First Semester Examination
2019/2020 Academic Session

December 2019 / January 2020

EME 411 – Numerical Methods for Engineers
[Kaedah Berangka Untuk Jurutera]

Duration : 3 hours
[Masa : 3 jam]

Please check that this paper contains **EIGHT [8]** printed pages including appendix before you begin the examination.

*[Sila pastikan bahawa kertas soalan ini mengandungi **LAPAN [8]** mukasurat bercetak beserta lampiran sebelum anda memulakan peperiksaan.]*

INSTRUCTIONS : Answer **ALL FIVE [5]** questions.
*[**ARAHAN** : Jawab **SEMUA LIMA [5]** soalan.]*

Answer Questions In **English** OR **Bahasa Malaysia**.
*[Jawab soalan dalam **Bahasa Inggeris** ATAU **Bahasa Malaysia**.]*

Answer to each question must begin from a new page.
[Jawapan bagi setiap soalan mestilah dimulakan pada mukasurat yang baru.]

In the event of any discrepancies, the English version shall be used.
[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. **Provide BRIEF answers to the following questions:**

Berikan jawapan-jawapan RINGKAS bagi soalan-soalan berikut:

[a] Provide an example where the Robin boundary condition is necessary in modeling a heat transfer problem.

Berikan satu contoh di mana syarat sempadan Robin diperlukan dalam pemodelan masalah pemindahan haba.

(5 marks/*markah*)

[b] Explain the meaning of "second order accuracy" in an FDM formulation.

Terangkan makna "kejituan tertib kedua" dalam formulasi FDM.

(5 marks/*markah*)

[c] Suppose the system matrix A and the system load vector b are Obtained by an FDM procedure in MATLAB. However, solving the linear system for x

Andaikan bahawa matriks sistem **A** *dan vektor beban sistem* **b** *diperolehi dari satu prosedur FDM dalam MATLAB. Walau bagaimanapun, penyelesaian bagi sistem linear bagi* **x**

```
x = inv(A)*b
```

yields an NaN result, i.e., x is not solvable. State a reason for this issue.

memberikan keputusan NaN, yaitu **x** *tiada penyelesaian. Nyatakan satu sebab bagi isu ini*.

(5 marks/*markah*)

2. **Consider the steady state heat conduction in a thin plate with a width of 3 m and height of 2 m with the following boundary conditions:**
   - **<u>Homogenous Neumann boundary condition</u> (BC) on the West wall.**
   - **50 °C on the East wall.**
   - **30 °C on the North wall.**
   - **10 °C on the South wall.**

Pertimbangkan aliran haba mantap di dalam sekeping plat segiempat dengan lebar 3 m dan tinggi 2 m dan mempunyai syarat-syarat sempadan berikut:

- *Syarat sempadan (BC) Neumann homogen pada dinding Barat.*
- *50 °C pada dinding Timur.*
- *30 °C pada dinding Utara.*
- *10 °C pada dinding Selatan.*

[a] **Write the complete mathematical statement of the problem.**

Tuliskan penyataan matematik lengkap bagi masalah itu.

(5 marks/*markah*)

[b] **Sketch and number the grid points with the step size $h = 1$ m for solving the problem.**

Lakarkan dan nomborkan titik-titik grid dengan saiz langkah $h = 1$ m yang digunakan untuk menyelesaikan masalah itu.

(5 marks/*markah*)

[c] **For the $u_{i,j}$ on the West wall, derive the FDM equation for this value to**

include the homogenous Neumann BC.

Bagi $u_{i,j}$ pada dinding Barat, terbitkan persamaan FDM bagi nilai tersebut berserta BC Neumann homogen.

(7 marks/*markah*)

[d] **Set up the linear system for this problem such that all the unknowns can be solved with $\mathbf{u} = \mathbf{A}^{-1}\mathbf{b}$. DO NOT solve for the values of $\mathbf{u}$.**

Binakan sistem linear bagi masalah ini supaya semua nilai dapat diselesaikan dengan $u = A^{-1}b$. JANGAN selesaikan bagi nilai-nilai u.

(8 marks/*markah*)

3. **Consider the weak Galerkin finite element formulation for the 1D axial displacement in a bar as in Figure 3. Discuss how the Galerkin formulation is considered "weak". Discuss also the implication of the weak formulation with respect to the stress and strain in the bar.**

Pertimbangkan formulasi unsur terhingga Galerkin lemah bagi sesaran paksi di dalam bar 1D seperti dalam Rajah 3. Bincangkan bagaimana formulasi Galerkin dianggap "lemah". Bincangkan juga akibat daripada formulasi lemah ini berkenaan tegasan dan terikan di dalam bar.

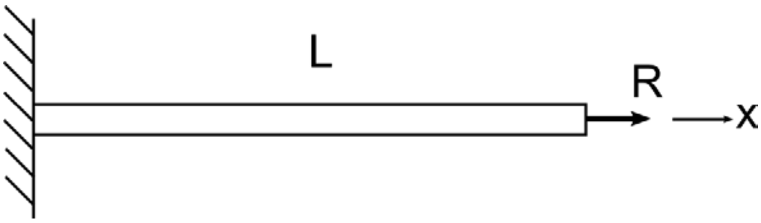

Figure 3
Rajah 3

(12 marks/*markah***)**

4. **Consider the displacement response of a $L = 1$ m bar due to the axial forces $P_1 = 60$ N at $x = 0.2$ and $P_2 = -40$ N at $x = 0.7$ as in Figure 4. The bar is fixed at both ends. Assume $E \cdot A = 100 \times 10^3$ N.**

Pertimbangkan tindak balas sesaran bagi sebatang bar $L = 1$ m oleh daya-daya paksi $P_1 = 60$ N pada $x = 0.2$ dan $P_2 = -40$ N pada $x = 0.7$ seperti di dalam Rajah 4. Bar itu ditetapkan pada kedua-dua hujung. Andaikan $E \cdot A = 100 \times 10^3$ N.

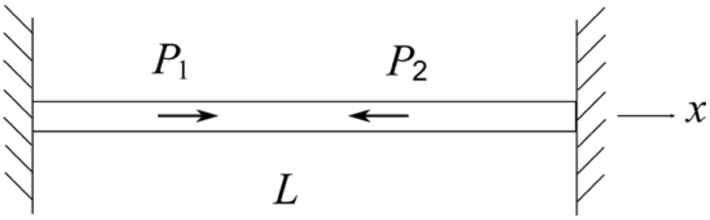

Figure 4
Rajah 4

[a] The above problem may be stated with 1-D Poison's equation. Sketch the 1-D domain and state the strong form and the weak form of the problem COMPLETELY (derivation not necessary).

Masalah di atas boleh dirumuskan dengan persamaan Poisson 1-D. Lakarkan domain 1-D dan nyatakan bentuk kuat dan bentuk lemah masalah itu SECARA LENGKAP (terbitan tidak perlu).

(8 marks/*markah***)**

[b] Sketch the 1-D mesh of 2 LINEAR elements where all the nodes are specified at $x = \{0, 0.4, 1.0\}$. Label completely all the DOFs and elements.

Lakarkan jejaring 1-D dengan 2 unsur LINEAR di mana semua nod ditetapkan pada $x = \{0, 0.4, 1.0\}$. Nomborkan semua DOFs dan unsur yang sesuai.

(5 marks/*markah***)**

[c] **Set up the global linear system for the problem without inclusion of the boundary data. Detailed derivation steps are not necessary.**

Binakan sistem linear global bagi masalah itu tanpa memasukkan data sempadan. Langkah-langkah terbitan yang terperinci adalah tidak perlu.

(15 marks/*markah***)**

[d] **Modify the matrix and load vector, then solve the linear system. Sketch the solution.**

Ubah suaikan matriks dan vektor beban, kemudian selesaikan sistem linear itu. Lakarkan penyelesaiannya.

(12 marks/*markah***)**

5. **The following is MATLAB code for 1D FEM that is written for linear elements of the same length. Rewrite the code so it can solve for a mesh of <u>25 QUADRATIC elements</u> of varying lengths. You may only assume:**

- **the domain is 1-unit length.**
- **the nodal coordinates and the element connectivity map have been defined.**
- **the function `getElCoord` is available and it returns `xe=[xi xj]` where `xi` and `xj` are the end points of the element.**
- **the functions `getLoadVec` and `getElMat` have been modified for the quadratic elements.**

Berikut ialah kod MATLAB bagi FEM 1-D yang ditulis untuk unsur-unsur linear yang sama panjang. Tuliskan semula kod itu agar ia boleh menyelesaikan bagi jejaring dengan <u>25 unsur KUADRATIK</u> yang berbeza panjang. Anda hanya boleh mengandaikan bahawa:

- *domain itu ialah 1-unit panjang.*
- *koordinat nod dan peta kesalinghubungan unsur telah ditakrifkan.*
- *fungsi* `getElCoord` *telah tersedia dan ia memulangkan* `xe = [xi xj]` *di mana* `xi` *dan* `xj` *ialah titik-titik hujung bagi unsur itu.*
- *fungsi-fungsi* `getLoadVec` *dan* `getElMat` *telah diubahsuai untuk unsur-kuadratik.*

```
elementDOF = 2
enum = 120
len = 1/enum
p = 2 % p is the RHS value of the D.E.
ndof = enum + 1

K = zeros(ndof,ndof)
bb = zeros(ndof,1)
for e = 1:enum
  be = (len*p/2)*getLoadVec
  ke = (1/len)*getElMat
  for ir = 1:elementDOF
    irs = elementDOFMap(e,ir)
    bb(irs) = bb(irs) + be(ir)
    for ic = 1:2
      ics = elementDOFMap(e,ic)
      K(irs,ics) = K(irs,ics) + K(irs,ics)
    end
  end
end
```

(8 marks/*markah*)

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APPENDIX 1
LAMPIRAN 1

Useful formulas

Forward differences:

$$u_i' = \frac{u_R - u_i}{h} + O(h)$$

Centered Differences:

$$u_i'' = \frac{u_R - 2u_i + u_L}{h^2} + O(h^2) \qquad u_i' = \frac{u_R - u_L}{2h} + O(h^2)$$

2D Stencil for Poisson's equation:

$$-u_{E,j} - u_{W,j} + 4u_{i,j} - u_{i,N} - u_{i,S} = \frac{h^2 f_i}{k}$$

Robin boundary condition

$$p \cdot u + q \cdot \nabla u \cdot \mathbf{n} = g$$

1D heat equation

$$-\alpha \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial t} = Q(x,t)$$

FDM equation with variable material

$$-k_{i-\frac{1}{2}} u_{i-1} + \left(k_{i-\frac{1}{2}} + k_{i+\frac{1}{2}}\right) u_i - k_{i+\frac{1}{2}} u_{i+1} = h^2 f_i$$

Element stiffness matrix:

$$\mathbf{K}^e = \frac{k}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \mathbf{K}^e = \frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\mathbf{K}^e = \frac{k}{l}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 16/3 \end{bmatrix} \qquad \mathbf{K}^e = \frac{EA}{l}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 16/3 \end{bmatrix}$$

Element load vector with point load *P* at ρ_0:

$$\mathbf{b}^{(e)} = P\begin{bmatrix} \xi_1(\rho_0) \\ \xi_2(\rho_0) \end{bmatrix} \qquad \mathbf{b}^{(e)} = P\begin{bmatrix} \xi_1(\rho_0) \\ \xi_2(\rho_0) \\ \xi_3(\rho_0) \end{bmatrix}$$

Basis functions in local coordinate

$$\xi_1(\rho) = \frac{1}{2}(1-\rho) \qquad \xi_2(\rho) = \frac{1}{2}(1+\rho) \qquad \xi_3(\rho) = \frac{1}{2}(1-\rho)^2$$

Local-global coordinates mapping

$$x = \frac{l}{2}\,\rho + \frac{1}{2}\left(x_i + x_j\right)$$

MATLAB statements

1. To build a 2 by 2 matrix

    A = [1 2; 3 4]

2. To take the inverse of a matrix: inv(A)

3. To take the transpose of a matrix: A'

4. To call a function: x = myFun(y,z)