EVALUATION OF COMPRESSED SENSING RECONSTRUCTION ALGORITHM IN OFDM SYSTEM

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EVALUATION OF COMPRESSED SENSING RECONSTRUCTION ALGORITHM IN OFDM SYSTEM

by

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This thesis is dedicated to everyone in the related field of wireless communication system who embarks the journey of extending the knowledge and passion for the future development of OFDM system.

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LIST OF ABBREVIATIONS

Orthogonal Frequency Division Multiplexing	OFDM
Inter Symbol Interference	ISI
Channel Estimation	CE
Channel State Information	CSI
Orthogonal Matching Pursuit	OMP
Compressed Sensing Matching Pursuit	CoSaMP
Subspace Pursuit	SP
Least Squares	LS
Compressed Sensing	CS
Inverse Fast Fourier Transform	IFFT
Mean Square Error	MSE
Bit Error Rate	BER
Addictive White Gaussian Noise	AWGN
Fast Fourier Transform	FFT
Quadrature Amplitude Modulation	QAM
Greedy Pursuit	GP
Symbol Error Rate	SER
Mean Square Error	MSE
Decibel	dB
Cyclic Prefix	CP
Signal To Noise Ratio	SNR
Restricted Isometry Property	RIP
Guard Interval	GI
Line of Sight	LOS

ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier modulation technique that had been adapted widely by high data rate wireless communication systems. This is due to its ability to reduce multipath channel fading, increase bandwidth efficiency, and elimination of inter symbol interference (ISI) which are favorable conditions in wireless signal transmission. OFDM divides the frequency selective fading channels into many narrow band flat fading sub channels to ease the equalization. The idea of orthogonality of this modulation allows all the sub carriers to be independent of each other and provides no interference among the sub carriers, and hence, this explained why the transmitted information can still be separated from the sub carriers. Channel estimation (CE) in OFDM system is compulsory to obtain the channel state information (CSI) at the receiver. Conventional channel estimation (CE) methods had been introduced to estimate CSI, but they are not able to exploit the wireless channel sparsity which causes reduction in bandwidth efficiency. The implementation of compressive sensing methods in OFDM was initiated that able to exploit the sparsity property of the signal and therefore, outperforms the conventional CE. Consequently, various widely used compressive sensing reconstruction algorithm such as: Orthogonal Matching Pursuit (OMP), Compressed Sensing Matching Pursuit (CoSaMP), and Subspace Pursuit (SP) will be evaluated to test their efficacy sparse estimation performances in OFDM system.

ABSTRAK

Orthogonal Frequency Division Multiplexing (OFDM) adalah teknik modulasi multipembawa yang sering digunakan oleh sistem komunikasi tanpa wayar terutamanya sistem yang mempunyai kadar data yang tinggi. OFDM berupaya untuk mengurangkan saluran pelbagai arah pudar, meningkatkan kecekapan jalur lebar, dan menghapuskan gangguan intersymbol (ISI) dalam penghantaran isyarat tanpa wayar. Ia membahagikan frekuensi saluran pudar terpilih kepada beberapa jalur sempit rata dengan saluran sub pudar untuk meringankan proses penyamaan. Idea ortogon yang diserapkan ke dalam modulasi ini telah memberikan subcarrier tersebut ciri bebas yang tersendiri dan sekaligus tidak memberikan gangguan di dalam kalangan subcarrier, Dengan ini, terjelaslah persoalan tentang aktiviti pengasingan maklumat yang dihantar daripada salurannya. Pelaksanaan channel estimation (CE) dalam sistem OFDM adalah penting bagi mendapatkan channel state information (CSI). Terdapat beberapa kaedah channel estimation (CE) yang konvensional telah diperkenalkan untuk mendapatkan CSI, tetapi mereka telah gagal dalam pengeksploitasian sparsity terhadap saluran tanpa wayar yang akan menyebabkan pengurangan dalam kecekapan lebar jalur. Pelaksanaan kaedah penderiaan mampatan dalam OFDM telah dilaksanakan bagi mengeksploitasi ciri sparsity yang terdapat pada isyarat tersebut. Pelaksanaan kaedah ini terbukti mempunyai keputusan evaluasi yang memuaskan jika dibandingkan dengan CE yang konvensional. Oleh itu, pelbagai algoritma pembinaan semula isyarat yang berkonsepkan penderiaan mampatan seperti: Orthogonal Matching Pursuit (OMP), Compressed Sensing Matching Pursuit (CoSaMP), dan Subspace Pursuit (SP) akan dinilai bagi menguji keberkesanan persembahan anggaran jarang mereka di dalam sistem OFDM.

CHAPTER 1
INTRODUCTION

1.1 Background

In wireless communications systems, the transmitted signal usually propagates through many distinct paths from the transmitter to the receiver. This multipath delay spread leads to time dispersion and frequency-selective fading [1]. To deal with this drawback in wireless communications system, orthogonal frequency division multiplexing (OFDM) system is introduced and recognized as a technique for subduing the problem of multipath fading channels. It converts the frequency-selective fading channels into several parallel flat-fading narrowband sub-channels while maintaining orthogonality among subcarriers [2]. The effect of flat fading channel permits the channel coherence bandwidth to be larger than the bandwidth of the signal. Consequently, all frequency components within the received signal will undergo the same degree of fading which makes it simpler to track the time- varying channel statistics. In coherent wireless communication, it is critical to obtain accurate estimation of the channel state information (CSI) at the receiver which describes the characteristics of channel in terms of signal propagation within transmitter and receiver. Conventional techniques for the channel estimation of CSI, such as “least squares,” (LS), results in the excessive overuse of the spectral resources and cannot exploit the wireless channels sparsity [3].

Recently, the advancements in compressed sensing (CS) has triggered the broad investigation on the application of sparse recovery algorithms to channel estimation, which requires fewer pilots and proves to be more accurate than the conventional least squares (LS) [1], [2], [4]. CS exploits the inherent sparsity property of a signal and asserts that certain signals can be reconstructed from far fewer samples or measurements than conventional methods of channel estimation when the original data signal and the compression matrix have some specific properties. Hence, a proper reconstruction algorithm is essential for accurate or even exact recovery of the target signal. In order to establish a perfect reconstruction of the continuous time signal from its discrete time samples, the continuous time signal is to be sampled with a sampling frequency of at least twice the highest frequency component of the signal being sampled so as to avoid information loss. This means, more samples will be acquired which are not fully utilized in the reconstruction process and however, places a huge burden on the analog-to-digital converters, storage, transmission and processing requirements [4], [5]. This means that, applications have to further process the sampled signal through compression for bandwidth-efficient transmission. Consequently, a significant measure of the acquired information which is the least essential information content is thrown away [6]. A possibility of overcoming the “Nyquist barrier” so as to compress the signals in the sampling process, one has to merge the traditional techniques from sampling theory contemporaneously with current advancements in compressed sensing (CS). CS concurrently executes sensing and compression in the same step; hence, the transmitted signal is sensed in a compressed form. This demonstrates a statistically significant reduction in the number of measurements that is required to be processed. CS is essentially suitable for sparse signals or for signals that are compressible with the sparsity basis as having only a few dominant paths in propagation. The originally transmitted signal can be fully reconstructed from its samples either by convex optimization techniques or greedy recovery algorithms.

Recently, many greedy reconstruction algorithms have since emerged for sparse signal recovery from what seems to be an incomplete set of measurements. Typically, these algorithms aim specifically to decrease the cost of the computational complexity of the optimum solution of the l_1 minimization, while still maintaining the same reconstruction accuracy. The greedy algorithms iteratively identify the locations of the signal support (i.e., index set of nonzero elements) by correlating columns of the measurement matrix with the measured signal. In each stage of iteration of the greedy algorithm, a number of correlation values are chosen and their indices are appended to an identified support set. Once the correct support set have been identified, the non-zero signal coefficients are determined by applying the pseudo-inversion process which subsequently leads to the reconstruction of the sparse signal.

1.2 Problem Statements

Orthogonal frequency-division multiplexing (OFDM) is a method of encoding digital data on multiple carrier frequencies which is widely used in various wireless communication standards. In OFDM, a high rate bit stream is partitioned into bit streams of a lower rate and are modulated over one of the orthogonal subcarriers [7], [8]. In a single carrier system, a single fade or interferer can cause the entire link to fail, but in a multicarrier system which is offered by OFDM , only a small percentage of subcarriers will be affected which can be corrected by properly estimating the channel at the receiver [5].

As channel estimation (CE) at the receiver of OFDM system is crucial in order to obtain reliable CSI for signal reconstruction purpose [9], however, the traditional CE method such as LS is not able to exploit the channel sparsity as prior information, where the channel is naturally supported by few non-zero coefficients. This condition may results to the estimation errors in the system which leads to inaccurate signal reconstruction at the receiver. Additionally, LS method offers less spectrum efficiency due to the large requirement of pilot symbols to be inserted into transmitted signal in order to obtain the CSI [9],[10].

Therefore, to increase the estimation performance in the OFDM system, compressed sensing (CS) based CE is implemented in order to take advantage of the channel sparsity. By exploiting the sparsity, the spectrum efficiency can be improved due to less number of pilot symbols required for the estimation purpose. CS reconstruction depends on the particular reconstruction algorithm being used. This research majorly focuses on the l_2 minimization adopting the greedy pursuit algorithm as it aims specifically to decrease the cost of the computational complexity of the optimum solution of the l_1 minimization, while still maintaining the same reconstruction accuracy. The current greedy algorithms select indices of correlation values from the proxy signal, to determine the identified support set for sparse signal reconstruction.

In this work, various compressed sensing greedy reconstruction algorithms such as the Orthogonal Matching pursuit (OMP), Compressed Sampling Matching Pursuit (CoSaMP) and Subspace Pursuit (SP) will be simulated and discussed for evaluation. Additionally, in order to increase the channel estimation accuracy, a deterministic pilot placement schemes are proposed to improve Mean Square Error (MSE) and Bit Error Rate (BER) performances.

1.3 Research Objectives

The aim of this work is to evaluate a number of greedy pursuit algorithms for signal recovery based on sparse channel estimation in OFDM system. The stated aim will be accomplished by fulfilling the project objectives as follow:

1. To implement and understand the structure of OFDM system in the recovering of the originally transmitted signal for sparse channel estimation.
2. To develop a MATLAB program which can be used to evaluate and simulate the results on MSE and BER of different reconstruction algorithms such as OMP, CoSaMP, and SP.
3. To analyze and evaluate the estimation performance of different reconstruction algorithms such as OMP, CoSaMP, and SP based on their MSE and BER simulation results by considering LS as the benchmark.

1.4 Scope of Research

The research will be focus on the evaluation of estimation performance of compressed sensing reconstruction algorithms which is also known as greedy pursuit algorithm in the OFDM system.

Various compressed sensing (CS) greedy reconstruction algorithms such as the Orthogonal Matching pursuit (OMP), Compressed Sampling Matching Pursuit (CoSaMP), and Subspace Pursuit (SP) will be simulated using MatLab and the resulting estimation performance from the simulation will be discussed in terms of MSE. The evaluation on estimation performance of the CS greedy reconstruction algorithms are to be conducted in order to examine the signal reconstruction accuracy at the receiver. Additionally, in order to increase the channel estimation accuracy, a deterministic pilot placement scheme is proposed to improve Mean Square Error (MSE) and Bit Error Rate (BER) performances.

1.5 Thesis Organization

The rest of the thesis is organized as follow:

In Chapter 2, theoretical background and literature review will be focused. The introduction to OFDM system in wireless system will be discussed along with the system block model of the system. The concept of wireless channel estimation and compressed sensing in OFDM system will also be studied. Finally, a summary of different compressed sensing based greedy algorithms will be presented.

In Chapter 3, the entire research methods required for the step by step implementation of the research study is proposed and explained.

Chapter 4 discusses the results of the simulation for LS, OMP, CoSaMP, and SP in terms of their MSE and BER performances.

Lastly, Chapter 5 presents the overall conclusion for the entire work along with project limitations and recommendation for future work. The MatLab program code is contained in the Appendices.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

There are several algorithms available for sparse signal recovery. These algorithms include convex minimization methods and the greedy pursuit algorithms [5], [11]. The fundamental concept of convex optimization algorithms is that it transforms the problem of solving the underdetermined system of linear equation to minimum l_1-norm convex optimization problem. The Basis pursuit algorithm (BP) [12] is a typical convex optimization algorithm, for solving convex feasibility problems. The major benefit of the BP algorithms is that the probability of reconstructing the original signal is very high [13]. However, it has a high computational complexity, making them unattractive for large scale problem. Driven by the need to develop computationally inexpensive solutions, several greedy algorithms have since been proposed in the literature for sparse signal recovery. Greedy recovery algorithms iteratively strive to obtain the signal support. In each stage of iteration, the sparse signal is estimated based on the identified support set via least square minimization.

In this chapter, the overview of basic principles of the OFDM system is studied. The introduction of OFDM system is discussed in Section 2.2 while the OFDM system block design for this research is presented in Section 2.3. Next, the introduction of the wireless channel estimation (CE) is explained in Section 2.4. The working concept of Compressed Sensing (CS) is introduced in Section 2.6. There are two main algorithmic strategies to find the possible solutions to this problem, a detailed literature review on both strategies are effectively addressed and presented in Section 2.5 and Section 2.8 respectively. Lastly, the summary of this chapter is presented in Section 2.9.

2.2 Introduction to OFDM System

Orthogonal frequency division multiplexing (OFDM) system is introduced and recognized as a technique in various wireless communication standards due to its high resistance to multipath fading, high information rate, and simple channel equalizers [5],[14]. OFDM divides wide frequency band of high data rate stream into multiple small frequencies band also known as subcarriers of low data rate stream, where user can make use of orthogonal subcarriers to transmit single data stream simultaneously at a time. This special characteristic of OFDM can be seen as an advantage over single carrier transmission where it allows only one subcarrier involves in the transmission of data stream at a time.

The orthogonality of the subcarriers mean their correlation is zero or in other words, they are all independent of each other and resulting in their capability to prevent interference between overlapping carriers. Also, orthogonality allows more subcarriers per bandwidth which resulting in high bandwidth efficiency, more information can be transmitted over given bandwidth. OFDM is resistance to multipath fading, which caused by multipath signal propagation by transmitting data over multiple subcarriers of slow data rate simultaneously. The multiple subcarriers also allowing simpler equalization instead of applying it to the whole channel which is much more complex.

2.3 OFDM System Block

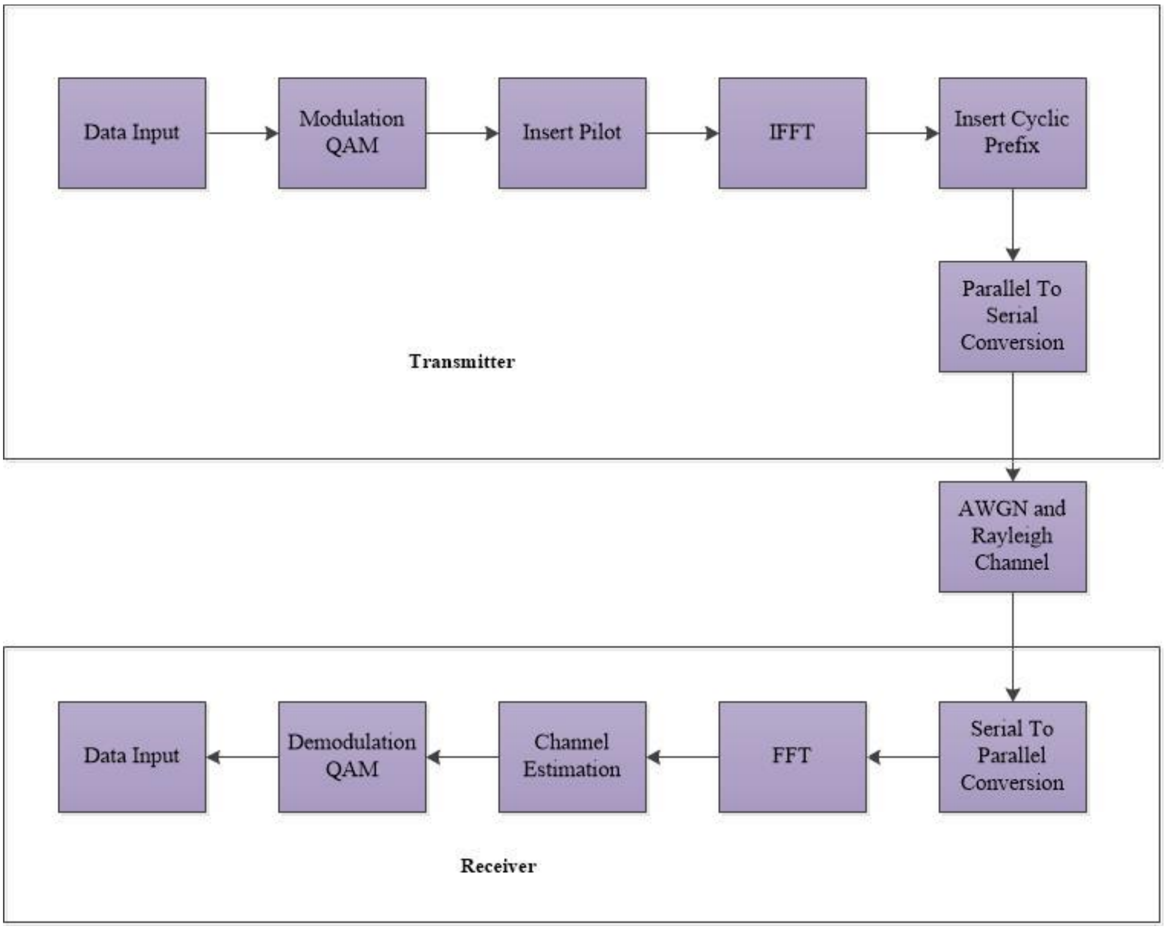

Figure 2.1: OFDM block diagram

Figure 2.1 above shows the block diagram of OFDM system model that is used in this research. The represented block in the OFDM model will be discussed as follows:

a) Modulation

The input signal is first modulated using QAM modulation technique that used four points on the constellation diagram, which spaced apart at equal distances around a circle. The four possible phases on the constellation diagram are mapped with two bit each. Higher implementation of order modulation formats, gives more points on the constellation, which enable the transmission of more bits per symbol. However, more constellation points give exposure to the points to stay closer to each other, and therefore, more susceptible to noise and data errors.

b) Pilot Insertion

Pilot are subcarriers of know data or known symbols that are used to perform channel estimation at the receiver. A pilot symbol is a complete OFDM symbol where the value of each subcarrier is predefined and known in transmitter and receiver. The received signal is then correlated with the pilot symbol to detect the channel estimation in OFDM system. Insertion of pilots into the OFDM system involved two major methods: block-type and comb type.

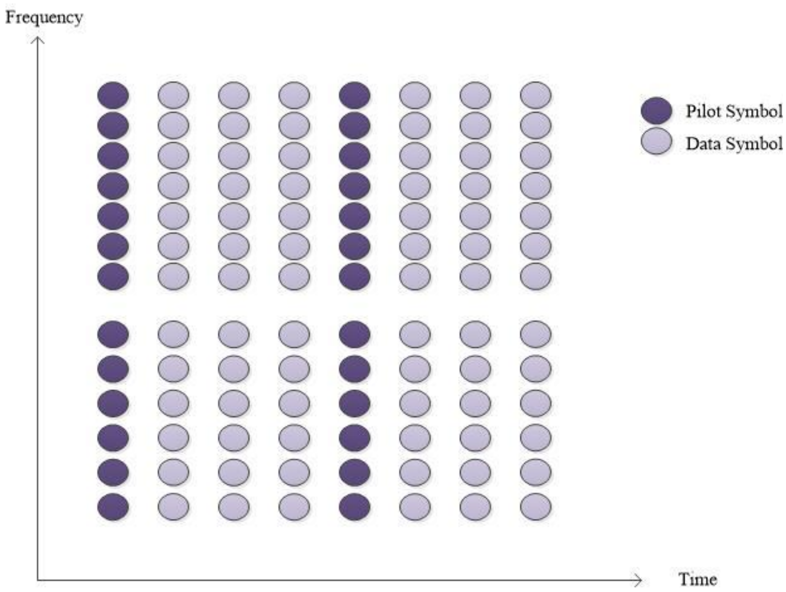

Figure 2.2: Block type pilot arrangement in OFDM CE

In Figure 2.2, the block type pilot arrangement is shown. This type of arrangement is performed by inserting pilots into all subcarriers of OFDM symbols or data symbols within a certain period [15],[16] and they are transmitted periodically for the purpose of CE. This approach is applicable for slow fading channel where the channel is considered to be constant within one OFDM symbol so that it is effective against frequency selectivity [17],[18]. Since all the subcarriers are used as pilots to estimate the channel error condition and the same estimate is used for the upcoming symbols under constant channel, therefore, there is no estimation error [18],[19].

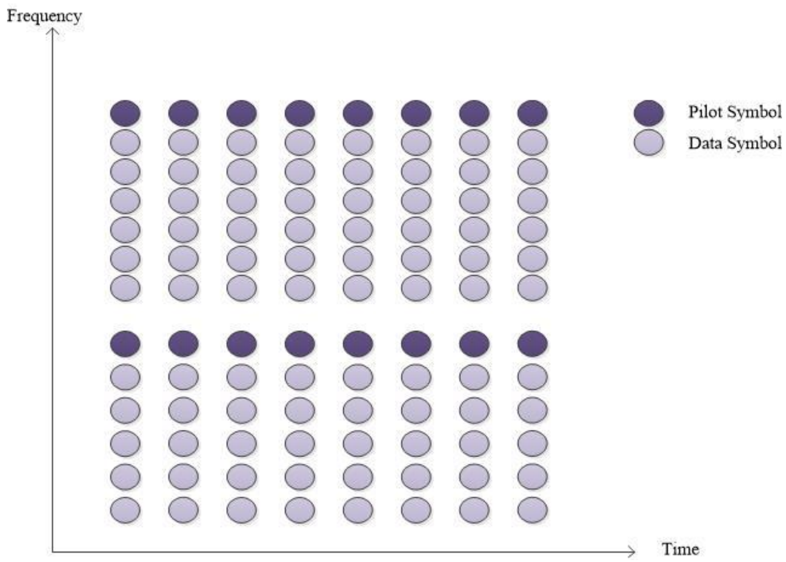

Figure 2.3: Comb type pilot arrangement in OFDM CE

In Figure 2.3, the comb type pilot arrangement is shown. For this type of arrangement, pilots are inserted uniformly into certain subcarriers within each OFDM symbol and it is designed for the implementation in fast fading channel where the channel condition is varied within neighboring OFDM symbols [16],[20]. Since only some of the subcarriers are used as pilots, therefore, the channel response of data subcarriers can be estimated using channel information of neighboring pilot subcarriers based on several interpolation techniques [21]. In this paper, comb type pilot arrangement is used to handle fast fading Rayleigh channel for the performance analysis of OFDM system.

c) Inverse Fast Fourier Transform (IFFT)

IFFT is implemented at the transmitter which is used to convert frequency domain to time domain. IFFT converts a complex data pints with the length that is power of 2 in frequency domain, into the same number of points in time domain. The output signal of IFFT is transmitted sequentially where each of the channel bits is appeared at different sub-carrier frequency to ease the received signal evaluation process.

d) Cyclic Prefix Insertion

Guard interval is used to retain the orthogonality of the data of the sub carriers through the process of data transmission. By means, guard interval allows time gap for multipath signals from the previous symbol to dissipate before the formation from the current symbol is started. The most common and effective guard band that is used is cyclic prefix. The working concept of cyclic prefix is that it copies a part of signal from the end of the OFDM symbol and paste it to the front of the symbol as shown in Figure 2.4, and hence, it creates a time gap between the delays of the signal of sub carriers. Insertion of cyclic prefix is able to tolerate the signal delay and inter symbol interference (ISI) although it may consumes bandwidth efficiency.

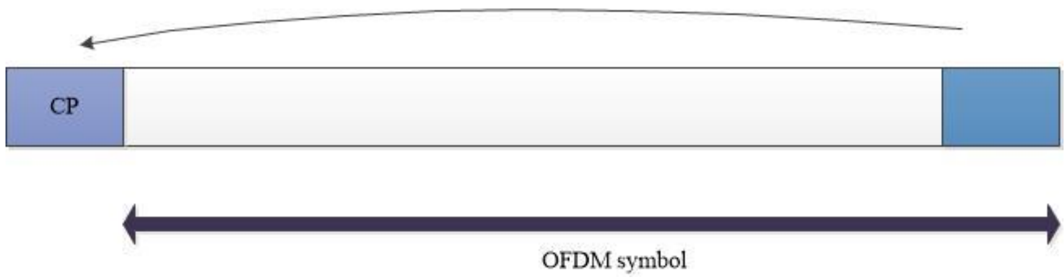

Figure 2.4: Cyclic Prefix (CP)

e) Parallel To Serial Conversion

Parallel to serial conversion allows the sub carriers to be transmitted as one signal after the insertion of cyclic prefix. Therefore, by combining all the sub carriers into one signal, they can transmitted simultaneously bit by bit to the receiver of the OFDM system.

f) Addictive White Gaussian Noise (AWGN) and Rayleigh Channel

AWGN is a basic random noise which contains a continuous frequency spectrum over a specific frequency band of a channel. It is used to model the effect of random noise that occurs in the transmission channel.

Rayleigh flat fading is modeled as the effect of multipath propagation where the Line of Sight (LOS) connection between transmitter and receiver is blocked by objects such as wall and building that cause the signal to be reflected form its direct path to the receiver.

g) Serial To Parallel Conversion

Serial to parallel conversion at the receiver of the proposed OFDM block diagram performs the opposite process of parallel to serial conversion at the transmitter side previously. It converts back the serial data bits into several parallel data bits, which then, divided among the individual sub carrier called symbols. The serial to parallel converter at the receiver has the function to receive the data that is going to be demodulated later.

h) Cyclic Prefix Removal

Cyclic prefix is removed from the information symbols before FFT to be prepared for the signal reconstruction process at the receiver.

i) Fast Fourier Transform (FFT)

FFT is implemented at the receiver of OFDM system to convert back time domain to frequency domain as the received symbol is in time domain where it does exactly the opposite of IFFT at the transmitter. The output of FFT will be then transmitted to channel estimation block for the process of implementation of reconstruction algorithm.

j) Channel Estimation (CE)

The implementation of CE at the receiver of the OFDM system is used to estimate the channel state information (CSI) by using the proposed compressed sensing reconstruction algorithms. This process is crucial to the reconstruction property of the transmitted signal.

k) Demodulation

Finally, the demodulation process at the receiver side is used to evaluate and convert back the received symbols to the data words. The data words are then combined back to the same word size as the original data.

2.4 Wireless Channel Estimation

Channel estimation (CE) plays an important role in the designing of receiver in OFDM systems for the application in wireless communication system [22]. Implementation of CE in OFDM system is compulsory to obtain the channel state information (CSI), reducing the bit error rate at the receiver and also to achieve a distortion less output data [23]. CE in OFDM is usually done in the frequency domain which is independently in all the sub-carriers. By exploiting the interrelationship among the channel estimates in the multiple sub-channels, a better quality of channel estimates can be obtained by transforming the frequency domain channel estimates to time domain which requires a windowing operation along with a correction operation. Lastly, channel estimates can be obtained once again by transforming back the resulting time domain to frequency domain which then, can be used directly in equalization and decoding at the receiver of the OFDM system [24]. By means, better receiver performance with higher signal accuracy can be obtained as the transmitted signals is reobtained from the demodulated signals [24],[25]. Generally, there are several techniques which are used to perform CE such as: blind channel estimation, semi blind channel estimation, and training based estimation which will be discuss later in the next subsection.

2.4.1 Blind Channel Estimation

For blind channel estimation (CE) technique, the term blind is applied to this particular CE technique when the signal input of the channel is not available for processing at the receiver [26]. Unlike semi blind CE and training based CE, blind CE used no training symbols which is used for channel identification and hence, it makes efficient use of the structures of the channel and properties of the input signal [27]. Blind CE is favourable to the increases of the system bandwidth efficiency or in other words, the information rate and system throughput due to the elimination of training symbols applied to the technique [28]. However, there is a drawback of this particular CE technique as it is not always efficient due to the fact that it may suffers from high computational complexity and critical performance degradation in fast fading channel when handling with large data amount [29].

2.4.2 Semi Blind Channel Estimation

Semi blind channel estimation (CE) requires the combination of both blind and training based methods to be implemented in OFDM system [5]. Therefore, it provides trade-off between high computational complexity and less bandwidth efficiency which are suffered by both blind CE and training-based CE. Semi blind CE gives out better information rate performance compared to training-based CE due to the reduction in the number of inserted pilots which reduces bit error rate (BER). This explained why better bandwidth efficiency and accuracy can be obtained by using semi blind CE instead of training-based method. While for the case with blind CE, semi blind CE provides less computational complexity compared to blind CE because most of the standard blind CE involves high computational complexity. Nevertheless, blind CE allowing higher spectral efficiency because no pilot symbols is required which means more data signal can be transmitted through the channel.

2.4.3 Training Based Channel Estimation

As for training based channel estimation(CE) also known as non-blind CE, the method involves the addition of pilot signals into either all of the subcarriers or some of the subcarriers of OFDM symbols with a specific period which enables estimation of the channel response at the pilot position [30],[29]. For insertion of pilot signals into all of the subcarriers of an OFDM symbol which is also known as training sequence is having the block-type pilot arrangement and it is viable in slow fading channel, where the channel is assumed to be constant over one OFDM period or more [29]. While the case of training based CE which is the insertion of pilot signals into some of the subcarriers of an OFDM symbol which having the comb-type pilot arrangement is applicable in fast fading channel, where the channel changes even in one OFDM period [29].

In general, training based CE can be studied into two approaches: Pilot Assisted Channel Estimation (PACE) and Decision Directed Channel Estimation (DDCE) [31],[5]. In PACE approach, training signals called pilot signals is known previously will be transmitted along with the data signals so that the receiver in OFDM system can easily estimates channel state information (CSI) from the received signals [5]. However, there is trade-off occurs in the implementation of this approach, the increasing in the pilot proportion in order to increase the estimation efficiency will lead to the reduction in the bandwidth efficiency or information rate. While for DDCE approach, it exploits both the pilot signals and the information signals for channel estimation [32]. This method allows the reduction of the number of pilot symbols required due to the absence of transmission errors, and therefore, more pilot information is benefited by using the detected subcarrier symbols as an a reference signal [32]. However, DDCE gets affected easily to the wrongly detected data which causes an error propagation, and therefore, this explained why the implementation of DDCE is suitable for slow fading channel [5] , [33].

2.5 Compressed Sensing

Compressed sensing (CS) has been widely applied in signal processing fields such as image processing and communication systems [23]. The application of CS in the communication system is focused in this research which can be applied to sparse channel estimation. Generally, Nyquist-Shannon sampling theorem defined the importance of sampling with frequency of at least twice the maximum frequency of the signal called Nyquist rate as shown in $Equation$ 2.1 in order to avoid the losing of information when capturing an original signal. Therefore, if the signal is sampled at a frequency that is lower than the Nyquist rate, aliasing may occurs, the presence of unwanted components in the reconstructed signal. However, due to the fact that the amount of data that can be acquired has grown tremendously with the upgraded capability of modern computational system, the sampling rate complexity in the practical applications increased which then, leads to major drawbacks such as costly data processing, difficulty in designing and building an appropriate sampling device and, limited storage as data amount increased in the real life processing [25]. This is when the data compression comes in handy. Data compression is used to reduce storage space and allows shorter and easier data transmission.

$$fs \geq 2fc \tag{2.1}$$

Equation 2.1 shows the mathematical terms of Nyquist-Shannon sampling theorem where fs is the sampling frequency and fc is the highest frequency contained in the signal respectively.

There are number of compression techniques have been proposed in the literatures [34]. However, nowadays, the attention of compression technique is placed on compressive sensing technique (CS) which is the alternative to Nyquist-Shannon sampling theorem. CS is the combination of compression and acquisition which is used to capture and represent compressible or sparse signals, defined as signal with very few large coefficients and more small coefficients at a rate significantly below the Nyquist rate while sustaining the relevant information in a given signal [35], [36], [37]. It provides solution to undetermined system of linear equation with the exploitation of sparsity or non-zeros of the signal as shown in Figure 2.5(a) and (b) [5].

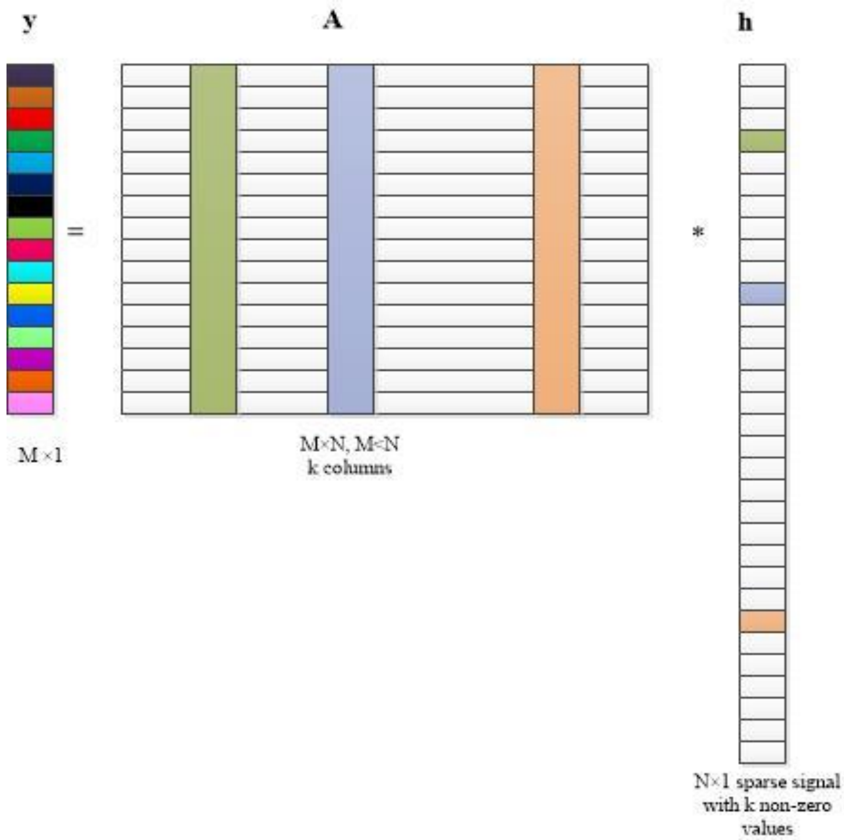

Figure 2.5(a): Undetermined system

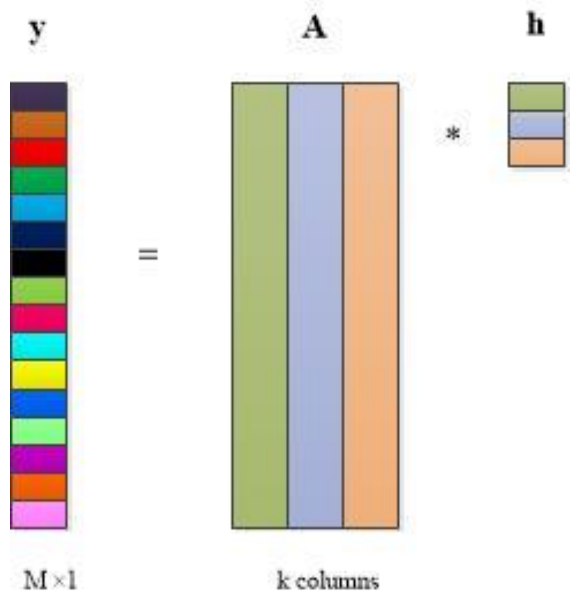

Figure 2.5(b): Linear combination

It can be stated mathematically as follow:

$$y = Ah + z \tag{2.2}$$

where y: observation vector of measurement matrix $M \times 1$

A: known measurement matrix $M \times N$

h: unknown sparse vector $N \times 1$

z: unknown vector of measurement noise

Vector h is said to be $k - sparse$ if at most k of its entries are consist of large coefficients or nonzero to be precise which is unknown, while the remaining coefficients are zero or negligibly small.

2.5.1 Restricted Isometry Property

Restricted Isometry Property (RIP) is an important property of measurement matrix A in CS technique [38], [39]. This property is a favorable condition for the recovery performance of the sparse signal [33], [34]. The definition of RIP can be presented as follows:

$$(1 - \delta_k \ )||h||_2^2 \ \leq || \ A_q h||_2^2 \ \leq \ (1 + \delta_k \ )||h||_2^2 \tag{2.3}$$

where δ_k : the k - RIP

A_q : reconstruction matrix (measurement matrix×representation matrix)

Based on $Equation$ 2.3, a matrix is said to obey the RIP of order k if δ_k is not too close to one [34]. RIP assures that all subsets of k columns is taken almost orthogonally from matrix A and the $k - sparse$ vector should not in null space of matrix A to sense it, or else vectors reconstruction will be failed [34], [40]. Likewise, if δ_{2k} is less than one, then all pair wise distances between k-sparse signals must be well preserved in the measurement space for k-sparse vectors h_1 and h_2, as shown by:

$$(1 - \delta_{2k})\|h_1 - h_2\|_2^2 \leq \| A_q(h_1 - h_2)\|_2^2 \leq (1 + \delta_{2k})\|h_1 - h_2\|_2^2 \qquad (2.4)$$

2.5.2 Incoherence

In spite of the fact that RIP can guarantee the recovery of sparse signals, it somehow having difficulty during the verification of given matrix to determine whether the matrix fulfil the property or not. Therefore, mutual coherence is exploited to solve the bottleneck of RIP. Incoherence of matrix is used to sample signal of interest, referred as measurement matrix A and the representation matrix, in which signal of interest is sparse which referred as representation matrix h. As the coherence property used to measure the performance of matrix A in terms of how well it preserves the information in $k - sparse$ vector signal, therefore, low coherence between A and h may results in fewer samples required for reconstruction of signal [34]. The coherence of a matrix A, (A), is the largest absolute inner product between any two columns A_m, A_n of A. The definition of coherence is mathematically represented as follow:

$$\mu(A) = \max_{0 \leq m < n \leq L-1} \frac{| A_m, A_n|}{\| A_m\|_2 \cdot \| A_n\|_2}$$

$$= \max_{0 \le m < n \le L-1} \frac{|\,(A_m^H A_n)\,|}{\|\,A_m\|_2 \cdot \|\,A_n\|_2} \qquad (2.5)$$

From Equation 2.5, when $(A) = 1,\ A_m$ and A_n are maximally incoherent.

It is important to have low level of similarity, the coherence or high level of dissimilarity which is known as incoherence between A_m and A_n in order to reconstruct a sparse signal.

2.6 Compressed Sensing Reconstruction Algorithm

CS contains a collection of methods in representing a signal on the basis of a limited number of measurements and then recovering the signal from these measurements. The approximation of a signal given noise vector is a major algorithmic challenge in CS technique [41]. To solve these complication, the literature discusses several related approaches which can be categorized into three main methods: Convex Relaxation Method, Greedy Pursuit Algorithm, and Combinational Algorithm. In this research Convex Relaxation Method and Greedy Pursuit Algorithm are considered for details discussion which will be presented next.

2.6.1 Convex Relaxation Method

Convex relaxation approach such as basic pursuit algorithm, solve a convex program whose minimizer is known to approximate the target signal [36]. Further discussion on basic pursuit algorithm will be presented in subsection 2.7.1(a). The number of