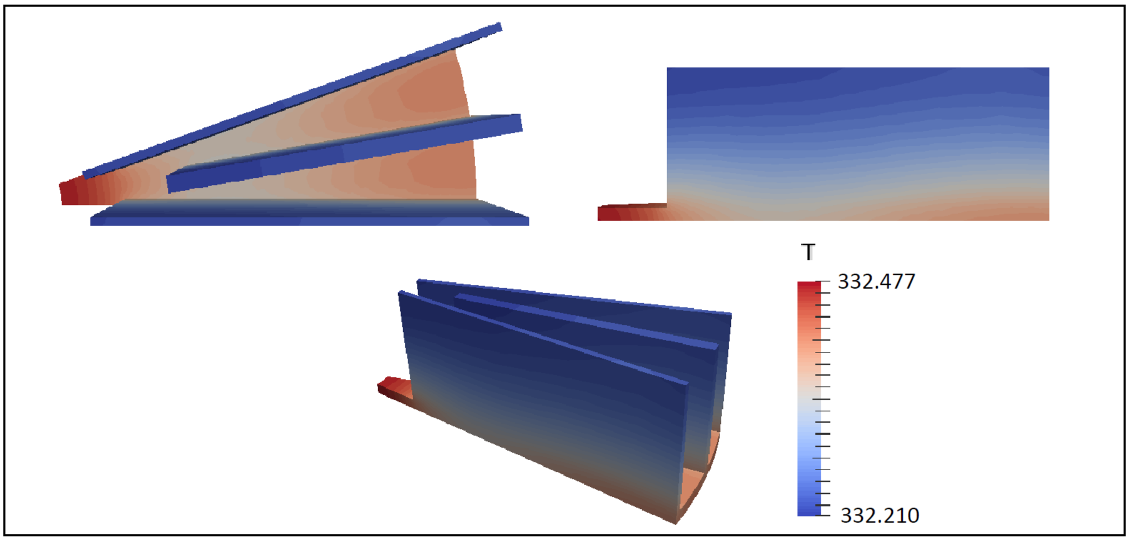


Figure 4.12: Temperature plot for L_M = 0.045m, T_{max} = 332.477 K, T_{min} = 332.210 K

Convergence Testing

Table 4.11 shows that for L_M = 0.005m, at 32332 degree of freedom or 85.69s CPU time, all the parameters T_{max}, T_{min} and R_{th} have converged at 6 significant figure accuracy.

Table 4.11: Convergence testing for middle fins length, L_M = 0.005m, maximum adaption step = 8

Adaption Step	NDOF	No. Matrix Entries	CPU time (sec)	T_{max} (K)	T_{min} (K)	R_{th} (K/W)
0	11272	259798	5.80	333.659	333.314	2.51101
1	11952	276696	11.84	333.659	333.314	2.51101
2	13521	315663	18.84	333.659	333.314	2.51101
3	16085	381563	27.71	333.659	333.314	2.51101
4	20413	493123	41.45	333.659	333.314	2.51101
5	25238	619724	57.76	333.659	333.313	2.51101
6	**32332**	**808042**	**85.69**	**333.660**	**333.313**	**2.51109**
7	43907	1117973	133.76	323.660	333.313	2.51109
8	57462	1482480	210.90	323.660	333.313	2.51109

From Table 4.12, for L_M = 0.025m, at 15715 degree of freedom or 23.32s CPU time, all the parameters T_{max}, T_{min} and R_{th} have converged at 6 significant figure accuracy.

Table 4.12: Convergence testing for middle fins length L_M = 0.025m, maximum adaption step = 7

Adaption Step	NDOF	No. Matrix Entries	CPU time (sec)	T_{max} (K)	T_{min} (K)	R_{th} (K/W)
0	12418	286606	6.74	331.251	330.992	2.31282
1	13560	315450	14.27	331.251	330.992	2.31282
2	**15715**	**370357**	**23.32**	**331.252**	**330.992**	**2.31290**
3	18862	450214	35.05	331.252	330.992	2.31290
4	23985	584433	53.41	331.252	330.992	2.31290
5	30172	748354	80.01	331.252	330.992	2.31290
6	39496	995662	128.69	331.252	330.992	2.31290
7	54404	1396016	210.91	331.252	330.992	2.31290

Table 4.13 shows that for L_M = 0.045m, at 18071 degree of freedom or 38.42s CPU time, all the parameters T_{max}, T_{min} and R_{th} have converged at 6 significant figure accuracy.