A MINIATURIZED 2.4 GHz WILKINSON POWER DIVIDER USING COMPLEMENTARY SPLIT RING RESONATOR

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by

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SYMBOLS

f_o	Operating frequency
dB	Decibel
Ω	Ohm
λ	Wavelength
Γ	Transmission coefficient
Z	Impedance
Z_o	Characteristic impedance
L	Inductance
C	Capacitance
B	Susceptance
c	Speed of light
ϵ_o	Permittivity in vacuum
ε_r	Relative permittivity of substrate
μ	Permeability
E_eff	Effective electrical length
^{o}	Degree
GHz	Giga Hertz
mm	millimetre

ABBREVIATIONS

ADS	Advance Design System
CST	Computer Simulation Technology
CSRR	Complementary Split Ring Resonator
OCSRR	Open Complementary Split Ring Resonator
RF	Radio Frequency
SRR	Split Ring Resonator
VSWR	Voltage Standing Wave Ratio
WPD	Wilkinson Power Divider
SMA	Sub-Miniature Version A
Etc	Et cetera
PTFE	Polytetrafluoroethylene
SMD	Surface Mount Device
S-Parameters	Scattering parameters
VNA	Vector Network Analyzer

PENGECILAN PEMBAHAGI KUASA 2.4 GHZ DENGAN MENGGUNAKAN PELENGKAP PENYALUN GELANG BELAH.

ABSTRAK

Pembahagi kuasa Wilkinson (WPD) adalah elemen penting dalam bidang kejuruteraan gelombang mikro. Walau bagaimanapun, WPD konvensional adalah tidak sempurna kerana ia mempunyai saiz yang besar disebabkan 90^0 garis cawangan yang membawa kepada kemahalan dan sukar diurus. Beberapa kaedah telah dicadangkan untuk mengatasi kelemahan ini dan yang paling popular di antaranya ialah dengan melaksanakan struktur penyalun berasakan bahan-meta. Dalam kajian ini, pelengkap penyalun gelang belah (CSRR) digunakan untuk mengecilkan WPD sambil mengekalkan ciri-cirinya. Pemodelan dan mereka bentuk WPD dan CSRR pada frekuensi 2.4 GHz dan mempunyai julat frekuensi yang tinggi adalah fokus utama dalam kajian ini. WPD dan CSRR telah dimodel dengan menggunakan perisian CST Studio. Reka bentuk WPD kemudiannya dieksport ke susun atur CSRR untuk digabungkan supaya menjadi pembahagi kuasa padat. Model-model ini dicetak atas papan litar RO4003C dengan pemalar elektrik 3.38 dan ketebalan sebanyak 0.813 mm. Model yang dicetak akan dianalisis dan diuji tentang fungsi dan prestasi dengan menggunakan alat penganalisis Agilent PNA-X Network. Parameter penyerakan seperti pekali pantulan, pekali penghantaran dan pengasingan yang tinggi dipertimbangkan dalam projek ini. Keseluruhan, WPD yang diusulkan beroperasi pada frekuensi 4.744 GHz dengan pembahagian kuasa yang hamper sama iaitu -4.3786 dB, pengasingan antara lorong pengeluar -29.58 dB dan pengurangan saiz adalah 58.82% lebih kecil dari konvensional.

MINIATURIZED A 2.4 GHz POWER DIVIDER USING COMPLEMENTARY SPLIT RING RESONATOR.

ABSTRACT

Wilkinson power dividers (WPD) are an essential element and most popular in the RF and microwave fields. However, a conventional power divider is imperfection as it has a large size because of the 90° branch line which leads to expensive and unmanageable device. Several methods have been proposed to overcome this flaw and the most popular of among these methods is by implementing metamaterial-based resonator. Thus, for this research, complementary split ring resonator (CSRR) is used to miniaturize the WPD while retaining the features. The CSRR is etched on the ground plane. Modelling and designing a conventional WPD and a compact WPD having a wider Bandwidth operate at 2.4 GHz are the main focus of this research. The WPD and CSRR have been designed by using CST Studio Suite software. The design of conventional WPD is exported to CSRR layout to combine for miniaturized WPD. The models are fabricated on Roger RO4003C printed circuit board (PCB) which has a dielectric constant of 3.38 and height of 0.813 mm. Fabricated model is analysed and tested for functionality and performance by using Agilent PNA-X Network Analyser. Scattering parameters (S-parameters) such as reflection coefficient, transmission coefficient and high isolation are considered in this project. Overall, the proposed WPD operates at 4.744 GHz with retun loss of -22.55 dB and almost equal power division of -4.3786 dB, the isolation between the output ports is -29.58 dB and the size reduction is 58.82% smaller than the conventional.

CHAPTER 1

INTRODUCTION

1.1 Motivation of the Project

In most circumstances, power dividers (also known as power splitter) are a passive transmission component that play an integral role in RF and microwave systems functioning as power amplifiers, balanced mixers, an antenna array feed networks for distribution of low-power signal, laboratory equipment and phase shifters. The fundamental function of a power divider is it is used for electromagnetic power division, where an input signal is divided into two (or more) output signals of lesser power and split specific amplitude and phase characteristics equally. In designing the microwave circuits, the size reduction, filtering unwanted harmonics and wide bandwidth need to be considered. Planar microwave components exhibit, generally, large dimensions. Thus, to obtain size reduction of the components, different techniques have been conducted. An example could be the use of EBG cell for the suppression of harmonics and size reduction as in [1]. However, in recent times, the alternative to achieve a high degree of miniaturization is by using metamaterial-based transmission lines, consisting on a host line loaded with reactive elements. As in [2], a Wilkinson power divider with CSRR is presented and 37% of size reduction is achieved. A compact 1:4 Lossless T-junction power divider with OCSRR is discussed in [3] and overall size is reduced to 51.94%. Planar microwave devices by means of CSRs to miniaturize is reported in [4]. In this project, a circular three ports equally Wilkinson power divider loaded with CSRR is proposed and analysed in order to minimize the dimension of divider. To design an ideal power divider, the desired requirements needed are it should be lossless, or very low-loss, reciprocal, and easily matched at each port, but this is not possible [1]. However, there are power dividers that demonstrate two of the three properties. T-junction dividers, resistive dividers, and the Wilkinson power divider are three common power dividers featuring unique characteristics. Among these, Wilkinson power divider is picked for this project as it has the best attribution such as achieving completely matched output ports with high isolation between them while CSRR is counterpart of resonant structure split ring resonator (SRR) [5] and mostly used in size reduction and performance enhancement of microwave devices.

1.2 Problem Statement

The Wilkinson power divider is often use in microwave applications due to its simplicity and a high isolation between ports. This device also potentially lossless provided no reflected power from output ports enter into it.

However, as the Wilkinson divider is based around the use of quarter wave transmission lines, it has a limited bandwidth. Quarter wavelength long transmission lines are used as the basic building section also resulting in a significant circuit size. In wireless communication systems, usually smaller device size is required in order to meet circuit miniaturization and cost reduction. Thus, size reduction is becoming major design considerations for practical applications even though the size of conventional Wilkinson power divider is smaller in low microwave frequency range, but it still too large for some applications.

The other drawback of conventional Wilkinson power divider is presence of spurious pass-band due to high orders harmonic suffered from even and odd-mode analysis employed for a single-section Wilkinson power divider which can affect the performance of the circuit drastically, especially when the dividers are applied in wireless system [6].

1.3 Objectives of the Project

The main purpose of this project is to design a circular Wilkinson power divider having smaller in size, low cost and wider Bandwidth by implementing CSRR structure without changing its properties.

- To design a compact Wilkinson power divider operating at 2.4 GHz and having wider bandwidth by using CSRR.
- To test and compare the performance between a conventional and a compact Wilkinson power divider.

1.4 Scope of the Project

There are three sections to be focused in this project and all sections involving designing, namely design a conventional WPD, design a CSRR and lastly design a compact WPD embedded with CSRR. There are many types of transmission lines can be designed such as waveguides, coaxial lines, microstrip lines and strip-lines. In this project, microstrip line is chosen because it is easily miniaturized and integrated with both passive and active microwave devices. Design of circular microstrip Wilkinson power divider must obtained the operating frequency at 2.4GHz while the resonant frequency of CSRR should be the same with the frequency of WPD. Return loss of the design is expected to be below -15 dB to meet the condition of good transmission lines and the insertion loss is expected to be nearly -3 dB so that the signal is divide equally between the output ports. Other expectation is the size of a compact WPD with CSRR must be small than the conventional.

1.5 Thesis Organization

This thesis is divided into five chapters. The first chapter is about the introduction of the project. The sub-topics include motivation in designing a power divider with CSRR, problem statement, objectives, scope of the project and the chapter's organization.

Chapter 2 presents the literature review of RF and microwave fields. The fundamental of power dividers is been discussed. Theory of SRR, CSRR and OCSRR are studied for more understanding. Lastly, related work of power divider embedded with CSRR is analysed.

For chapter 3, the method used to develop the project is presented. The flowchart of the design is provided for better understanding on how this project is conducted. Development of each model is explained in very details and the software used for this project is stated in this chapter.

Results and discussions are placed in Chapter 4. This chapter give information about the results obtained from the analyser and how much the size of a compact design reduced compared to the conventional.

The last chapter in this thesis is Chapter 5. This chapter will conclude the overall works done over the time. It will describe how the project can be improved for the future work.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

In designing power divider implemented with CSRR, three main topics are be addressed which are power divider, CSRR and the structure of power divider with CSRR. The study of fundamental background of different types of power dividers is performed in order to choose the best performance of power divider. For CSRR part, the theory and design of SRR is reviewed as CSRR is a counterpart of SRR resonator. Next, the characteristics of CSRR that can miniaturize and the combination of power divider and CSRR are analysed. Apart from that, some of published works that are related to power dividers and split ring resonators are consulted in this chapter in order to get some ideas and enhance understanding on this project.

2.2 Power dividers

In most circumstances, power dividers are a passive transmission structure that divide radio frequency (RF) input power among several output signals of lesser power and split specific amplitude and phase characteristics equally. In addition, power dividers can accomplish several functions of signal processing such as:

- Add or subtract signals vectorially.
- Obtain multi in-phase output signals (with an equal power division ratio (3 dB) or unequal power division ratios) with proportional to the level of a common input signal.
- Provide a capability to obtain RF logic arrangements.

Theoretically, the characteristics of the output signals in power divider are equal amplitude, high isolation between each output signal and 0^{o} phase relationship between any two output signals [7]. As power divider is a reciprocal passive device, it may be used as a power combiner simply by applying each signal singularly into each of the divider output ports but in this work, the focus will be given to the power divider with equal split power division (3.dB) only. The power divider may have three ports, four ports or more and may be (ideally) lossless. Three port networks take form from of T-Junctions and other power dividers, while four-port networks take the form of directional couplers and hybrids. For this project, three port power divider is to be designed. Various type of waveguide couplers and dividers were invented and characterized at the MIT Radiation Laboratory in the 1940s. These included E- and H-*plane* waveguide T-junction, the Bethe hole coupler, multirole directional coupler, the waveguide magic-T and others kind of couplers using coaxial probes [8]. Stripline or microstrip technology is then introduced in the mid-1950s through the 1960s for reinvention of those power dividers and couplers. To design an ideal power divider, the desired requirements needed in the circuit design are it should be lossless, or very low-loss, reciprocal, and easily matched at each port, but this is not possible. However, there are power dividers that demonstrate two of the three properties which is three-port dividers, as depicted in the Figure 2.1. T-junction dividers, resistive dividers, and the Wilkinson power divider are three common power dividers featuring unique characteristics thus these dividers are nominated in this project. Based on the Figure 2.1, P_1 is the input port whereas P_2 and P_3 are the output ports with equally power division.

Figure 2.1 Block diagram of power divider [8].

2.2.1 Basic Properties of Three-Port Networks

Basically, there are nine independent elements in the scattering matrix (S-matrix) of three-port network [8]. The scattering matrix is as in equation (2.1) [8].

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \tag{2.1}$$

Since power dividers are a passive component and if it contains no anisotropic materials, thus it must be reciprocal and the scattering matrix will be symmetric ($S_{ij} = S_{ji}$). For an ideal power divider, the desired requirements needed in the circuit design are it should be lossless, or very low-loss, reciprocal, and easily matched at each port, but this is not possible for three-port networks. If all ports are matched, then $S_{ii} = 0$ while the S-matrix of (2.1) will reduce to (2.2) if the network is reciprocal.

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \tag{2.2}$$

For lossless network, the unitary properties need to be satisfied by S-matrix which required by energy conservation that leads to the following condition [8]:

$$|S_{12}|^2 + |S_{13}|^2 = 1 \tag{2.3a}$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \tag{2.3b}$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \tag{2.3c}$$

$$S_{13}^{*}S_{23} = 0 \tag{2.3d}$$

$$S_{23}^{*}S_{12} = 0 \tag{2.3e}$$

$$S_{12}^{*}S_{13} = 0 \tag{2.3f}$$

From the above equations, it shows that three-port network cannot be simulteneously lossless, reciprocal and matched at all ports because the condition will be always inconsistent unless if one of these three conditons is at rest, then a physically realizable device is possible. If the three-port network is matched and lossless but not reciprocal, thus the device is known as *circulator* which makes $S_{ij} \neq S_{ji}$. To indicate the circulator is nonreciprocal, the S-matrix of a matched three-port network has the following term [8]:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \tag{2.4}$$

And the [S] must be unitary as shown below for lossless network.

$$S_{31}^{*}S_{32} = 0 \tag{2.5a}$$

$$S_{21}^{*}S_{23} = 0 \tag{2.5b}$$

$$S_{12}^{*}S_{13} = 0 \tag{2.5c}$$

$$|S_{12}|^2 + |S_{13}|^2 = 1 \tag{2.5d}$$

$$|S_{21}|^2 + |S_{23}|^2 = 1 \tag{2.5e}$$

$$|S_{31}|^2 + |S_{32}|^2 = 1 \tag{2.5f}$$

The above equations are simplified into:

$$S_{12} = S_{23} = S_{31} = 0, \qquad |S_{21}| + |S_{32}| + |S_{13}| = 1 \tag{2.6}$$

This result prove that $S_{ij} \neq S_{ji}$ which implies that the device must be non-reciprocal. Next, if the three-port network is set to be lossy, it can be reciprocal and matched at all ports; and this case is called as resistive divider. Additionally, a lossy three-port network can be made to have isolation between its output ports.

2.2.2 The T-Junction Power Divider

A simple "T' connection is the most basic form of a power divider which has one input and two outputs as can be seen in Figure 2.2. It can be implemented in virtually any type of transmission line medium.

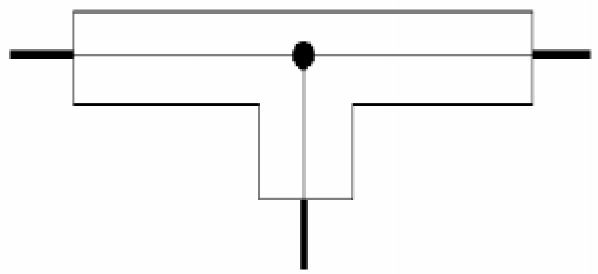

Figure 2.2 Block diagram of basic T-junction power divider [8].

General form of T-junction structures used in waveguide and microstrip-line are presented as in Figure 2.3 [8]. However, these are the lossless junctions as it is caused by the absence of transmission line loss.

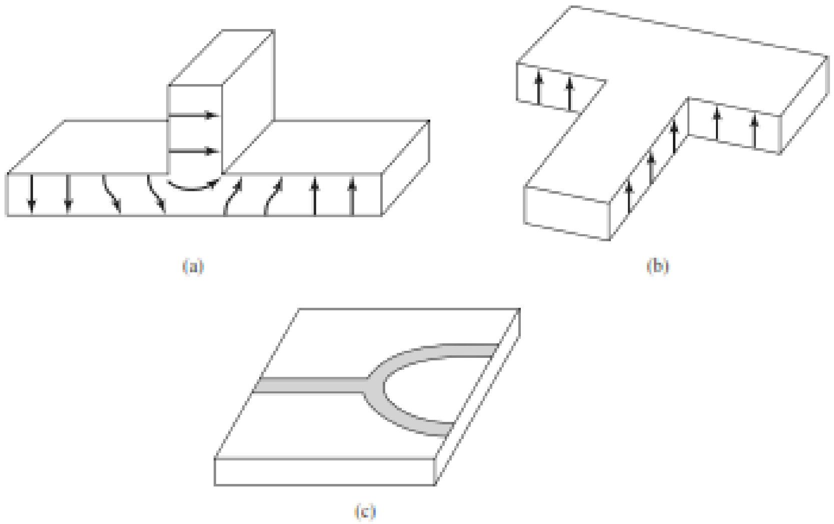

Figure 2.3 Different types of T-junction power dividers. (a) E-plane waveguide T. (b) H-plane waveguide T. (c) Microstrip line T-junction divider [8].

These lossless junctions can be replicated as a junction of three transmission lines as shown in Figure 2.4 [8].

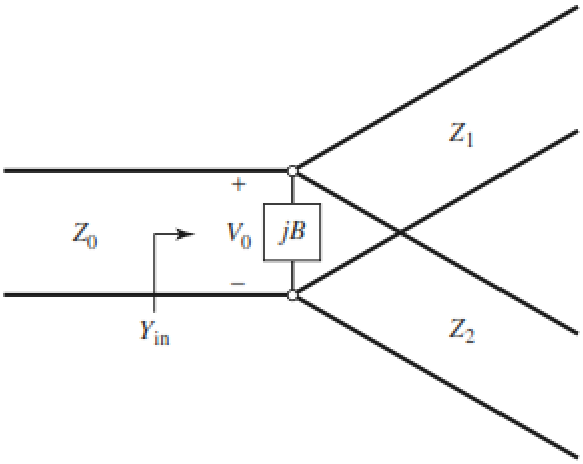

Figure 2.4 Model of a lossless T-junction divider transmission line [8].

Basically, there may be fringing fields and higher order modes associated with the discontinuity at such a junction, leading to stored energy that can be accounted for by a lumped susceptance, *B*.

In order for the input line of characteristic impedance Z_o to be matched with power divider,

$$Y_{\text{in}} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}. \tag{2.7}$$

The characteristic impedances are real if the transmission lines are assumed to be lossless. If let *B*=0, then the equation (2.7) can be simplified into

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0} \tag{2.8}$$

Literary, some type of discontinuity compensation or a reactive tuning element can usually be used to cancel susceptance if *B* is not negligible, at least over a narrow frequency band. To provide various power division ratios, the output line impedances, Z_1 and Z_2 can be selected. In addition, quarter-wave transformers can be used to bring the output line impedances back to the desired levels. The input line will be matched if the output lines are matched. It is capable of being reciprocal, but has the drawbacks of lacking isolation between the output ports and an inability to be matched at all ports. This lack of isolation between ports can limit the usefulness of three-port junctions, particularly in power monitoring, combining and divider applications.

2.2.3 Resistive Power Divider

In a resistive power divider, both output signals are 6dB lower than the input signal, and they are in phase. This type of an equal-split divider (-3dB) is shown in Figure 2.5 and is constructed from three resistors. A three-port divider can be made to be matched at all ports if it contains lossy components even though the two output ports may not be isolated.

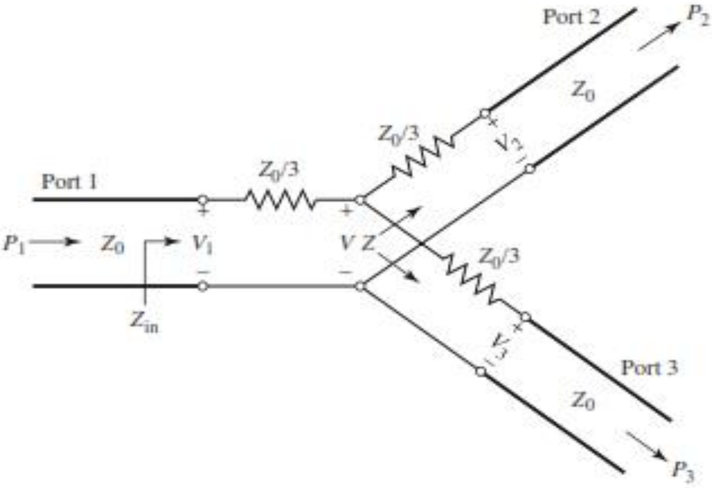

Figure 2.5 An equal-split three-port resistive power divider [8].

The circuit from Figure 2.5 can be analysed by using fundamental of circuit theory. All ports from divider are assumed to be terminate in the characteristic impedance Z_0, the impedance Z, seen looking into the $\frac{Z_0}{3}$ resistor followed by a terminated output line, is [8]

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3} \tag{2.9}$$

Thus the input impedance Z_{in} of the divider is

$$Z_{in} = \frac{Z_0}{3} + \frac{2Z_0}{3} = Z_0 \tag{2.10}$$

which shows that the input is matched to the feed line. As the network is symmetric from all three ports, the output ports are also matched. So $S_{11} = S_{22} = S_{33} = 0$. When the voltage, V_1 is at port 1, then the voltage V at the centre of the junction is as in equation (2.11) by using voltage division.

$$V = V_1 \frac{\frac{2Z_0}{3}}{\frac{Z_0}{3}+\frac{2Z_0}{3}} = \frac{2}{3} V_1 \tag{2.11}$$

and the output voltages are,

$$V_2 = V_3 = V_1 \frac{Z_0}{Z_0+\frac{Z_0}{3}} = \frac{3}{4} V = \frac{1}{2} V_1 \tag{2.12}$$

Hence $S_{21} = S_{31} = S_{23} = \frac{1}{2}$, so the output powers are 6dB below the input power level. This network become reciprocal, thus the scattering matrix is symmetrical, and it can be write down as

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}. \tag{2.13}$$

The power delivered to the input of this divider is

$$P_{in} = \frac{1}{2}\frac{V_1^2}{Z_0} \tag{2.14}$$

while the output powers are

$$P_2 = P_3 = \frac{1}{2}\frac{\frac{1}{2V_1^2}}{Z_0} = \frac{1}{8}\frac{V_1^2}{Z_0} = \frac{1}{4} P_{in} \tag{2.15}$$

which shows that half of the supplied power dissipated in the resistors. For resistive power dividers, they can be implemented to ensure that the same impedance is achieved at all ports, however loss is then introduced to the power divider.

2.2.4 Wilkinson Power Divider

Differ with T-junction power divider and resistive power divider, Wilkinson power divider is the main device in the RF and microwave system due to its simplicity and sufficiently high isolation between two divider output ports. Basically isolation between the output ports of a power divider can be described as the ability of a signal at one port does not affect the signal at another port [8]. Isolation between two output ports are very important for a three-port network in order to reduce the cross-talk due to the coupling between the ports. The performance of power divider can eventually be improved with isolation between the output ports.

Wilkinson power divider is an example of a reciprocal matched three port network. It was invented by an engineer named Ernest Wilkinson. It splits an input signal into two equal phase output signals. Wilkinson relied on quarter-wave transformers to match the split ports to the common port. Because a loss-less reciprocal three-port network cannot have all ports simultaneously matched, thus a resistor is added. The resistor does a lot more than allow all three ports to be matched, it fully isolates port 2 from port 3 at the center frequency. The resistor adds no resistive loss to the power split from port 1, so an ideal Wilkinson splitter is 100% efficient.

Theoretically, the Wilkinson power divider can be made with arbitrary power division and equal-split (3 dB) division but in this project, an equal-split division is considered.

Microstrip line form and transmission line circuit of this divider is shown in Figure 2.6.

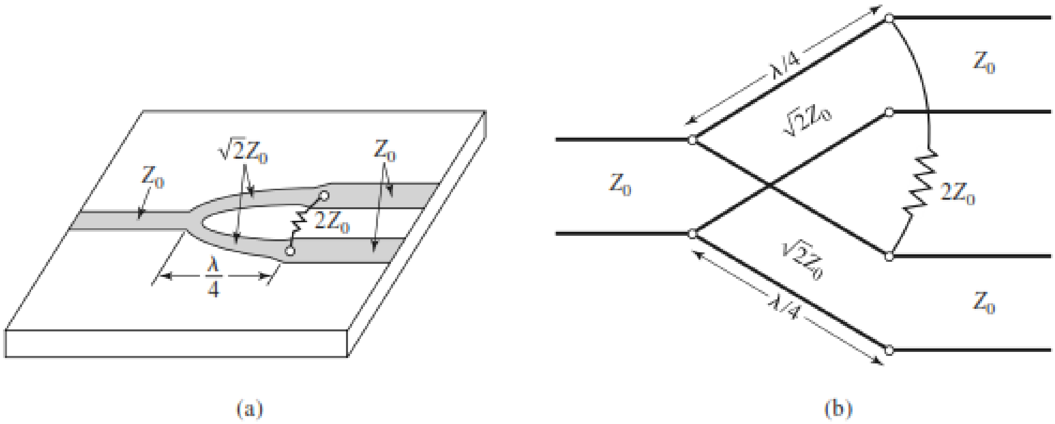

Figure 2.6 The Wilkinson power divider. (a) An equal-split Wilkinson power divider in microstrip line form. (b) Equivalent transmission line circuit [8].

The "even-odd" mode analysis technique is used to analyse the Wilkinson circuit. The technique is by reducing the circuit to two simpler circuits driven by symmetric and anti-symmetric sources at the output ports. To make it easier, all the impedances can be normalize to the characteristic impedance Z_0 and the circuit from Figure (2.6b) is been redraw with voltage generator at the output ports as dedicated in Figure (2.7).

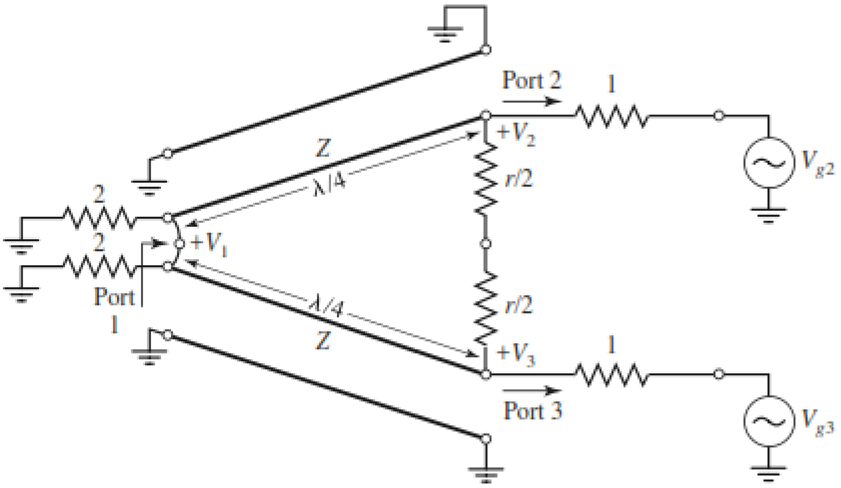

Figure 2.7 Normalized and symmetric form of Wilkinson divider circuit [8].

To sum up, based on [8], the scattering parameters of Wilkinson power divider can be summarized as follow [8]:

$$S = \begin{bmatrix} 0 & {}^{-j}\!/\!_{\sqrt{2}} & {}^{-j}\!/\!_{\sqrt{2}} \\ {}^{-j}\!/\!_{\sqrt{2}} & 0 & 0 \\ {}^{-j}\!/\!_{\sqrt{2}} & 0 & 0 \end{bmatrix} \tag{2.16}$$

where

$S_{11} = 0$; $Z_{in} = 1$ at port 1 (2.17)

$S_{22} = S_{33} = 0$; ports 2 and 3 matched for even and odd modes (2.18)

$S_{12} = S_{21} = \frac{V_1^e + V_1^0}{V_2^e + V_2^0} = -\frac{j}{\sqrt{2}}$; symmetry due to reciprocity (2.19)

$S_{13} = S_{31} = -\frac{j}{\sqrt{2}}$; symmetry of ports 2 and 3 (2.20)

$S_{23} = S_{32} = 0$; due to short or open at bisection (2.21)

Take note that no power is dissipated in the resistor when the divider is driven at port 1 and the outputs are matched. Only reflected power from ports 2 or 3 is dissipated in the resistor. Port 2 and port 3 are isolated as $S_{23} = S_{32} = 0$.

The S-parameters of WPD is shown in Figure 2.8. Note that ports 1, 2 and 3 are matched. The equal power distribution can be seen by the overlapping curves of S_{21} and S_{31} and the isolation is shown by graph S_{32} [9].

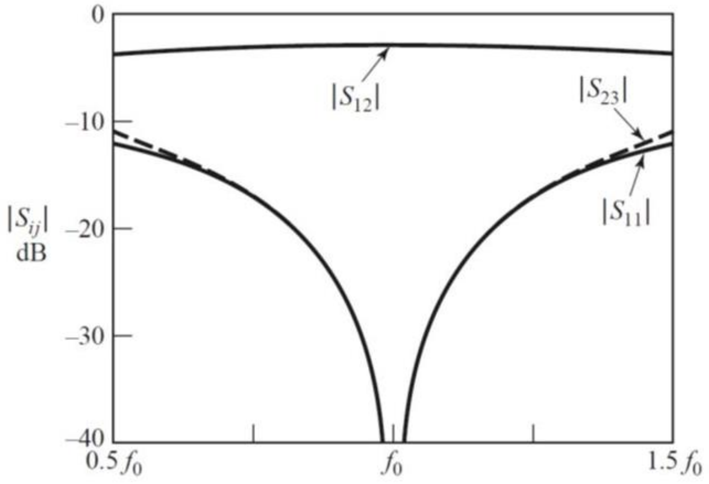

Figure 2.8 Frequency response of WPD [8].

2.3 Resonators

Resonators are key element in radio frequency (RF) and microwave engineering. Most of passive and active devices and antennas contain resonant elements in their design. There are various types of resonators of interest for RF and microwave applications such as cavity resonators, dielectric resonators, acoustic resonators, wave resonators, semi-lumped resonators and others. In this project, semi-lumped resonators, namely, planar resonators, are used because of their special behaviour of electrically small resonant elements which can reduce the dimension to be much smaller than the signal wavelength, λ, at desired resonance frequency. This is because such resonators are made up from a unique material known as *metamaterial*. Basically, metamaterial is an artificial structure having electromagnetic properties generally not exist in nature. The structure behave as an effective medium with negative values of permittivity (ε) and permeability (μ) at interest frequencies [5]. Most essential of this material is by tuning the electrical or mechanical characteristics of the elements, the wave propagation can be manipulated. So, there are several topologies of semi-lumped resonant particles like split ring resonators

(SRRs), complementary split ring resonators (CSRRs), double split ring resonators (DS-SRRs), open complementary split ring resonators (OCSRRs) and many more. The SRR and CSRR will be discussed in details in next sub-topics since CSRR is the main design in this project and SRR is the basic structure of CSRR.

2.3.1 Split Ring Resonator (SRR)

Generally, SRR is a structure from a single ring resonator turned into a double rings resonator. This happened when a λ/2 transmission line is closed in a ring configuration, referred to Figure 2.9(a), the resonator can be excited by means of an axial time varying magnetic field. The electrical length of the ring will be λ/2 at resonance and the diameter will be λ/2π at frequency, relatively small compared to the wavelength. Thus, the best solution to reduce the size of a split ring is by adding an inner ring with the gap on the opposite side, shown in Figure 2.9(b).

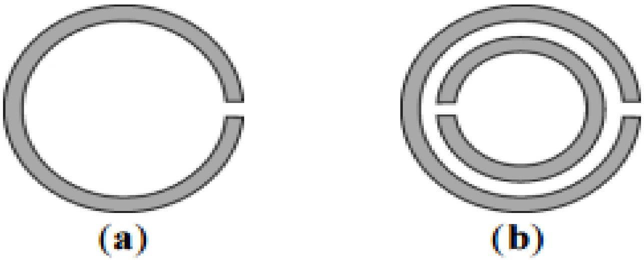

Figure 2.9 (a) λ/2 ring resonator (b) Split ring resonator [10].

To ensure the fundamental resonance of the structure to be lower than the fundamental resonance of any single rings so that the structure is electrically smaller than a single ring resonator, mutual coupling is been done between both rings [10]. From view of circuit, SRR structure behaves as an oscillatory L-C circuit. Figure 2.10 shows the equivalent circuit model of SRR. The induced electromotive force around the SRR causes a current when exposed under external magnetic field. The currents then will passes from one ring to the other through the inter ring spacing. This structure is useful for the implementation of effective media with controllable metamaterial as long as its diameter can be made much smaller than the wavelength. In microstrip technology, SRRs is etched on the surface of upper substrate side, near to the conductor strip.

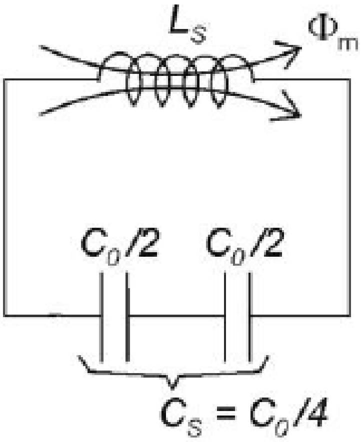

Figure 2.10 Equivalent circuit of the SRR [10].

From only a circular configuration, different configuration of SRR such as square-SRR (S-SRR), hexagonal-SRR (H-SRR) and other have been analysed by researchers for realizing artificial magnetic material or employing the SRR as perturbations for different passive planar circuit design. Different configurations of SRR is shown in Figure 2.11 [11].

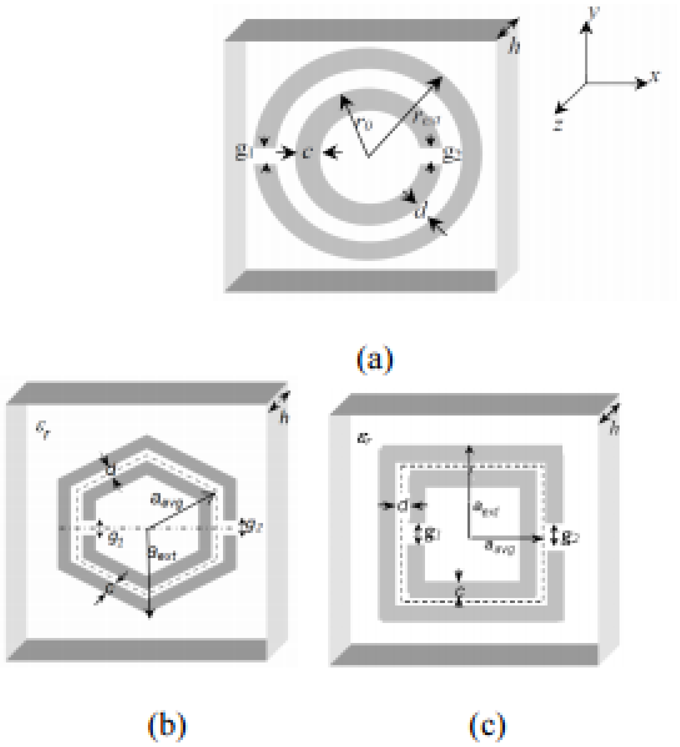

Figure 2.11 Schematic view of split ring resonators (a) Circular (b) Hexagonal (c) Square [11].

From Figure 2.11, the structure of SRRs formed with metallic strips of width, c, dimension, r_{ext} (for circular) and a_{ext} (for square and hexagonal), inter ring spacing, d, and split gap, $g_1 = g_2$. Generally, the equation of resonant frequency for SRR is written as

$$\omega_{o}= \sqrt{\frac{1}{L_T C_{eq}}} \tag{2.22}$$

The equation is then specified into three different expressions based on different configurations. The expression is given as [11]

For Circular-SRR:

$$f_{0C} = \frac{1}{2\pi\sqrt{L_T\left[\frac{(\pi r_{avg} - g)C_{pul}}{2} + \frac{\varepsilon_o\, ch}{2g}\right]}} \tag{2.23}$$

For Square-SRR:

$$f_{0C} = \frac{1}{2\pi\sqrt{L_T\left[\left(2a_{avg} - \frac{g}{2}\right)C_{pul} + \frac{\varepsilon_o\, ch}{2g}\right]}} \tag{2.24}$$

For Hexagonal-SRR;

$$f_{0C} = \frac{1}{2\pi\sqrt{L_T\left[\frac{(3a_g - g_1)C_{pul}}{2} + \frac{\varepsilon_o\, ch}{2g_1}\right]}} \tag{2.25}$$

where

h is substrate thickness

c_o is speed of light in free space, $3\text{x}10^8$

Z_o is impedance of the medium.

C_{eq} is the equivalent capacitance of the structure

C_{pul} is per unit length capacitance between the rings, given as

$$C_{pul} = \sqrt{\frac{\varepsilon_e}{c_o Z_o}} \tag{2.26}$$

where effective permittivity of the medium, ε_e is calculated as [12]

$$\varepsilon_e = 1 + \frac{\varepsilon_r - 1}{2} \frac{K(k')K(k_1)}{K(k)K(k_1')} \tag{2.27}$$

where

$$k = \frac{c/2}{\frac{c}{2}+d}, \quad a = c/2, \quad b = \frac{c}{2} + d \tag{2.28}$$

$$k_1 = \frac{\sinh(\frac{\pi a}{2h})}{sinh(\frac{\pi b}{2h})} \tag{2.29}$$

$$k' = \sqrt{1-k^2} \tag{2.30}$$

K is a complete elleptic function of the first kind and K' is its complimentary function. An expression for K/K' is given as [12]

$$\frac{K(k)}{K(k')} = \left[\frac{1}{\pi} \text{ In } \left(2\frac{1+\sqrt{k\prime}}{1-\sqrt{k\prime}}\right)\right]^{-1} \quad \text{for } 0 \leq k \leq 0.7 \tag{2.31}$$

$$\frac{K(k)}{K(k')} = \frac{1}{\pi} \text{ In } \left(2\frac{1+\sqrt{k\prime}}{1-\sqrt{k\prime}}\right) \quad \text{for } 0.7 \leq k \leq 1 \tag{2.32}$$

L_T is total inductance of the SRR, computed as

$$L_T = 0.00508l\,(2.303 \log_{10} \frac{4l}{d} - \theta) \tag{2.33}$$

where *l* and *d* are the wire length and width in mm. The length of wire, *l* is straightforward as

$$l = 2\pi r_{ext} - g \tag{2.34}$$

The constant θ varies with wire geometry and is given by

θ = 2.41 for C-SRR

2.853 for S-SRR and

2.636 for H-SRR. (2.35)

r_{avg}/a_{avg} is the distance between two constituent rings and centre given as

$$r_{avg} = r_{ext} - c - d/2 \tag{2.36}$$

Different geometries will have different resonant frequencies at the same values of parameters. From [11], the C-SRR resonates at higher frequency compared to S-SRR but resonates at lower frequency when compared to H-SRR.

2.3.2 Complementary Split Ring Resonator (CSRR)

For SRR, it is etched in the upper substrate side, in proximity to the conductor strip and it is been provide the same effect [13]. However, the used of shunt inductances are required for the implementation of an associated effective negative ε, which are associated to metallic vias to the ground. To overcome this problem, the concept of introducing an effective negative permittivity in microstrip devices was demonstrated by some authors [14] that by periodically etching the negative image of SRRs on the ground plane of a microstrip line underneath, the conductor strip give exactly a resonant frequency of conventional SRR etched on the same substrate. The stopband characteristics obtained was interpreted as due to a negative effective permittivity introduced by these new elements named as CSRR, shown in Figure 2.12.

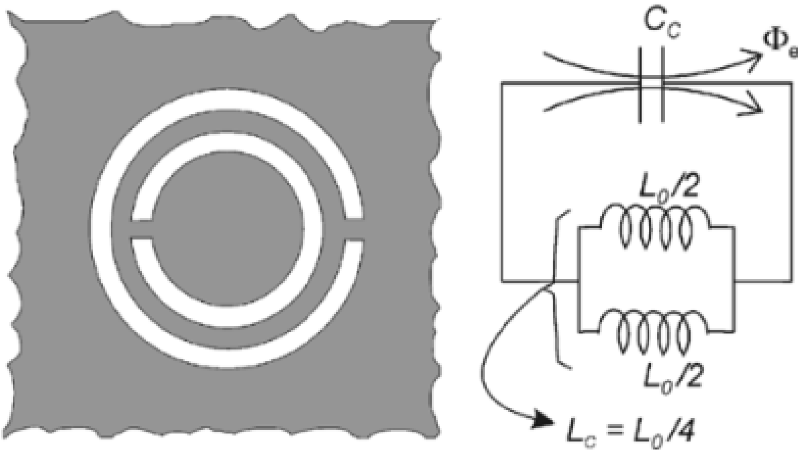

Figure 2.12 (a) Topology of CSRR (b) Equivalent circuit of CSRR [10].

If the effects of metal thickness and losses, and the dielectric substrate are neglected, a perfect dual behaviour is expected from the CSRR and it essentially behaves as an electric dipole (with the same frequency resonance) that can be excited by an axial electric field. From equivalent circuit of CSRR in Figure 2.12(b), the inductance of L_s of the SRR model is replaced with the capacitance C_c of a disk of radius $r_o - c/2$ surrounded by a ground plane at a distance *c* of its edge. The parallel of two inductances connecting the inner disc to the ground is replace after the series connection of two capacitances $C_o/2$. Given each inductance is $L_o/2$, where $L_o = 2\pi r_o L_{pul}$ and L_{pul} is the per unit length inductance of the CPWs connecting the inner disk to the ground [10].