First Semester Examination
2018/2019 Academic Session

December 2018/January 2019

EUM113 – ENGINEERING CALCULUS
(Kejuruteraan Lanjutan)

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **EIGHT (8)** pages and **ONE (1)** page of printed appendix material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN (8)** muka surat dan **SATU (1)** muka surat lampiran yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: This question paper consists of **FOUR (4)** questions. Answer **ALL** questions. All questions carry the same marks.

*[**Arahan:** Kertas soalan ini mengandungi **FOUR (4)** soalan. Jawab **SEMUA** soalan. Semua soalan membawa jumlah markah yang sama.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

1. (a) Find $c \in \mathbb{R}$ such that the function

Cari nilai $c \in \mathbb{R}$ supaya fungsi berikut

$$f(x) = \begin{cases} x^2 + 2x, & \text{if } x \geq 2 \\ 2x + c, & \text{if } x < 2 \end{cases}$$

is continuous at $x = 2$.

adalah selanjar pada $x = 2$.

(4 marks/*markah*)

(b) (i) Differentiate the following function:

Bezakan fungsi yang berikut:

$$y = x^{e^{-x^2}}$$

(4 marks/*markah*)

(ii) Integrate the following function:

Kamirkan fungsi yang berikut:

$$\int sin^{-1}x \ dx$$

(4 marks/*markah*)

(c) Show that $f(x) = -4x^4 + 4x^3 - 10x$ has two roots. The first root is 0 and the second root is in the interval [−1,−2].

Evaluate the second root using Newton-Raphson's method correct to 3 decimal points. Use the initial guess $x_0 = -1.0$

Tunjukkan bahawa $f(x) = -4x^4 + 4x^3 - 10x$ mempunyai dua punca. Punca pertama ialah 0 dan punca kedua adalah dalam julat [−1,−2].

Nilaikan punca kedua dengan menggunakan kaedah Newton-Raphson tepat kepada tiga tempat perpuluhan. Gunakan tekaan awal $x_0 = -1.0$.

(7 marks/*markah*)

(d) Evaluate the double integral below by first converting it to polar coordinates.
Nilaikan kamiran berganda di bawah dengan terlebih dahulu menukarkannya kepada koordinat kutub.

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{0}\cos(x^2+y^2)\,dy\,dx$$

(6 marks/*markah*)

2. (a) Find the general solution for the Bernoulli equation below:
Cari penyelesaian am bagi persamaan Bernoulli berikut:

$$x\,dy + 2y\,dx = 2x\sqrt{y}$$

(8 marks/*markah*)

(b) Solve the non-exact differential equation by using the integrating factor:
Selesaikan persamaan tak-tepat dengan menggunakan faktor kamiran:

$$(xy^2 - 2y^3)dx + (3 - 2xy^2)dy = 0$$

(8 marks/*markah*)

(c) Using Euler's numerical method with step size 0.1, obtain the approximate numerical solution of the ordinary differential equation below when $x = 0.5$.
Dengan menggunakan kaedah berangka Euler berlelaran 0.1, dapatkan anggaran penyelesaian persamaan pembezaan biasa peringkat pertama di bawah apabila $x = 0.5$.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{x^2 + y}$$

Given the initial value $y(0) = 0.8$, work out your answer accurate to 4 decimal points.
Diberi nilai awal $y(0) = 0.8$, *dapatkan jawapan anda tepat kepada 4 titik perpuluhan.*

(9 marks/*markah*)

3. (a) Solve the following linear differential equation based on method of undetermined coefficient.
Selesaikan persamaan pembezaan linear berikut berdasarkan kaedah pekali tak tentu.

$$2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 4 = 6x - 1$$

(8 marks/*markah*)

(b) Obtain the solution for ordinary differential equation below using method of variation of parameter.
Dapatkan penyelesaian bagi persamaan pembezaan biasa di bawah dengan menggunakan kaedah ubahan parameter.

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + y = \cos x$$

(9 marks/*markah*)

(c) Solve the following initial value problem using Laplace Transform method
Selesaikan masalah nilai awal berikut dengan kaedah jelmaan Laplace

$$y'' - 2y' + 10y = 0\,, \quad y(0) = -1 \text{ and } y'(0) = 11$$

(8 marks/*markah*)

4. (a) A flat circular plate has the shape of the region $\{(x,y): x^2 + y^2 \le 1\}$ in xy plane. The plate, including the boundary where $x^2 + y^2 = 1$, is heated so that the temperature (in °C) at the point (x,y) is given by:

$$T(x,y) = x^2 + 2y^2 - x$$

Find the temperature at the hottest and coldest points on the plate.

Sebuah plat bulat rata mempunyai bentuk kawasan $\{(x,y): x^2 + y^2 \le 1\}$ pada satah xy. Plat, termasuk sempadan $x^2 + y^2 = 1$, dipanaskan supaya suhu (dalam °C) pada titik (x,y) diberikan oleh

$$T(x,y) = x^2 + 2y^2 - x$$

Cari suhu pada titik paling panas dan paling sejuk di atas plat.

(7 marks/markah)

(b) The height of a cone is increasing at the rate of 3 mm/s and its radius is decreasing at the rate of 2 mm/s. If the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, determine the rate (in cm^3/s) at which the volume is changing when the height is 3.2 cm and the radius is 1.5 cm.

Ketinggian sebuah kon menokok pada kadar 3 mm/s dan jejarinya menyusut pada kadar 2 mm/s. Jika isipadu kon ialah $V = \frac{1}{3}\pi r^2 h$, tentukan kadar (dalam cm^3/s) dimana isipadunya berubah apabila ketinggiannya ialah 3.2 cm dan jejarinya 1.5 cm.

(7 marks/*markah*)

(c) A salt solution with concentration of $0.25\ kg/l$ is set to flow at the rate of $8\ l/min$ into a tank containing 80 litre of pure water. The liquid in the tank is kept homogeneous by constant stirring. At time $t = 0$ minute, the flow of salt solution into the tank is started and also the valve at the bottom of the tank is opened so that liquid flows out from the tank at a volumetric rate of $12\ l/min$. The overall rate of change of the amount of salt, m (in kg), in the tank, is fundamentally given by material balance equation:

Larutan garam dengan kepekatan $0.25\ kg/l$ *ditetapkan untuk mengalir pada kadar* $8\ l/min$ *ke dalam tangki yang mengandungi 80 liter air tulen. Cecair di dalam tangki dikekalkan homogen dengan pengadukan berterusan. Pada masa t = 0 minit, aliran larutan garam ke dalam tangki dimulakan dan juga injap di bahagian bawah tangki dibuka supaya cecair keluar dari tangki pada kadar volumetrik* $12\ l/min$. *Kadar keseluruhan perubahan jumlah garam, m (kg), dalam tangki, pada asasnya diberikan oleh persamaan keseimbangan bahan:*

$$\frac{dm}{dt} = \left(\frac{dm}{dt}\right)_{in} - \left(\frac{dm}{dt}\right)_{out}$$

where:

$\left(\frac{dm}{dt}\right)_{in}$ is the mass flow rate of salt into the tank,

$\left(\frac{dm}{dt}\right)_{out}$ is the mass flow rate of salt out of the tank,

as in Figure 4(c).

di mana:

$\left(\frac{dm}{dt}\right)_{in}$ *ialah kadar aliran jisim garam ke dalam tangki,*

$\left(\frac{dm}{dt}\right)_{out}$ *ialah kadar aliran jisim garam keluar dari tangki,*

seperti di bawah Rajah 4(c).

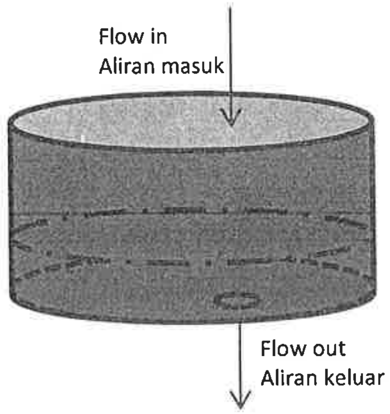

Figure 4(c)

Rajah 4(c)

The instantaneous concentration of salt solution, C within liquid volume, V in the tank, at any time, t is given by: $C(t) = \frac{m(t)}{V(t)}$ and $V(t) = V_0 + \left[\left(\frac{dV}{dt}\right)_{in} - \left(\frac{dV}{dt}\right)_{out}\right]t$,

Kepekatan seketika larutan garam C dalam cecair berisipadu V, dalam tangki, pada bila-bila masa diberikan oleh: $C(t) = \frac{m(t)}{V(t)}$ *dan* $V(t) = V_0 + \left[\left(\frac{dV}{dt}\right)_{in} - \left(\frac{dV}{dt}\right)_{out}\right]t$,

where :

V_0 is the initial volume of liquid in the tank,

$\left(\frac{dV}{dt}\right)_{in}$ is the liquid volumetric flow rate into the tank,

$\left(\frac{dV}{dt}\right)_{out}$ is the liquid volumetric flow rate out of the tank.

di mana:

V_0 *ialah isipadu awal cecair dalam tangki ,*

$\left(\frac{dV}{dt}\right)_{in}$ *ialah kadar aliran isipadu cecair ke dalam tangki,*

$\left(\frac{dV}{dt}\right)_{out}$ *ialah kadar aliran isipadu cecair keluar dari tangki.*

(i) Show that the amount of salt *m* (in kg) in the tank over the time *t* (in minute), until the tank is virtually emptied can be described by differential equation:
Tunjukkan bahawa jumlah garam m (dalam kg) dalam tangki sepanjang masa t (dalam minit), sehingga tangki hampir dikosongkan boleh dijelaskan oleh persamaan kebezaan:

$$\frac{dm}{dt}+\frac{3m}{20-t}=2$$

(4 marks/*markah*)

(ii) Solve the differential equation in part (i) to obtain the algebraic expression of amount of salt in the tank as a function of time. Note that this is an initial value problem with *m* = 0 at *t* = 0.
Selesaikan persamaan pembezaan dalam bahagian (i) untuk mendapatkan ungkapan algebra jumlah garam dalam tangki sebagai fungsi masa. Perhatikan bahawa ini adalah masalah nilai awal dengan m = 0 pada t = 0.

(5 marks/*markah*)

(iii) Determine the amount of salt in the tank (in kg) after 5 minutes.
Tentukan jumlah garam dalam tangki (dalam kg) selepas 5 minit.

(1 marks/*markah*)

(iv) At what time the tank is emptied?
Pada masa bila tangka tersebut dikosongkan?

(1 marks/*markah*)

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APPENDIX
LAMPIRAN

Course Outcomes (CO)- Programme Outcomes (PO) Mapping

Pemetaan Hasil Pembelajaran Kursus – Hasil Program

Question	CO	PO
1	1	1
2	2	1
3	3	1
4	4	2

UNIVERSITI SAINS MALAYSIA

PUSAT PENGAJIAN KEJURUTERAAN ELEKTRIK DAN ELEKTRONIK
UNIVERSITI SAINS MALAYSIA, KAMPUS KEJURUTERAAN
SEMESTER PERTAMA, SIDANG 2018/2019

MAKLUMAT KERTAS PEPERIKSAAN

A.	Kod Kertas	EUM113	
	Tajuk Kertas	ENGINEERING CALCULUS	
	Pensyarah	PM DR DZATI	DR NORAMALINA
	Jumlah Salinan Asal		
	Tarikh Terima		
B.	Jurutaip	NURUL LELA	
	Tarikh Penaipan		
	Jumlah Salinan Bertaip	8	
C.	Tarikh Semakan	24-10-18 23/10/18	
	Salinan Lampiran Pembetulan		
	Tarikh Penaipan Pembetulan		
	Tarikh Semakan Kedua	29/10/18	
D.	Tarikh Vetting	24/10/2018	24/10/2018
	Pensyarah-pensyarah yang Terlibat		Nor Asiah.
	Tarikh Tandatangan Pengesahan		
	Tarikh Penyerahan ke Bahagian Peperiksaan		
	Kakitangan Terlibat		

NLS/Borang Maklumat Kertas Peperiksaan