ENHANCED ALGORITHMS BY COMBINING GAUSS-SEIDEL AND NEWTON-RAPHSON IN LOAD FLOW ANALYSIS

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***Abstract*— Traditional load flow solution methods like Newton-Raphson has a great convergence characteristics with regards to its number of iterations and computing time, but suffers from poor convergence when used to solve ill-conditioned networks or if the starting initial values are far from solution. To overcome these concerns, we present enhanced algorithms for load flow analysis by combining Gauss-Seidel and Newton-Raphson methods that incorporate constant Jacobian to give a more dependable method with tolerable accuracy and shorter computation time.**

***Index Terms*— Gauss-Seidel method, Newton-Raphson method, Initial estimation, Constant Jacobian and Convergence**.

I. INTRODUCTION

Load flow analysis studies a systematic mathematical method to determine different parameters of electric power flows under steady state conditions. It is a tool that contains numerical methods for analyzing the electrical power system.

In the previous studies that were carried out on the traditional load flow solution methods such as Gauss-Seidel (GS) and Newton-Raphson (NR) methods, it was observed that there are possibilities for such methods to have poor convergence or even divergence from solution, particularly in ill-conditioned systems. Moreover, modern power systems have become very large, and thus lead to longer execution time which might be not acceptable when running load flow solution on-line.

Algorithm with better convergence capability is developed to provide solution of ill-conditioned systems with short execution time. In this paper, the idea of the enhanced load flow analysis by combining GS and NR methods is proposed to solve the gap of solution by implementing GS as a starting application to NR process.

II. LITERATURE REVIEW

Olukayode A. Afolabi et al. focused on determining the suitable and efficient method for efficient operation in the system load flow analysis [1]. It mentioned that power flow analysis method may take long execution time and therefore fail to achieve an accurate result for power flow solution because of continuous changes in power demand and generations.

NR method in load flow analysis may have difficulties to obtain convergence to unfeasible solutions using the traditional approach [2]. As a solution to the convergence problem, various techniques such as update truncation and factor relaxation were applied to increase the reliability of the results obtained. This paper discussed about the one-shot iteration of GS method to obtain the starting point estimation and use it for a better approximation of the initial value in NR method.

Y.Wang, Silva, L.C.P.D., W.Xu & Yzhang investigated the relationship between power flow ill conditional case and voltage instability, and carried out the critical review on load flow methods for well, ill and unsolvable condition [3].

Semlyen, A. & De Leon, F. demonstrated the successful use of Quasi-Newton power flow method with substantial computation time saving by applying Constant Jacobian and Partial Jacobian updates in Newton Raphson method [4]. The methodology has the provision of selecting the next step by examining the residuals. They indicated that Fast Decoupled Power Flow method fails to converge when the power system is close to its operating limits and when the R/X ratio is high for long transmission lines. Newton Power Flow (NPF) has excellent convergence properties even under stressed conditions, but Jacobian matrix needs to be factorized at every iteration and is time consuming. Therefore a constant Jacobian for a number of iterations to reduce the computational burden of a NPF solution is used.

III. REVIEW OF GAUSS-SEIDEL AND NEWTON-RAPHSON METHODS

A. Abbreviations and Acronyms

GS	Gauss-Seidel
NS	Newton-Raphson

NR-Jc	Newton-Raphson with constant Jacobian
GSNR	Gauss-Seidel- Newton-Raphson
GSNR-Jc	GS- NR with constant Jacobian
Ybus	Admittance Matrix
dP	Change in Real Power
dQ	Change in Reactive Power
dδ	Correction in Voltage Angle
dV	Correction in Voltage Amplitude
δ	Voltage Power Angle
ε	Specified Tolerance Value (Epson)
k	Iteration Number

B. Gauss-Seidel Methodology

This method uses successive displacement in solving the non-linear equation, such that the latest value of the bus voltage is immediately substituted in the equation of subsequent rows. The solution for the bus voltage and power flow equations is obtained when the difference between the voltage values of the successive iteration is less than a specified tolerance value ε.

Gauss-Seidel power flow equations:

$$(\boldsymbol{V_i})^{\boldsymbol{k+1}} = \frac{\frac{P_i^{sch} - jQ_i^{sch}}{V_i^{*(k)}} + \sum y_{ij} V_j^{(k)}}{\sum y_{ij}} \qquad \boldsymbol{j \neq i} \qquad \textbf{(1.1)}$$

$$\Delta V_i^{k+1} = (V_i)^{k+1} - (V_i)^k \qquad (1.2)$$

$$\Delta V_{i,acc}^{k+1} = (V_i)^k + \alpha\Delta(V_i)^{k+1} \qquad (1.3)$$

$$P_i^{(k+1)} = \Re\left\{V_i^{*(k)}\left[V_i^{*(k)}\sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j^{(k)}\right]\right\} \qquad (1.4)$$

$$Q_i^{(k+1)} = -\Im\left\{V_i^{*(k)}\left[V_i^{*(k)}\sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j^{(k)}\right]\right\} \qquad (1.5)$$

$$\Delta V_i^{k+1} \leq \varepsilon \qquad (1.6)$$

C. Newton Raphson Methodology

This method is an iterative method which approximates a set of non-linear simultaneous equations to a set of linear simultaneous equations. The solution for the bus voltage and power flow equations is obtained when the maximum power mismatch in dP and dQ values of the successive iteration is less than a specified tolerance value ε.

NR's method is mathematically superior to GS method because of its quadratic convergence property when near the solution, this method is found to be more efficient and practically used for large power system as the number of iteration is independent of the power network size.

Newton Raphson power flow equation:

Calculate, $P_i^{[k]}, Q_i^{[k]}$ (2.1)

$$\text{Calculate, } \Delta P_i^{[k]} = P_{i\,sch} - P_i^{[k]} \qquad (2.2)$$

$$\Delta Q_i^{[k]} = Q_{i\,sch} - Q_i^{[k]} \qquad (2.3)$$

Solve equation for correction vector

$$\begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \qquad (2.4)$$

Update new Voltage and Angle

$$\delta_i^{[k+1]} = \delta_i^{[k]} + \Delta\delta_i^{[k]} \qquad (2.5)$$

$$\left|V_i^{[k+1]}\right| = \left|V_i^{[k]}\right| + \Delta\left|V_i^{[k]}\right| \qquad (2.6)$$

Check for convergence

$$\left|\Delta P_i^{[k]}\right|, \left|\Delta Q_i^{[k]}\right| \leq \varepsilon \qquad (2.7)$$

IV. ALGORITHIMS ENHANCEMENT STRATEGIES

The strategy used in this paper to reduce the computational time of NR process is by keeping the Jacobian matrix fixed as constant after the first iteration and then same values are to be used in the following iterations to approach solution. With applying constant Jacobian, the number of iterations usually increases but overall computing time is reduced since the computation burden at each iteration is less compared to full Jacobian calculation.

The strategy used to improve NR sensitivity to starting values is implemented by using GS as a starting application to NR so that a better approximation values are used as initial point for NR. Poor convergence and the method more robust to solve the load flow problem is avoided which are kept on their operation condition.

V. COMBINED GAUSS-SEIDEL WITH NEWTON RAPHSON-CONSTANT JACOBIAN METHOD

This is implemented by modifying the traditional Gauss-Seidel process, removing the stage where the process examine for the maximum mismatch in change in voltage and replacing it with a break statement to exist the iteration process after the 1st or 2nd iteration at which an updated value of bus voltages are obtained from the flat estimated ones. The results are closer to the desired solution and these updated results are used as an input starting point for Newton's method iterative process to find the solution with the required criteria when the maximum change in residual power values become less than the required tolerance.

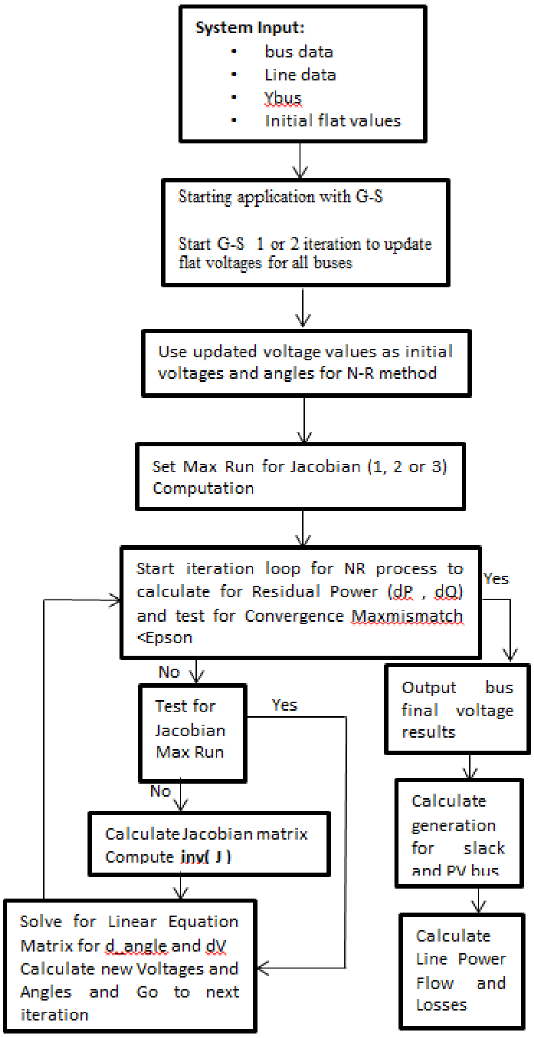

Figure 1: Abbreviated flow chart for GSNR combined System

Since most of the time in the Newton-Raphson iteration is spent in computing the Jacobian and factorization.

When first iteration is completed the Jacobian matrix has now been initialized, after the first or second iteration Jacobian matrix shall be maintained constant and used for following iterations, this is allowed because Jacobian elements values remains almost the same and that will give good approximation [4]. Keeping constant Jacobian matrix aims to reduce the unnecessary computational burden encountered with computing its elements and finding the matrix inverse when solving for the simultaneous equations in every iteration.

VI. Performance Evaluation / Testing

4.1 The "Combined GS with NR-Jc Method" has been tested with well-conditioned systems in table 1 and results were compared with the traditional GS and NR standalone methods. The proposed method is found to be very accurate and with improved total number of iteration, computational time as per Figure 2.

Table 1: Description of three test systems

Number of Description	IEEE 14bus	IEEE 30 bus	IEEE 39 bus
Buses	14	30	39
Generators	5	6	10
Slack bus	1	1	1
Load buses	9	24	29
Generation buses	4	5	9
Transformers	3	4	12
Branches	20	41	46
Inject capacitor	1	2	0

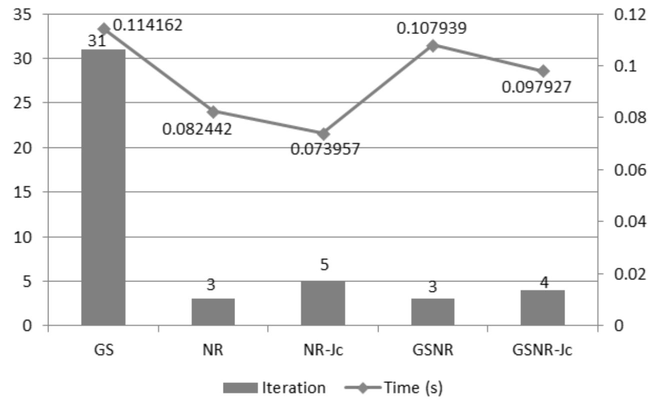

Figure 1: Iterations and Time Comparison of 30Bus System

For IEEE 30bus combined system, the convergence plot and performance table are shown in Figure 3 and Table 2 respectively.

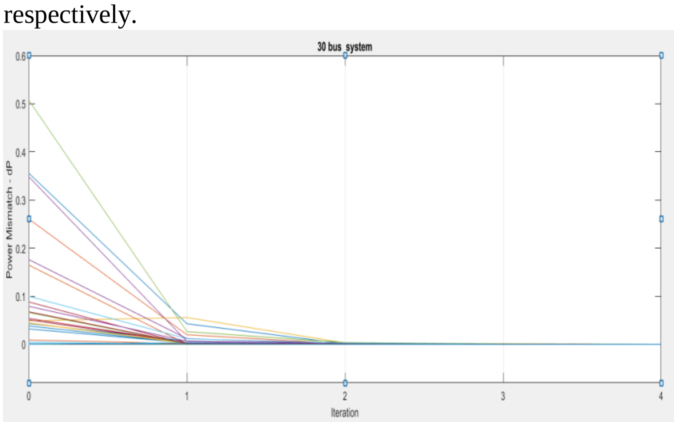

Figure 2: Convergence plot of GSNR-Jc for IEEE 30bus system

Table 2: GSNR-Jc Performance Table for IEEE 30Bus

Iter.	Time (s)	Tolerance	Jacobian Run	GS Run	Convergence
4	0.0979	0.001	1	1	Yes

Results and Discussion:

- GS shows highest number of iterations for convergence to solution and number of iterations increases with size of network.
- NR method is fast and has least number of 3 iterations because of its quadratic convergence property.
- Slight time saving is observed with applying constant Jacobian compared to traditional NR method. Number of iteration increases when applying constant Jacobian in the calculation but with reduced computational burden for each iteration.

- The time saving as effect of applying constant Jacobian is small, this is because the tests were conducted on small size networks. However, considerable time saving will be realized with large bus systems, since Jacobian factorization accounts for about 85% of the CPU time [4].

The combined system also has been tested for robustness performance under starting initial guess and ill-conditioned system with high R/X ratio cases on IEEE 30 bus system:

4.2 For initial voltage stability test, the method performance results as per Table 3.

Test for underestimated and overestimated initial bus voltage is carried out the IEEE 30bus system by considering the heavy loaded bus in the system. PQ bus no.21 is selected and the initial voltage values in the practical region 1.0±6% of the nominal voltage.

Table 3 shows a comparison between GS, NR and GSNR algorithms. It is showing the initial voltage ranges under which convergence to solution, poor convergence or divergence occurs, noting that some values in the table are beyond the practical voltage which was rarely used. This is for numerical methods testing purpose only to find the extreme limits.

Table 3: Test results for load flow solution with initial values 1.0 ± 6 % of the nominal voltage

Bus 21 initial voltage	**GS**	**NR**	**GSNR**
1.06	YES	Poor convergence	YES
1.05	YES	Poor convergence	YES
1.04	YES	YES	YES
1.03	YES	YES	YES
1.02	YES	YES	YES
1.00	YES	YES	YES
0.99	YES	YES	YES
0.98	YES	YES	YES
0.97	YES	YES	YES
0.96	YES	YES	YES
0.95	YES	Poor convergence	YES
0.94	YES	Poor convergence	YES

Table 4 shows, NR starts to give poor convergence with initial values higher or equal to 1.05 and lower or equal to 0.95 pu V which are still in the practical region. But GSNR higher or equal to 1.39 and lower or equal to 0.71 pu V. NR provide correct solution within a narrow range between 0.96 to 1.04 pu V but GSNR can give solution with wider range of initial voltages that covers the nominal voltage 1.0±6% and even beyond the practical margin for $1.38 \geq V \geq 0.72$ pu V.

Table 4: Voltage range comparison of NR and GSNR programs

Voltage range	**Correct solution**	**Poor convergence**	**Divergence**
NR	$1.04 \geq V \geq 0.96$	$1.05 \leq V \leq 0.95$	$1.17 \leq V \leq 0.84$
GSNR	$1.38 \geq V \geq 0.72$	$1.39 \leq V \leq 0.71$	$V \leq 0.42$

Results and Discussion:

- GS have converged to solution.
- GSNR technique performs better in term of convergence stability than NR and it is proven to be more robust with underestimated and overestimated initial voltage value.
- NR diverts when less than 0.85 or above 1.16 but GSNR diverts when less than 0.42.

4.3 For ill-conditioned system with high R/X ratio For IEEE 30bus system data is used with modification to line 1 by raising the resistance value in multiple to make high R/X ratio. A test is carried out with different program to find their stability under high R/X ratio and results as in Table 5.

Table 5: 30bus system high R/X ratio test for convergence performance

Line 1 - IEEE 30bus	**Iterations**		
new R/X	**GS**	**NR**	**GSNR**
0.33	37	3	3
0.67	55	8	6
1.00	62	8	8
1.34	72	N/C	6
1.67	85	N/C	7
2.00	141	N/C	7
2.34	157	N/C	8
2.67	177	N/C	10
3.01	194	N/C	N/C
3.34	224	N/C	N/C
3.67	276	N/C	N/C
4.01	N/C	N/C	N/C

Where N/C is no convergence

NR method successfully converges to solution for R/X ratio equal to 1.0 or less, but failed to converge at R/X ratio 1.34 and higher. GSNR combined method remains stable with ratio increased 8X, but diverged beyond 9X when R/X ratio is 3.0. GS method continues to provide solution until R/X ratio is 3.67.

Results and Discussion:

- Networks with high R/X ratio can significantly influence the convergence. The number of iterations increase as R/X ratio increases and this leads to slower convergence or may cause divergence [3].
- Failure of traditional NR method is due to the instability of the numerical method.
- GSNR technique performs better in term of convergence stability and it is proven to be more robust with ill-conditioned systems with much higher line R/X ratio than for NR method.

VII. Conclusion & FUTURE WORKS

A. Conclusion:

Newton Raphson method is generally implemented due to its convergence characteristics such as the number of iterations and computing time for convergence is excellent and independent of network size. However, it fails to converge to a solution with underestimated initial values or under ill-conditioned systems. Gauss-Seidel method is found to be successful in solving nonlinear power equations but number of iterations and CPU time is relatively high. Besides, it gives inaccurate result when the initial values are far from the results. In this paper GSNR combined method proves to be a robust algorithm and can give better convergence performance with underestimated initial values and also under high R/X ratio ill-conditioned cases. Besides, it gives a considerable computational time saving by applying constant Jacobian technique with acceptable solution accuracy.

B. Future Works:

In this paper, introductory of different stability method are reviewed. However, further investigations are need to consider taking advantage of the sparsity found with the Ybus Admittance matrix and Jacobian matrix to avoid the unnecessary computation with the zeros elements and to save large memory storage.

Further researches are recommended for other nature of ill-conditioning causes to overcome difficulties with voltage instability in load flow solution and instability of the numerical methods.

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