COMPARISON OF VARIOUS ALOGRITHMS IN DIFFERENT LOADING OF LOAD FLOW ANALYSIS

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by

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ABSTRAK

Analisis aliran kuasa adalah tulang belakang analisis sistem kuasa dan reka bentuk. Mereka perlu untuk perancangan, operasi, penjadualan ekonomi dan pertukaran kuasa antara utiliti. Maklumat utama analisis aliran kuasa adalah untuk mencari sudut magnitud dan fasa voltan di setiap bas dan kuasa sebenar dan reaktif yang mengalir di setiap talian penghantaran. Analisis aliran kuasa adalah alat penting yang melibatkan analisis berangka yang digunakan untuk sistem kuasa. Dalam analisis ini, teknik berulang digunakan kerana tidak ada kaedah analisis untuk menyelesaikan masalah. Banyak algoritma yang digunakan untuk menyelesaikan masalah aliran kuasa dalam sistem kuasa tetapi algoritma utama yang selalu digunakan adalah kaedah Newton Raphson, kaedah Gauss Seidel dan kaedah Fast Decoupled. Tujuan projek ini adalah perbandingan pelbagai algoritma dalam memuatkan beban beban analisa aliran tenaga yang berbeza. Kaedah Newton Raphson, Kaedah Gauss Seidel dan Kaedah Fast Decoupled dalam analisis aliran beban untuk beban yang berbeza adalah pelaksanaan dalam projek ini menggunakan MATLAB R2017a. Set persamaan algebra bukan linear diselesaikan menggunakan kaedah lelaran. Hasilnya dibandingkan dengan bilangan lelaran dan masa pengiraan yang diambil. Kerja-kerja penyelidikan ini dilaksanakan menggunakan sistem IEEE 14-Bus, IEEE 30-Bus dan IEEE-57-Bus.

ABSTRACT

Power flow analysis is the backbone of power system analysis and design. They are necessary for planning, operation, economic scheduling and exchange of power between utilities. The principal information of power flow analysis is to find the magnitude and phase angle of voltage at each bus and the real and reactive power flowing in each transmission lines. Power flow analysis is an important tool involving numerical analysis applied to a power system. In this analysis, iterative techniques are used due to the unavailability of analytical method to solve the problem. Many algorithms are being used to solve the problem of power flow in power system but the main algorithms always used are Newton Raphson method, Gauss Seidel method and Fast Decoupled method. The aim of this project is the comparison of various algorithms in different load loading of power flow analysis. Newton Raphson method, Gauss Seidel method and Fast Decoupled method in load flow analysis for different loading are implementation in this project using MATLAB R2017a. The set of nonlinear algebraic equations is solved using iteration methods. The results ware compared for number of iteration and the computational time taken. This research work are implemented using IEEE 14-Bus, IEEE 30-Bus and IEEE 57-Bus system.

CHAPTER 1

INTRODUCTION

1.1 Background

Power flow analysis is a standard instructional tool especially in advance course such as engineering [1]. Involving numerical analysis applied to a power system is importance in power flow analysis study of power flow usually uses simplified focuses on various forms of AC power. Power system is basically modelled by electric networks that consist of generators, distribution networks and transmission networks.

The power flow equation will solve using numerical methods. There are three methods are the Newton Raphson, Gauss Seidel and Fast Decoupled. Procedures faster convergences characteristics is Newton Raphson. However the starting values of Newton Raphson method are very crucial since the solution at the beginning can oscillate without converging towards the solution. Fast Decoupled method is variation of Newton Raphson method [2].

A bus voltages, phase angles, active and reactive power flows for different branches, generators, transformer settings and load under steady state condition will determine from a systematic mathematical approach using this analysis [3]. Find the magnitude and phase angle of voltage at each bus and the real and reactive power flowing in each transmission lines is the principal information of power flow analysis [4].

In reality, the demand from society in industrial sector is increasing. The power system will keep increasing and cause the increasing of dimension of load flow equation to several thousand since the demand is increasing [3]. But theoretically, the active and reactive power demands are assumed to have specified constant value and it is independent of the voltage magnitudes [4]. These will cause any numerical methods fail to converge to correct solution.

One of powerful tool used to calculate and develop the power flow analysis is MATLAB software. Technical computation that can integrate calculation and visualize and programming in a user friendly environment is high performance language. Power flow analysis becomes a standard instructional tool especially in advance course such as engineering.

1.2 Motivation

The power flow study is necessary for planning, economic scheduling and control of existing system because it is very important for future expansion [4]. It also can determine whether the system voltages remain within specified limits under various conditions whether the equipment is overloaded [2].

1.3 Problem Statement

The purpose of work is to study of different load flow algorithms. Different loading is important so that the suitable algorithm can be determined. The consumer is usually in high quality to power generated. However, loads of electrical consumers distort it progressively due to manner [5]. The industry will cause instability in the system because demand is increasing. Most of the time loading changes. Some of load flow algorithms may perform efficiently under normal loading. The load flow algorithm may fail to converge to obtain the accurate result when the loading is increased or decreased drastically.

1.4 Objectives

The objectives in this study are:

i. To apply three methods in Newton Raphson, Gauss Seidel and Fast Decoupled using MATLAB R2017a for load flow analysis in three IEEE Bus System.
ii. To compare the results in load flow methods under different loading.

1.5 Scope of Work

MATLAB R2017b is used to calculate the load flow of three IEEE standard bus systems under different loading using three different algorithms in order to achieve the project objectives. The scopes of this project are as follow:

i. Studies of the MATLAB R2017b programming for Newton Raphson, Gauss Seidel and Fast Decoupled
ii. Changing the load by increasing 20%, 40% and 60%
iii. Changing the load by decreasing 20%, 40% and 60%
iv. Check number of iteration and time elapsed for the three methods under all conditions.

1.6 Chapter Organization

This thesis begin with abstract and consists of six chapters.

Chapter 1 is introduction of this thesis. This chapter consists of background, motivation, problem statement, objective and scopes of project.

Chapter 2 is literature review of this thesis. The research work about power system analysis using Gauss-Seidel, Newton-Raphson and Fast Decoupled are present in this chapter. Explain about the basic equation of load flow and the bus system.

Chapter 3 is about the methodology of this thesis. Method of Newton Raphson, Gauss Seidel and Fast Decoupled are explain about the power flow process. The three load flow algorithms are includes the flowchart.

Chapter 4 is the simulation result of the three methods using MATLAB programming. The result are displayed in tables and graphs for comparison.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The literature review will covers in this chapter. The power flow analysis algorithms are explains to describe about the comparison of various algorithms in different loading of load flow analysis using three different methods are Newton Raphson, Gauss Seidel and Fast Decoupled.

Three methods (Newton Raphson, Gauss Seidel and Fast Decoupled) are commonly used iteration method applied in solving the power flow analysis than another methods [3]. Works done by other researcher are explain in this chapter.

2.2 Flow Analysis

In power engineering, the power flow analysis (also known as load-flow study) is an importance tool involving numerical analysis applied to a power system. Unlike traditional circuit analysis, a power flow study usually uses simplified notation such as a one-line diagram and per-unit system, and focuses on various form of AC power (ie: reactive, real and apparent) rather than voltage and current.

The advantage in studying power flow analysis is in planning the future expansion of power systems as well as in determining the best operation of existing systems. Power flow analysis is being used for solving power flow problem. There are three methods can be used to solve power flow analysis. The methods are Newton-Raphson method, Fast-Decoupled method and Gauss-Seidel method. This sub chapter will discuss all three methods generally on formula or mathematical step in order to solve power flow problem.

Some purposes of load flow analysis as follows [6]:

- To determine the magnitude of voltage and phase angles at all nodes of the feeder.
- To determine the line flow in each line section specified in Kilo Watt (KW) and KVAr, amperes and degrees or amperes and power factor.
- To determine the power loss.
- To determine the total input to the feeder Kilo Watt (KW) and KVAr.
- To determine the active and reactive power of load based on the defined model for the load.

2.2.1 **Bus Admittance Matrix**

The simple power system of impedance diagram is shows in figure 2.1 to obtain the node voltage equations. Express the impedance on a common MVA base in per unit and neglected the resistance [4].

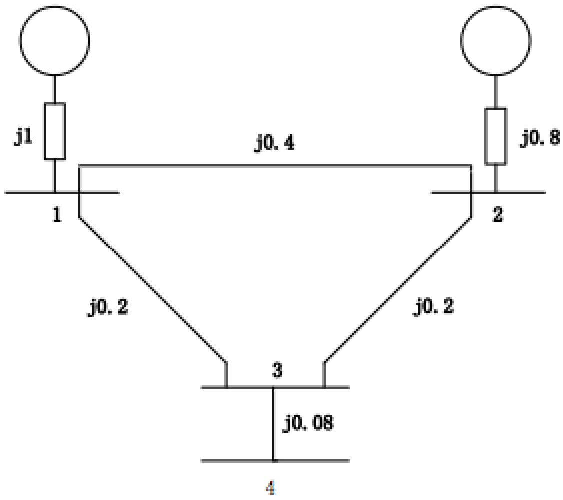

Figure 2.1: Simple system of impedance diagram

2.2.2 **Bus Classification**

Bus is connection of a point or node in which one or many transmission lines, loads and generators [3]. The four quantities that associated with the bus are magnitude of voltage (V), phase angle with voltage (°), active power P and reactive power Q in power system study. From the four quantities, two quantities are determined in the solution of equation while another two quantities are specified [3]. The types of the bus are load bus, slack bus and generator bus shown in Table 2.1.

Table 2.1: The types of the bus

		Variables	Variables	Variables	Variables
No	Type of bus	[V]	[°]	P	Q
0	Load Bus (PQ)	Unknown	Unknown	Known	Known
1	Slack Bus	Known	Known	Unknown	Unknown
2	Generator Bus (PV)	Known	Unknown	Known	Unknown

2.2.2.1. Load Bus (PQ)

Bus 0 is identified of load bus (PQ). From historical data record and measurement or forecast this bus is a non-generator bus [3]. This bus the known variables are active and reactive power while the unknown variables are voltage magnitude and phase angles of voltage. The power system of power supplied are defined to be positive and the power consumed are defined to be negative.

2.2.2.2. Slack Bus

This bus is identified as bus 1 in the bus data. Supply the losses to the network using effective generator of this bus. To meet the power balance condition slack bus as reference bus and to ensure power balanced can adjusted generating unit [3]. In this bus, the voltage magnitude and phase angles of voltage are known variables and the unknown variables are active and reactive power.

2.2.2.3. Generator Bus (PV)

Generator bus is identified as bus 2. The output generated can be controlled by adjustment because this bus is connected to generator. By adjusting the excitation of generator the prime mover and voltage can be control. The known variables in this bus are active and magnitude of voltage while the unknown are reactive power and the phase angle of voltage [3]. A limit reactive power of this bus depends on the characteristic of machine.

2.3 MATLAB

MATLAB is a multi-paradigm numerical computing environment. There are various typical use MATLAB such as

- Algorithm development
- Scientific and engineering graphics
- Math and computation
- Modelling simulation and prototyping
- Data analysis, exploration and visualization
- Application development, include graphical user interface building.

MATLAB does not require dimensioning of basic data element. MATLAB also allows to solve many technical computing problems include problem with vector and matrix formulations. Toolboxes are feature a family of application specific solutions in

MATLAB. Collections of MATLAB functions (m-files) that MATLAB environment to solve particular classes of problems are also called toolboxes [7].

The advantage of MATLAB are basic data element in matrix, graphical output is optimized for interaction and MATLAB functionality can be expanded by additional toolboxes while disadvantages are real time application are complicated, using large amount of memory and difficult to use in slow computer [8].

2.4 Case Study

2.4.1 Load Flow Analysis

First study case is paper by Olukayode A. Afolabi, Warsame H. Ali, Penrose Cofie, Pamela Obiomon and Emmanuel S. Kolawole is study of analysis of the load flow problem in power system [3]. The objective in this paper to determine the best operation for a power system and exchange of power between utility companies. In order to have an efficient operating power system, it is necessary to determine which method is suitable and efficient for the system's load flow analysis. Newton Raphson, Gauss Seidel and Fast Decoupled methods were compared for a power flow analysis solution using IEEE 9-Bus, IEEE 30-Bus and IEEE 57-Bus system to show results of simulation. The simulation results were compared for number of iteration, computational time, tolerance value and convergence. For this project, the tolerances are 0.00001 for all simulation. The result show that in Gauss Seidel is longest compared to Newton Raphson and Fast Decoupled and Gauss Seidel method is increase the computational time for iteration. Advantage of Gauss Seidel method is that the computational time for iteration is lower compared to Newton Raphson and Fast Decoupled. The more computational time for iteration is Newton Raphson because the complexity of

Jacobian Matrix in every iteration. The Newton Raphson method increase in quadratic progression, Gauss Seidel increase in arithmetic progression and Fast Decoupled increase in geometric progression. Newton Raphson is small number of iteration but still converges fast. The Gauss Seidel method can be used due to the good computational characteristics for small system. The compared results can be conclude that Newton-Raphson is the most reliable method because it has the least number of iteration and converges faster.

2.4.2 Newton Raphson Method

Study case is paper by Nivedita Nayak, Dr A.K Wadhawani and Dr Sulochanawani Wadhawani. The study is about the different techniques for solving a set of nonlinear equation with many assumption that a solution exist [9]. This paper is used iterative technique and the objective is to develop a toolbox for power flow analysis that help to check the performance of calculation. MATLAB programming is used to solve the load flow analysis. This study case paper realized the importance of power flow or load flow study is necessary for the load flow system. Analysing the load flow of bus system is suitable for Newton Raphson. The number of iteration is maintain when the number of bus increases for complex calculations. Newton Raphson method is found to be more efficient and practical from point of view of computational technique and convergence characteristics for large power systems.

2.4.3 Gauss Seidel Method

Study case paper by G.M Gillbert, D. E Bouchard and A.Y. Chikhani about a comparison Distflow method, Gauss Seidel method and Optimal Load Flow method in power flow analysis [10]. The calculation of load flow analysis using traditional algorithm are Gauss Seidel method and Optimal Load Flow method.

The linear system repeatedly find a solution to solve until the iteration is found are called an iterative numerical procedure that used in Gauss Seidel. When the matrix cannot be inverted it problem because the requirement two algorithms is to determine of admittance matrix. The accuracy and computational speed is being compared in this paper. The result shows that Distflow method is greater speed of convergence that another two method but the number of iteration of Gauss Seidel method and Optimal Load Flow method is large than Distflow method. Gauss Seidel are general method that can be used to solve most linear system but still have better method than Gauss Seidel method that can be conclude of this paper.

2.4.4 Fast Decoupled Method

Study case paper by Keter Samson Kipkirui about to develop a reliable and effective program based on Fast Decoupled method [11]. The programming platform that will be run on IEEE 14 Bus system is using MATLAB. Compared the result with other method such as Newton Raphson and Gauss Seidel. Method of Fast Decoupled and Newton Raphson will shows the same result. Fast Decoupled is shorter time taken than Newton Raphson. Fast Decoupled method of calculation at each iteration is simple that Newton Raphson because Newton Raphson need to use Jacobian at every iteration. Therefore, Fast Decoupled method takes more iteration compare than Newton Raphson method. Fast Decupled method is effective method in solving load flow problem that can be conclude in this case paper.

2.4.5 Load Change

Study case paper by P. Gayatri, G. Durga Sukumar and J. Jithendranath. This paper about effect of load change on source parameters in power system [5]. Some problems cannot be neglected in practical sense when source side is heavy loads. When taking rectifier and four DC motors as load are a method of a simulation study where during sudden load change variations are observed. The variations involved are current, power factor, voltage, total harmonic distortion, and reactive power. When the sudden load is increasing the current also increase. During normal condition, the current had been increased from 5 Amp to 21 Amp. The single phase loads current will also tends to be unbalance. When the overloading of the equipment and the unevenly distributed load in all three phase system occurs that means it is the unbalance current. The decrease in power factor is sudden change of load. A system of power factor should be maintained constant. When the result show high current that means the low power factor will present. The total harmonic distortion with respect to fundamental had been raised from 20% to 25% but the increasing of loss in the system. During sudden loading the source voltage was distorted normally the nature of source voltage was sinusoidal. The distorted will give effect to other loads by degrading performance. Reactive power demand also increases. The network need sufficient reactive power to avoid voltage unbalanced and other severe power quality issue. From this paper, small changes in load can cause drastic change in load and also all other loads connected to that source will affect the source.

Another study case paper is similar about the change of load. This paper by Sibasish Kanungo is about modelling of load in power flow analysis. The incorporation of load model in optimal power flow analysis and comparing the results obtained with those obtained from optimal power flow studies with incorporating load models are focusing [6]. Nowadays distribution system and power generation have develop from time to time. In theory, load flow analysis of the reactive and active power are constant. In real application, the magnitude of voltage and frequency deviations are a load power consumption. In this paper used IEEE 14-Bus for both with load flow method and load flow incorporating voltage dependent load models. MATLAB Power System Toolbox will present simulations using Newton Raphson, Gauss Seidel and Fast Decoupled. The voltage independent load and the magnitude voltage are less compared to the voltage dependent load that present in result analysis. When the magnitude below 1 p.u the active power generation is increases while the magnitude above 1 p.u is decreases. The active power for swing bus is 2.5% and the value is quite high. Active power difference of swing bus is dependent both on voltage and phase angle difference. The voltage dependent load case in active power consumption is less compare to the voltage independent loads. The active power consumption is decrease make system stable and less losses. The active power modelling has greatly affected the phase angles differences and the reactive power modelling affects the voltage differences that can be conclude in this paper.

2.5 Summary

This chapter explains about type of bus classification that are implemented in the MATLAB coding. Different format will be used in every coding. Next, six study case that explaining the work done and results by other for load flow analysis, Newton Raphson method, Gauss Seidel method and Fast Decoupled method and load change were presented. Newton Raphson method is the best method that can be conclude from many paper since the algorithm is the most stable and the number of iteration is smaller compared to Gauss Seidel method and Fast Decoupled method. A small size system can used Gauss Seidel because very convenient but for a large bus system it need to use Newton Raphson method as the number of iteration is not depending on the size of the bus while Fast Decoupled is shown as an unstable method.

CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter is about the methodology. Newton Raphson, Gauss Seidel, and Fast Decoupled are three types of IEEE bus that tested using algorithm. Since no known analytical method to solve the problem therefore iteration technique was used. The size of the system to solve the load flow problem are used method that involving plenty of calculation steps. There are a lot of calculation steps to be done that make it quite tedious and it takes a lot of time [12]. In this project, MATLAB programming was used to develop the power flow analysis. The bus systems that used in this project are IEEE 14-Bus, IEEE 30-Bus and IEEE 57-Bus. The standard data bus and line data based on the old research work done by other is collected at the internet. A flow chart of the project shown all the work done in this project and also all the formula for three method. Lastly, this chapter will explain about solving the load flow analysis programs in the MATLAB.

3.2 Project Flow Chart

The flow chart of this project show in figure 3.1. The flow chart shows the simplified flows of the method used from start until end in this project.

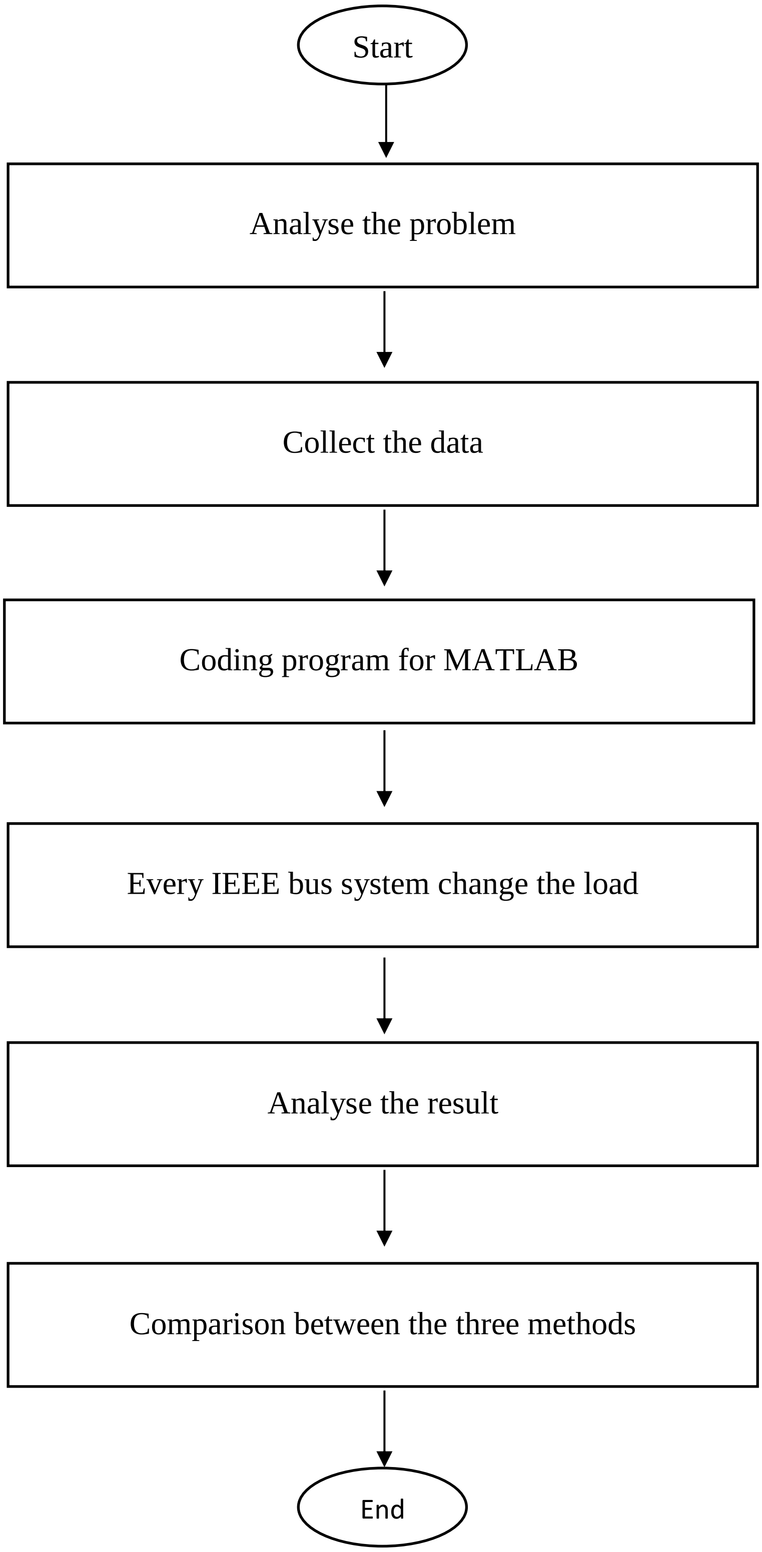

Figure 3.1: Flow chart

The whole project shows the flow chart in figure 3.1. The case study is start in this project. After the case study, the process of the coding the MATLAB started. Newton Raphson, Gauss Seidel and Fast Decoupled are three methods that have been used. These methods are implement on three bus system in example IEEE 14-Bus, IEEE 30-Bus and IEEE 57-Bus system. Lastly, the simulation of the MATLAB coding were carried out for the three methods.

3.3 Power Flow Analysis

The first step in load flow analysis is to form the Y-bus admittance using transmission line and transformer input data [3]. For power system network using Y-bus of the nodal equation is written as:

$$I = Y_{Bus}V \tag{3.1}$$

The nodal equation can be written in a generalized from for an *n* bus system

$$I_i = \sum_{j=1}^{n} Y_{ij}\ V_j \ for\ i = 1, 2, 3, n \tag{3.2}$$

The complex power delivered to bus *i* is

$$P_i + jQ_i = V_i I^*_i \tag{3.3}$$

$$Ii = \frac{Pi-jQi}{V*i} \tag{3.4}$$

Substituting for I_i in terms of P_i & Q_i

$$\frac{Pi-jQi}{V*i} = V_i \sum_{j=1}^{n} Y_{ij} - \sum_{j=1}^{n} Y_{ij}\ V_j \ \ j \neq I \tag{3.5}$$

In equation in (3.5) uses iteration techniques to solve load flow problems. Hence, it is necessary to review the general forms of the various solution methods such as Newton Raphson, Gauss-Seidel and Fast decoupled load flow.

3.4 Data for Load Flow

Data used for any load flow is standard discounting the types of algorithm that being used for the analysis. The data involved is usually are in per unit and the bus admittance matrix is formulated from these data. The system data, load data, transmission line data, generator bus data, shunt element data and transformer data. The entire algorithm in the MATLAB have four variables that should define are power system base MVA, power mismatch accuracy, acceleration factor and maximum number of iteration [4].The typical values are as follows :

- Base MVA = 100
- Acceleration = 1.8
- Accuracy = 0.00001
- Max iteration = 80

3.4.1 System Data

System data were required the data as follow:

- Number of buses
- Number of PV buses
- Number of loads
- Number of transformer
- Number of transmission lines
- Number of shunt elements
- Slack bus number
- Voltage magnitude of slack bus
- Tolerance limit
- Base MVA
- Maximum number of iteration

3.4.2 **Load Data**

All load data were required the data as follow:

- Bus number
- Active power demand
- Reactive Power demand

3.4.3 **Transmission Line Data**

The transmission line connects between buses were required the data as follow:

- Starting bus number
- Ending bus number
- Resistance of line
- Reactance of line half line changing admittance

3.4.4 **Generator Bus Data**

Data required for every PV bus are as follow:

- Bus number
- Active power generation
- Specified voltage magnitude
- Minimum reactive power limit
- Maximum reactive power limit

3.4.5 **Shunt Element Data**

Data required for shunt element are as follow:

- Bus number where element is connected
- Shunt admittance (G_{sh} + j B_{sh})

3.4.6 **Data Transformer**

Data required for transformer connected between buses are as follow:

- Starting bus number

- Ending bus number
- Resistance of transformer

3.5 Power Flow Programs

Each method of solutions are four program in MATLAB programming. The commons programs that will be used are Ifybus, Busout and Lineflow. Ifgauss is for Gauss Seidel program, Ifnewton is for Newton Raphson and Decoupled is for fast decoupled method. Firstly, Ifybus is run and followed by the program for each method such as Ifgauss, Ifnewton and Decoupled. Then followed by Busuout and Lineflow. Ifgauss is used before Ifybus and then Busout and lastly Lineflow. All method will use the same arrangement of program in the MATLAB [4]. The description of program used explain below:

i. Ifybus

This program contains bus admittance matrix that can converts impedances to admittances. It requires line and transformer parameters and transformer tap settings specified in input file named line data [4]. Design of the program is to control the parallel line in the system.

ii. Busout

This program if for the bus output result. A table form will shows the result. Bus output results include the voltage magnitude and angle, real and reactive power of generators and loads and shunt capacitor or reactor Mvar [6].

iii. Lineflow

This program prepares the line output data. It designed to display the active and reactive power flow entering the line terminals and line losses as well as the net power at each bus. Included also the total real and reactive losses in the system. The output of this portion is shown in sample case [4].

3.6 Newton Raphson Method

Newton Raphson method is commonly used and it is also the most common method introduced [1]. It is a successive approximation procedure based on initial estimate of one-dimensional equation given by series expansion and also to solving simultaneous nonlinear algebraic equations.

Bus admittance matrix in either first or second order expansion of Taylor series are used in Newton Raphson. It is evaluated as a best solution for the reliability and rapid convergence [1].

$$f(x)=c \tag{3.10}$$

If

$x^{(0)}$ is an initial estimate of the solution

$\Delta x^{(0)}$ is a small deviation from the correct solution

We must have

$$f(x^{(0)}+ \Delta x^{(0)})=c \tag{3.11}$$

Expand the left-hand side of the above equation in Taylor's series about $x^{(0)}$ yields

$$f(x^{(0)}) + (df/dx)^{(0)} \Delta x^{(0)} +1/2!\ (d2f/dx2)^{(0)} (\Delta x^{(0)})^2+\ldots=c \tag{3.12}$$

Assume the error $\Delta x(0)$ is very small, the higher-order terms can be neglected,

$$\Delta c^{(0)} \approx (df/dx)^{(0)} \Delta x^{(0)} \tag{3.13}$$

Where

$$\Delta c^{(0)}=c - f(x^{(0)}) \tag{3.14}$$

Adding $\Delta x(0)$to the initial estimate will result in the second approximation

$$x^{(1)}= x^{(0)} + \Delta c^{(0)}/ (df/dx)^{(0)} \tag{3.15}$$

Successive use of this procedure yields the Newton-Raphson algorithm

$$\Delta c^{(k)}=c - f(x^{(k)}) \tag{3.16}$$

$$\Delta x^{(k)}= \Delta c^{(k)}/ (df/dx)^{(k)} \tag{3.17}$$

$$x^{(k+1)} = x^{(k)} + \Delta x^{(k)} \tag{3.18}$$

(3.17) can be rearranged as

$$\Delta c^{(k)} = j^{(k)} \Delta x^{(k)} \quad \text{where } j^{(k)} = (df/dx)^{(k)} \tag{3.19}$$

In power system analysis, $J^{(k)}$ is called the Jacobian matrix. Element of this matrix are the partial derivatives evaluated at $X^{(k)}$. It is assumed that $J^{(k)}$ has an inverse during each iteration. Newton's method, as applied to a set of nonlinear equations reduces the problem to solving a set of linear equations in order to determine the values that improve the accuracy of the estimates [1].

3.7 Gauss Seidel Method

Gauss-Seidel method is also known as the method of successive displacements. It is named after the German mathematicians Carl Friedrich Gauss and Philip Ludwig Seidel [2]. Gauss Seidel methods is similar to Jacobi method.

This technique, consider solution of the nonlinear equation given [1]:

$$F(x)=0 \tag{3.20}$$

A function is rearrange and writes as

$$x=g(x) \tag{3.21}$$

If $x=(k)$ is an initial estimate of the variable x, the following iterative sequence is formed

$$X^{(k+1)} = g(x^{(k)}) \tag{3.22}$$

When the difference between the absolute value of the successive iteration is less than a specified accuracy

$$|x^{(k+1)} - x^{(k)}| \leq \varepsilon \tag{3.23}$$

ε is the desire accuracy

3.8 Fast Decoupled

Fast Decoupled method is an alternative strategy. When solving large scale power transmission systems is improving efficiency and reducing computer storage requirements is the decoupled power flow method [1]. Newton Raphson method is an approximate version.

The Fast decoupled power flow solution requires more iterations than the Newton-Raphson method, but requires considerably less time per iteration and a power flow solution is obtained very rapidly and useful in contingency analysis [1]. The contingency analysis where numerous outages are to be simulated.

The transmission lines have very high X/R ratio for large scale power system. Real power changes ΔP are most sensitive to changes in phase angle $\Delta\delta$and less sensitive to changes in voltage magnitude. For reactive power is less sensitive to changes in angle and most sensitive on changes in voltage magnitude. The elements of the submatrices J_{12} and J_{21} is zero these approximations into the Jacobian matrix.

$$\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_n} \\ \vdots & J_{11} & \vdots \\ \frac{\partial P_n}{\partial \delta_2} & \cdots & \frac{\partial P_n}{\partial \delta_n} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \vdots \\ \Delta\delta_n \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta P_n \end{bmatrix} \tag{3.24}$$

$$\begin{bmatrix} |V_2| \frac{\partial Q_2}{\partial |V_2|} & \cdots & |V_n| \frac{\partial Q_2}{\partial |V_n|} \\ \vdots & J_{22} & \vdots \\ |V_2| \frac{\partial Q_n}{\partial |V_2|} & \cdots & |V_n| \frac{\partial Q_n}{\partial |V_n|} \end{bmatrix} \begin{bmatrix} \frac{\Delta |V_2|}{|V_2|} \\ \vdots \\ \frac{\Delta |V_n|}{|V_n|} \end{bmatrix} = \begin{bmatrix} \Delta Q_2 \\ \vdots \\ \Delta Q_n \end{bmatrix} \tag{3.25}$$

In well-designed and properly operated power transmission system:

i) Angular differences between typical buses of the system are usually so small.

$\delta_{ij} = (\delta_i - \delta_j)$ very small that results,

$\cos \delta_{ij} \approx 1$

$\sin \delta_{ij} \approx 0$

ii) The line susceptances B_{ij} are many times larger than the line conductances G_{ij} so that $G_{ij} \sin \delta_{ij} << B_{ij}$.

iii) The reactive power Qi injected into any bus i of the system during normal operation is much less than the reactive power which would flow if all lines from that bus were short circuited to reference.

That is $Q_i << |V_i|^2 B_{ii}$.

$$\frac{\partial P_i}{\partial \delta_j} = -|Y_{ij} V_i V_j| \sin(\theta_{ij} + \delta_j - \delta_i) \tag{3.26}$$

$$|V_j| \frac{\partial Q_i}{\partial V_j} = -|V_j||Y_{ij} V_i| \sin(\theta_{ij} + \delta_j - \delta_i) = \frac{\partial P_i}{\partial \delta_j} \tag{3.27}$$

In Eq.(3.26) and Eq.(3.27), the off diagonal elements of J11 and J22 are given by

$$|V_j| \frac{\partial Q_i}{\partial V_j} = -|V_j||Y_{ij} V_i| \sin(\theta_{ij} + \delta_j - \delta_i) = \frac{\partial P_i}{\partial \delta_j} \tag{3.28}$$

Use the identity sin(α + β) = sin α cos β + cos α sin β in Eq.(3.28) gives us

$$\frac{\partial P_i}{\partial \delta_j} = |V_j| \frac{\partial Q_i}{\partial |V_j|} = -|V_i V_j| B_{ij} \cos(\delta_j - \delta_i) + G_{ij} \sin(\delta_j - \delta_i) \tag{3.29}$$

The approximation listed above then yield the off diagonal elements

$$\frac{\partial P_i}{\partial \delta_j} = |V_j| \frac{\partial Q_i}{\partial |V_j|} = -|V_i V_j| B_{ij} \tag{3.30}$$

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} |Y_{ij} V_i V_j| \sin(\theta_{ij} + \delta_j - \delta_i) \tag{3.31}$$

$$|V_i| \frac{\partial Q_i}{\partial V_i} = -\frac{\partial P_i}{\partial \delta_i} - 2|V_i|^2 B_{ii} = Q_i - |V_i|^2 B_{ii} \tag{3.32}$$

The diagonal elements of J11 and J22 are shown in Eq. (3.18) and Eq. (3.19) respectively.