

**EFFECTIVENESS OF
BLENDED DISCRETE MATHEMATICS MODEL
IN THE TEACHING & LEARNING OF CALCULUS**

RAJASEGERAN A/L RAMASAMY

UNIVERSITI SAINS MALAYSIA

2020

**EFFECTIVENESS OF
BLENDED DISCRETE MATHEMATICS MODEL
IN THE TEACHING & LEARNING OF CALCULUS**

by

RAJASEGERAN A/L RAMASAMY

**Thesis submitted in fulfilment of the requirements
for the degree of
Doctor of Philosophy**

November 2020

ACKNOWLEDGEMENT

First and foremost, I thank to Mighty god for all his blessings and guidance in helping me to complete this doctoral thesis.

I would like to express my appreciation and gratefulness to my main supervisor Dr. Hazrul bin Abdul Hamid for his supervision, advice, encouragement, guidance, influence and help during my research work and preparation of this thesis.

I would like to express my special thanks to the faculty and administrative staff of the School of Distance Education Studies, Universiti Sains Malaysia for providing the facilities, advice and support. My grateful thanks go to the administrative staff of the Institute of Postgraduate Studies (IPS), USM, for their assistance and support.

Also, I would like to gratefully acknowledge the wife, Kasthuri for her patient, encouragement and support throughout my doctoral studies. Not forgetting my two angels, Mardhev and Vishallini for their understanding and support throughout this study. To all my friends and colleagues thank you for all the encouragement and moral support given during the duration of my studies.

TABLE OF CONTENTS

ACKNOWLEDGEMENT.....	ii
TABLE OF CONTENTS.....	iii
LIST OF TABLES.....	vii
LIST OF FIGURES.....	xi
LIST OF ABBREVIATIONS.....	xii
ABSTRAK.....	xiv
ABSTRACT.....	xvi
CHAPTER 1 INTRODUCTION.....	1
1.1 Background of the Study1.....	1
1.1.1 Mathematics Ability.....	2
1.1.2 Mastery in Fundamental Skills and Heterogeneity of Mastery Level	3
1.1.3 Sequencing of Mathematics Courses	6
1.1.3(a) Spiral Curriculum.....	6
1.1.3(b) Sequential Curriculum	7
1.1.4 Traditional & Alternative Approaches in Teaching Mathematics....	10
1.2 Blended Discrete Mathematics Model.....	13
1.3 Statement of the Problem.....	13
1.4 Objectives of the Study.....	17
1.5 Research Questions.....	18
1.6 Theoretical & Conceptual Framework.....	21
1.7 Significance of the Study.....	24

1.8	Limitation of the Study.....	26
1.9	Definition of terms.....	26
1.10	Summary.....	30
CHAPTER 2 LITERATURE REVIEW.....		31
2.1	Introduction.....	31
2.2	Poor Achievement in Mathematics Course.....	32
2.3	Mathematics Ability.....	34
2.4	Mastery of Fundamental Mathematics Skills	38
2.4.1	Intervention to Poor Mastery in Fundamental Skills	43
2.5	Constructivism, Connectivism & Gagne Learning Hierarchy.....	45
2.6	Mathematics Teaching Methods.....	50
2.6.1	Traditional Mathematics Teaching Method.....	50
2.6.2	Alternative Mathematics Teaching Methods.....	52
2.7	Mode of Delivery.....	65
2.8	Discrete Clinical Trial Method.....	73
2.9	Blended Discrete Mathematics Model.....	80
2.9.1	Exploration Stage.....	80
2.9.2	Construction Stage.....	81
2.10	Summary.....	83
CHAPTER 3 RESEARCH METHODOLOGY.....		86
3.1	Introduction.....	86
3.2	Study Design.....	87
3.3	Study Variables.....	88
3.4	Study Sample and Sampling.....	90

3.4.1	First Case Study.....	90
3.4.2	Second Case Study.....	91
3.5	Procedure on Teaching & Learning Method.....	92
3.5.1	BDMM Method.....	93
3.5.2	Conventional Method.....	93
3.5.3	Assessment.....	93
3.5.3(a)	Pre-Test.....	94
3.5.3(b)	Formative Assessment.....	95
3.5.3(c)	Summative Assessment.....	99
3.6	Data Analysis.....	102
3.6.1	Item Response Theory (IRT) & Rasch Analysis.....	102
3.6.2	Welch t-test.....	104
3.6.3	Independent two sample t-test.....	105
3.6.4	Mann Whitney Test.....	106
3.7	Steps to Ensure Validity of Study Findings.....	107
3.8	Blended Discrete Trial Mathematics Model.....	108
3.9	Calculus in BDMM	124
3.10	BDMM as Blended Learning.....	134
3.11	Summary.....	135
CHAPTER 4 RESULTS & DISCUSSIONS.....		136
4.1	Introduction.....	136
4.2	Description of Study Participants.....	136
4.2.1	First case study.....	137
4.2.2	Second case study.....	138

4.3	Pre-Quasi Experimental Study Result	138
4.3.1	Description Statistics of Pre-test Achievement	139
4.3.1(a)	First case study.....	139
4.3.1(b)	Second case study.....	139
4.3.2	Hypothesis 1 Testing.....	140
4.3.2(a)	First case study.....	140
4.3.2(b)	Second case study.....	141
4.3.3	Summary of Hypothesis 1.....	141
4.4	Research Question 1.....	143
4.5	Research Question 2.....	143
4.5.1	Hypothesis 3 Testing.....	149
4.5.1(a)	First case study.....	149
4.5.1(b)	Second case study.....	151
4.5.2	Summary of Research Question 2.....	151
4.6	Research Question 3.....	153
4.6.1	Hypothesis 4 Testing.....	155
4.6.1(a)	First case study.....	155
4.6.1 (b)	Second case study.....	156
4.6.2	Summary of Research Question 3.....	157
4.7	Summary.....	159
	CHAPTER 5 RECOMMENDATION & CONCLUSION.....	163
5.1	Introduction.....	163
5.1.1	Diverse Mathematics Ability.....	163

5.1.2	Comparable Groups.....	165
5.1.3	Mastery of Fundamental Skills.....	166
5.1.4	E-learning Influences.....	167
5.1.5	Impact of BDTMM on Calculus Achievements.....	168
5.2	Recommendation.....	168
5.3	Conclusion.....	169
	REFERENCES.....	171
	APPENDICES	

LIST OF TABLES

		Page
Table 2.1	Summary of studies on Poor Achievement in Mathematics Course.....	34
Table 2.2	Summary of studies on Mathematics Ability	37
Table 2.3	Meta-analytic summary of studies on Prior Knowledge.....	42
Table 2.4	Summary of Studies on Prior Knowledge.....	44
Table 2.5:	Summary of studies on Constructivism, Connectivism & Gagne’s Learning Hierarchy.....	49
Table 2.6	Effect Size of studies in Alternative Teaching Methods.....	63
Table 2.7	Summary of Studies on Alternative Teaching Methods.....	64
Table 2.8:	Summary of studies on Mode of Delivery.....	72
Table 2.9:	Summary of comparison Between Study to show Research Gap.....	84
Table 3.1	Study Design.....	88
Table 3.2	Sample description (First case study).....	90
Table 3.3	Sample description (Second case study)	91
Table 3.4	Distribution of Groups.....	91
Table 3.5	Description of the Methods.....	93
Table 3.6	Description of Pre-Test.....	95
Table 3.7	Formative Assessment (First case study)	95
Table 3.8	Description of Formative Assessment (First case study)	96
Table 3.9	Formative Assessment (Second case study)	97
Table 3.10	Description of Formative Assessment (Second case study)	98
Table 3.11	Comparison of Formative Assessment in Case Study 1 & 2	99
Table 3.12	Description of Summative Assessment (First case study)	99

Table 3.13	Description of Summative Assessment (Second case study)	100
Table 3.14	Comparison of Summative Assessment in Case Study 1 & 2	101
Table 3.15	Data Analysis Framework of Research Questions.....	107
Table 3.16	Comparison between DCT and BDMM.....	113
Table 3.17	Example of BDMM Plan.....	114
Table 3.18	BDMM Teaching Plan in First case study	125
Table 3.19	BDMM Teaching Plan in Second case study	128
Table 4.1	Descriptive Statistics of Pre-Test Achievement (first case study)	139
Table 4.2	Descriptive Statistics of Pre-Test Achievement (Second case study)	139
Table 4.3	Welch t-test Result of Pre-Test Score (First case study)	140
Table 4.4	Independent t-test Result of Pre-Test Score (Second case study)	141
Table 4.5	Summary of Hypothesis 1	142
Table 4.6	Mathematics Ability based on Pre-Test Achievement (First case study)	144
Table 4.7	Mathematics Ability based on Pre-Test Achievement (Second case study)	144
Table 4.8	Summary of Hypothesis 2	145
Table 4.9	Type of Formative Assessment (First case study)	147
Table 4.10	Type of Formative Assessment (Second case study)	147
Table 4.11	Descriptive Statistics of Formative Assessment (First case study)	148
Table 4.12	Descriptive Statistics of Formative Assessment (Second case study)	149
Table 4.13	Mann Whitney Test for Formative Assessment	

	(First case study)	150
Table 4.14	Independent t-test Result of Formative Assessment (Second case study)	151
Table 4.15	Summary of Hypothesis 3	152
Table 4.16	Summative Assessment	153
Table 4.17	Descriptive Statistics of Summative Assessment (First case study)	154
Table 4.18	Descriptive Statistics of Summative Assessment (Second case study)	155
Table 4.19	Welch t-test Result for Summative Assessments (First case study)..	156
Table 4.20	t-test Result for Summative Assessments (Second case study)	157
Table 4.21	Summary of Hypotheses in Research Question 4	158

LIST OF FIGURES

	Page
Figure 1.1 Quantitative Course Flow Chart in AiU & APU	9
Figure 1.2 Prerequisite Flow Chart in College of Science University of North Texas	10
Figure 1.3 Research Framework	17
Figure 1.4 Relation between the study’s objectives, research questions and hypotheses	20
Figure 1.5 Theoretical Framework	23
Figure 2.1 Forest plot of Studies on Prior Knowledge	42
Figure 2.2 Forest plot of Studies on Alternate Teaching Methods	63
Figure 2.3 Flowchart of Discrete Clinical Trial Method	78
Figure 2.4 National Constructment Centre of ASD Suggested Steps	79
Figure 3.1 Variables of the Study	89
Figure 3.2 Flowchart of BDMM	112
Figure 3.3 Animation on Application of Integration.....	115
Figure 3.4 Powerpoint on Derivatives of Exponential Function	116
Figure 3.5 Video: Rules of Derivatives	116
Figure 3.6 Slideshow on Techniques of Integration	117
Figure 3.7 Animation on Changes of Rates	117
Figure 3.8 Full Worked Example	119
Figure 3.9 Partial Worked Example	120
Figure 3.10 Independent Example (Total Prompt Removal)	120
Figure 3.11(a) Example of Feedbacks	121

Figure 3.11(b) Example of Feedbacks	122
Figure 3.12 Extract of Online Quiz via Microsoft Form	123
Figure 3.13 Extract of Online Quiz	124

LIST OF ABBREVIATION

BDMM	Blended Discrete Mathematics Model
DCT	Discrete Clinical Trial

**KEBERKESANAN MODEL MATEMATIK
DISKRIT TERADUN DALAM PENGAJARAN DAN PEMBELAJARAN
KALKULUS**

ABSTRAK

Tujuan utama kajian ini adalah untuk membina and mengkaji keberkesanan Model Matematik Diskrit Teradun (BDMM) dalam pengajaran dan pembelajaran kursus Kalkulus. Kaedah ini berasaskan teknik ‘Discrete Clinical Trial Method’ yang diamalkan dalam dunia perubatan untuk mengajar kanak-kanak ‘autism’. BDMM menggunakan kaedah pengajaran bersemuka dengan pelajar (fizikal) dengan disokong melalui platform pembelajaran elektronik khususnya dalam pembelajaran kemahiran asas matematik. BDMM mengutamakan penguasaan masteri kemahiran-kemahiran asas mathematics sebelum seseorang pelajar memperolehi sebarang kemahiran Mathematics yang tinggi. Kajian ini menggunakan kaedah ‘studyal’ kedua dalam dua kajian kes iaitu di Universiti Antarabangsa Albukhary dan Universiti Asia Pacific. Faktor pertama dalam kajian ini ialah Tahap Kemampuan Matematik dan factor kedua ialah kaedah pengajaran kursus Kalkulus. Pencapaian pelajar dalam Ujian Pra, Pentksiran Fomatif dan Sumatif ialah pembolehubah bersandar dalam kajian ini. Seramai 143 pelajar dalam kajian kes pertama dan 94 pelajar dalam kajian kes kedua telah dipilih sebagai sample kajian. Kajian ini dijalankan selama 14 minggu. Analisa Rasch memberikan Tahap Kemampuan Matematik pelajar dan Indek kepelbagaian Simpson menunjukkan kepelbagaian Tahap Kemampuan Matematik di kalangan pelajar. Pencapaian pelajar dalam pentaksiran dianalisa melalui ‘Welch Test, Mann-Withney Test’ dan ‘Independent t-test’. Dalam kajian kes pertama,

pelajar BDMM menunjukkan prestasi yang ketara lebih baik daripada pelajar tradisional dalam pentaksiran formatif dan sumatif. Manakala dalam kajian kes kedua, prestasi pelajar BDMM ketara lebih baik berbanding dengan pelajar tradisional. Walaubagaimanapun, tidak wujud perbezaan yang ketara dalam prestasi petaksiran sumatif dalam kajian kes kedua. Ini menunjukkan bahawa BDMM berkesan meningkatkan pencapaian pelajar dalam kursus Kalkulus hingga tahap kesukaran yang terhad. Kajian ini juga menunjukkan bahawa penguasaan masteri kemahiran asas matematik sahaja tidak mencukupi untuk membantu pemerolehan kemahiran Matematik yang tinggi.

EFFECTIVENESS OF BLENDED DISCRETE MATHEMATICS MODEL IN TEACHING AND LEARNING OF CALCULUS

ABSTRACT

This purpose of this study to construct and evaluates the effectiveness of Blended Discrete Mathematics Model (BDMM) in the teaching and learning Calculus course. This model was based on the Discrete Clinical Trial Training engaged by medical personal to teach autism students. BDMM utilise face to face platform and supported by e-learning platform to especially in the learning of fundamental mathematics skills. BDMM emphasis the mastery of fundamental mathematics skills before a student progress to acquire new Mathematics skills. This study was a second studyal method in learning Calculus in two case studies at Albukhary international University (AiU) and Asia Pacific University. The first factor is the mathematics ability of the students and the second factor is the method of teaching. The dependent variable is the students' achievement in the Pre-Test, Formative and Summative Assessment. The sample were 143 students in first case study and 94 students in second case study. The study was conducted over a 14 weeks period. Rasch Analysis showed the mathematics ability among the students and Simpson Diversity Index showed that the students were diverse in terms of Mathematics Ability. The achievement of the students were analyses through Welch Test, Man-Whitney Test and independent t -test. In the first case study, the BDMM students performed significantly better than the traditional students in both Formative and Summative Assessments. Meanwhile in the second case study, BDMM students performed significantly better than the traditional students in Formative Assessments. There was

insufficient evidence to both group performance were significantly different in the Summative Assessments. This study showed that BDMM is effective to improve the performance in Calculus course for certain level of difficulty. It also showed that the mastery of fundamental mathematics skills alone might not enough to acquire higher difficulty Mathematics skills.

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Quantitative and mathematical competencies have become a must-have skill in every graduate to fit in the real working world especially in industries. Quantitative competency requires knowledge in mathematics and statistics at or above basic algebra level. This includes the ability to perform a comprehensive analysis of data and to interpret the data (Mcdermott, K. (2013, September 17) Quantitative Competencies, 2019 Retrieved from <https://geoc.uconn.edu/competency-resources/>). 'Knowledge-based industries' demanded more than basic education as it is posed an inaccurate indicator of individual capability. Industries require higher-level competencies and abilities such as analytical thinking developed through mathematics courses to adapt to a changing world and performing the challenging task (Blazquez et al., 2018). Computational thinking (CT) which is the basic concept in Information Process Theory Mathematics and CT requires mathematical skills such as problem-solving, abstraction, algorithmic thinking, creative thinking, logical thinking, and analytical thinking (Durak & Saritepeci, 2018). Mathematics is important not only in the field of science and technology but also in management and business. Most of the programs in these fields include Mathematics and Statistics as a compulsory course in their structure.

Instructors and students faced numerous obstacles to achieve a good result in Mathematics courses (Arshad et al., 2018). As for Mathematics instructors (teachers, lecturers, and tutors), teaching Mathematics c

an be challenging due to various reasons (Herbst & Milewski, 2018). This study is triggered by the following factors described in the following subsection 1.1.1 to 1.1.4.

1.1.1 Mathematics Ability

Mathematics Ability is defined as the ability of a person to perform mathematics tasks and solve basic mathematics problems (Karsenty et al., 2015). Hong et al. (2009) study showed that the lack of basic concepts in mathematics leads to the decreasing level of mathematical competencies and analytical ability among students. Fuch et al. (2005) stated that a simple but underappreciated fact is the answer to the question: preparation level of learners in the core number skills that empower the learners to handle basic or minimum requirements at their respective grade is not good enough as an adequate foundation as the learners' progress and face more challenging concepts and mathematics operations. Lack of adequate basic mathematics skills in lower levels leads to low performance in higher-level mathematics concepts (Lehner, 2008). This level of the challenge increases especially in a classroom with great heterogeneity in the prior knowledge of basic mathematics skills. The deficiencies of basic mathematics content in the first-year student cause a major problem to university authorities (Goyder & Miller, 2010).

A study was conducted in Wawasan Open University to investigate whether the poor performance in Engineering Mathematics 1 examination is due to the poor grasp of the tested mathematics skill or the fundamental mathematics skill. The study was based on responses of incomplete items (questions). The result showed that the failure to execute

the fundamental/basic mathematical skill is one of the main factors that contribute to poor performance in higher-level mathematics courses (Ramasamy, 2011).

1.1.2 Mastery in Fundamental Skills and Heterogeneity of Mastery Level

Biemans and Simons (1996) conceive of prior knowledge as 'all knowledge learners' have when entering a learning environment that is potentially relevant for acquiring new knowledge. In this study, prior knowledge refers to the fundamental mathematics skills acquired at an earlier level of study. Associating new skills with prior knowledge and previously acquired skills is one of the characteristics of learning new skills. The primary, secondary, and tertiary mathematics curriculum stresses the requirement of acquiring basic mathematics skills especially before progressing to the next higher-level mathematics courses. This requirement is in place to ensure the smooth progression of students in acquiring new mathematics skills. Therefore, the solution to proper mathematics teaching and learning must start from the basic foundation level for other mathematical concepts to build upon (Ahiakwa, 2005). This characteristic is very important especially in learning Mathematics as Mathematics new skills cannot stand alone. Goyder & Miller (2010) concluded that schools are under preparing the students to succeed in a mathematics course at the tertiary level. The question is not how much is taught at school but how the mathematical content is constructed at the school level? The high rate of not able to acquire high-level mathematics successfully at the tertiary level due to lack of basic mathematical concept mastery makes the mathematics department rethink alternate approaches to deliver the teaching and learning process effectively (Prendergast, 2016). All the advanced skills of mathematics irrespective of any level require some fundamental Mathematics skills to acquire them (Ramasamy, 2011).

Learning basic mathematical skills by acquiring the skills to be reapplied and be associated with the new mathematical skill is the desired outcome of pre-requisite course or prior knowledge skills. However, the system in Malaysia universities and schools allows students to take the next level mathematics course as long as the students met the minimum grade requirement. Students who experienced significant problems in learning mathematics variously manifest their learning problems. There several reasons why these students have trouble learning mathematics (Mercer, Jordan & Miller, 1996; Mercer et al., 1996; Mercer & Mercer, 2004; Miller & Mercer, 1997). One of the contributors to mathematics disability failures is the poor mastery of fundamental mathematics skills. National Center of Excellence in Teaching Mathematics, United States of America (2014) describes mastery in mathematics as the ability to develop conceptual understanding, recall and reapply the skills or knowledge rapidly and accurately. In this study, poor mastery level of mathematics is defined as unable to recall and apply the skill/knowledge accurately. Students with below minimal performance in mathematics often exhibit repeated difficulties in computation and problem solving (Fuchs et al., 2005) and require intervention and alternative strategies to support their academic deficit (Calhoun et al., 2007). Students' mathematics achievement will be severely affected if the issue of lack or poor mastery of basic mathematics skills is not addressed and being remediated by effective interventions that will place these students at-risk for academic failure (Sarrell, 2014).

Apart from poor mastery level of fundamental mathematics skills, the homogeneity of mastery in fundamental mathematics skills level among students in a class or group can pose a challenge to the teaching and learning process. Students in a

homogeneous class inability benefit from tailored instruction in classes more compared to heterogeneous class (Hoxby & Weingarth, 2006; Zimmer, 2003). One of the troublesome and enduring problems of mathematics instruction is accommodating heterogeneity in student preparation and learning rate (Koppersteiner, 2018; Slavin & Karweit, 1985).

This challenge is more obvious in an environment where students come from various backgrounds such as in an international group of learners with various entry requirements. The mathematics courses in the education system in various countries have variation in terms of topics, skills, and depth of knowledge. Different countries have a different combination of mathematics topics, organisation elements such as the amount of time allocated to specific topics, types of activities used in the teaching and learning process, the practice of reviewing the past lesson and introducing new skills, roles of homework, and the enhancement of key ideas and lesson flow that shapes the learning process (Hiebert et. al., 2004). The preparation level, weekly allocated hours, and awareness of current ideas on teaching mathematics are among the obvious differences found in the studies conducted by TIMSS Video Study in 1999. The variation in topics, the approach to teaching certain topics including time allocated to problem-based learning existed among the seven countries studied (Hiebert et al., 2004). The mastery level of each identified mathematics attributes is varying from country to country based on the content, teaching methods, classroom environment, and other factors (Tatsuoka et al., 2004). In a class of the multinational population, the possibility of having students with a heterogeneous level of mastery in various basic mathematics skills is high. The variance in prior knowledge mastery level as in a heterogeneity classroom leads to difficulties in the delivery in the classroom due to numerous levels of instructions required to address

the students (Kramarski, 1997). The poor mastery of fundamental Mathematics skills may cause a big threat to the acquisition of new advanced Mathematics skills. This factor will contribute to the slower pace and disability of learners to acquire new or advanced Mathematics skills. The emphasis on mastery learning of fundamental mathematics is being a key indicator of the success of acquiring new mathematics skills (Baldwin & Squires, 2019).

1.1.3 Sequencing of Mathematics Courses

The problem of classroom heterogeneity posed challenges in the teaching and learning mathematics course as the courses are structured in a sequential or spiral design. The level of challenges in the teaching process of Mathematics courses increased over time as a student progresses to a higher level where the degree of classroom heterogeneity in terms of prior knowledge in basic mathematics skills enlarges. Often the poor performance in achieving learning outcomes of any Mathematical courses will be associated with the difficulties of the course as the complexity and the level of the course getting higher. Salami & Omiteru (2017) mentioned that the complexity of higher-level mathematics is due to the nature of mathematics' spiral curriculum. Nevertheless, the curriculum designer prepared the complexity level in the context of age appropriateness (Kruger & Wessel, 2015).

1.13(a) Spiral Curriculum

The mathematics curriculum in Malaysia has the features of a spiral curriculum. Bruner's Theory of Development states that cognitive skills and techniques developed

gradually forms the basis of a spiral curriculum (Johnston, 2012). Harden (1999) had defined the spiral curriculum as a curriculum with the following features

- (i) topics will be revisited periodically i.e. every term or year,
- (ii) the level of difficulty of topics will increase gradually or successively at every visit,
- (iii) new skills or information will be introduced based and linked on previous skills,
- (iv) the competencies of students' topics will gradually be enhanced.

The essential core of the spiral curriculum, the competencies of mathematics skills are gradually increased as they were heavily linked or related to the previously learned skill. However, the student who moves to the next level or period without acquiring the current skills will lose out, and most probably will have problems in acquiring higher-level mathematics skills. The problem will be accumulated at every stage. The need in acquiring relevant mathematics skills is essential and fundamental in the process of acquiring new mathematics skills. In other words, the success of acquiring new mathematics skills is highly dependable on the level of mastery in acquired basic or prior mathematics skills (Snider, 2014).

1.13(b) Sequential Curriculum

As at the tertiary level, the mathematics curriculum is not a spiral curriculum as in the Malaysian school curriculum. The university will schedule the mathematics courses sequentially from low-level mathematics courses leading to a higher level. The skills in

lower-level mathematics courses are often required to acquire mathematics skills at the next level. Some universities make it compulsory student to sit and pass the lower-level mathematics courses before access to higher-level mathematics courses. These courses are known as pre-requisites courses. Terry N.B. et al. (2016) emphasis that the mastery of pre-requisite courses is critical to subsequent learning as their study shows a learning gap between students with strong pre-requisite skills and those with weak pre-requisite skills. They concluded that the structuring of the pre-requisite course must accompany by pre-requisite course grade requirements. The performance in pre-requisite courses correlated positively with the performance in the core courses (Islam et al., 2018). This study found out those who have taken calculus and algebra courses as pre-requisite courses performed better in Business Statistics as a core course compared to those who choose to take the Foundation of Business course as a pre-requisite. This study concludes that curriculum planning should be done in a way to create a learning continuity by preparing relevant pre-requisite courses. The performance in mathematics courses is determined by prior success in particular courses often referred to as prerequisites that systematically regulate student performance and progress in mathematics (Oakes, 1990). This requirement shows the importance of knowledge or mastery in respective basic mathematics skills to follow and acquire higher-level mathematics skills. The requirement of pre-requisite courses before signing up a higher-level Mathematics course is common in many universities regardless of Malaysia or global. In Wawasan Open University, students must sit and pass the University Mathematics courses before they were allowed to sign up for Calculus and Linear Algebra courses. Students must pass these two courses before they sign up for Engineering Mathematics I and a pass in this course is a must for a student who signs for Engineering Mathematics II. In Albukhary International University, the Mathematics

(including Statistics) courses were offered sequentially where the Algebra course serves as a pre-requisite course for Statistics and the Statistics course serves as a pre-requisite to Calculus course. In Asia Pacific University. Students are required to take Linear Algebra course before Calculus I and II. Mathematics has a specific sequence of topics that begins from Algebra followed by Geometry before ending up with Calculus (Cometto, 2008). The sequence of Mathematics course offerings at Albukhary International University and Asia Pacific University adhered to Cometto's statement. Pinker, 1997 found that students faced difficulty in Calculus, not because of the abstruse concepts in it but poor mastery of Algebra knowledge. Figure 1.1 shows the sequence of quantitative courses in the Foundation program at Albukhary International University (AiU), Alor Setar, Kedah, and Asia Pacific University. Figure 1.2 shows the flow chart of the connection between pre-requisites courses and next-level courses in mathematics discipline in one of the universities in the USA.



Figure 1.1 Sequential of Quantitative Courses in AiU and APU

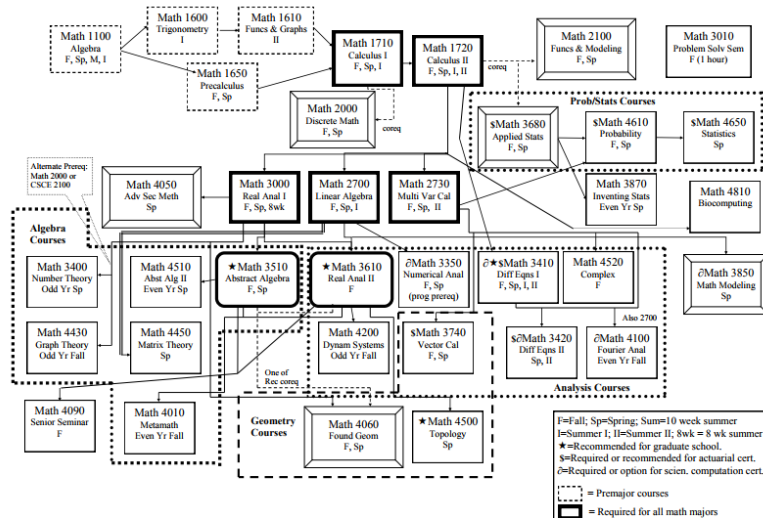


Figure 1.2 Pre-requisite Flow Chart in College of Science, University of North Texas

1.1.4 Traditional & Alternative Approaches in Teaching Mathematics

The traditional approach of learning and teaching mathematics involves direct instruction approach such as lecturing, tutoring, and teachers' lead demonstration are commonly used methods in the classroom. This approach has the element that all students be taught the common material at a common pace at the same point. Traditional teaching methods with constraints on delivery styles unable to cater to students with various levels of prior knowledge which leads to a learning burden to lower mastery level of basic mathematics skills' students (Zhou Lan, 2019). The good students in terms of mastery of mathematics skill might felt bored where the teaching approach unable to cater to their demand but will successfully acquire the knowledge without challenging themselves to acquire a higher level skill. Meanwhile, those with a poor grasp of mathematics might feel that the course is beyond their capabilities, fail to acquire the taught knowledge, and lost their confidence. Traditional teaching and learning processes might not suitable to cater to groups with various levels of mastery of basic mathematics skills. In the traditional

education process, the curriculum and the teachers used to dump common cognitive load to the learners regardless of the various level of mathematical ability of the group (Van Merriënboer et al., 2003). Teachers might have to lower the level of teaching and depth of the content to cater to the weaker group. This practice will jeopardise the quality of the course and the time allocated will be insufficient for the teacher to cover the necessary content. The institutions might not be able to lower the level of the mathematics skills in the courses as the quality of the programme will be questionable and lead to a problem with the accreditation body. The challenges in teaching mathematics in heterogeneous class are to design and deliver a curriculum that will be able to cater to a population with various level of basic mathematics mastery but at the same time maintain the standard of the curriculum at a level fixed by the authorities such as accreditation and professional bodies. The curriculum too should not jeopardise or demotivate the learning ability of students with a higher mastery level of mathematics skills. Another popular solution to tackle this challenge is the students being streamed or grouped according to their basic mathematical skills mastery level. It may be possible but the required fundamental mathematical skills for every new mathematics skill vary where will lead to the group's membership being fluid based on the actual mathematical skills going to be learned. The composition of the group varies each time a new topic or skill is being introduced. This will be an administrative headache to group and regroup learners at every skill taught in each grade. Kajander (2006) mentioned that institutions realise the limitation of traditional methods and at the same time lean towards the constructivism pedagogy.

The challenges posed by poor mastery of basic mathematics skills in addition to the heterogeneous group in terms of prior knowledge in basic mathematical skills leads to

this study. Alternative methods were developed and tested with mixed results in the successful acquisition of higher mathematics skills. Constructivism in mathematics was defined as a process where the student needs to construct their understanding of a concept. One of the critical paths in approach according to the study is the decomposition of mathematics concepts into development steps as in Piaget's theory of development. Gagne's model of learning hierarchy is based on a 'top-down' approach where the final task is defined as capabilities to perform a task (Talbi, 1990). The final task is analysed by identifying the prerequisite capabilities to perform the task which defines as the knowledge that one should have to perform the task (Gagne, 1962). The process will be repeated on the identified prerequisite task. The process ends when a complete hierarchy of basic simple skill is achieved (Sarat, 1988). Constructivism theory and Gagne's theory of learning, an alternative method to address the challenges mentioned earlier is constructed.

NCTM (2017) mentioned that technology usage in the teaching and learning mathematics will able to assist the student to understand, acquire new skills, and increase their competencies in mathematics. These two aspects of technology-assisted learning strategy; one is on the exploration of mathematical concepts and another on the communication and collaboration of the concepts (delivery).

E-learning is one of technology-assisted deliveries method can be utilised to accommodate alternate approaches for students with different prior knowledge and level of fundamental mathematics concept. E-learning, an electronic platform which is widely utilised as a platform in the teaching and learning process is being used in education institution globally (Aljawarneh, 2019). The platform has changed the ways and approach

of teaching and learning processes. The technology has created unlimited opportunities for users to try and practise various methods to improve the process of teaching and learning. Plenty of electronics resources have been produced and various techniques have been tried and practised through this platform.

E-learning can increase the quality of education and increase achievement in many subject's area (Diana, 2009). E-learning is defined as a new technology for delivering online, hybrid, and synchronous learning (Nagi & Vate-U-Lan, 2009). Blended learning was defined as learning that mixes various event-based activities, including face to face classrooms, live e-learning, and self-paced learning. This often is a mix of traditional instructor-led training, synchronous online conferring or training, asynchronous self-paced study (Singh, 2003). Blended learning refers to an integration of online activities and traditional face to face activities (Graham, Allen & Ure, 2005). Blended Learning is definite as learning where a portion of face to face is replaced by online activity in a planned, pedagogically valuable manner (Laster et al., 2005; Picciano, 2006). A direct translation of traditional materials to online will in no way yield a successful program as it does not provide sufficient opportunities to facilitate successful learning and performance (Singh, 2003).

1.2 Blended Discrete Mathematics Model

The Blended Discrete Mathematics Model (BDMM) is constructed and implemented in the teaching of Calculus to evaluate its effectiveness. BDMM is based on the Discrete Clinical Trial method (DCT), one of the recommended training methods by the medical practitioners and also known as one of the effective evidence-based teaching

skills in guiding the autism children to acquire new skills (Fraser, 2018; Matson & Smith, 2008; Matson & Sturney, 2011; Shilingsburg, Hansen & Wright, 2018). In this method, any new skill will not be taught at one go. Each new skill will decompose into smaller teachable skills (Cohen et al, 2006; Eikeseth et al., 2002). Each smaller skill will be in a list-steps from development to mastery level. These smaller skills will be related to prior learned skills. The new skill will be constructed based on the successful acquisition of each smaller skill. One of the features in BDMM is the decomposition of skill into structured development steps and integration of the mastered subskills to prepare the students to acquire new mathematics skills.

In this study, the BDMM will be constructed and examined on its effectiveness in improving the performance of the first-year Calculus course in two case studies. The first case study was conducted at Albukhary International University and the second case study was conducted at Asia Pacific University. The course has the following characteristics; (i) students with the diversified mastery level of fundamental mathematics skill and (ii) the content (mathematics skills) of the course requires several fundamental mathematics skills.

1.3 Statement of the Problem

The liberalisation and globalization of education decades ago create opportunities and access for many individuals to further their studies to the next level (Chinnamai, 2005). Individuals around the world with a different set of qualifications and diverse prior knowledge have access to further studies around the world (Zajda, 2020).

Mathematic Ability is defined as the capability of a person to perform basic mathematics tasks and solve mathematical problems (Karsenty et al., 2014). The mathematics ability is said to be a fixed trait in a person and had a strong impact on the acquisition of mathematics skills (Rattan, Good & Dweck, 2012; Ndung & Nendi, 2018). In a population of international students, the diversity of mathematics ability is high. As the mathematics ability is a complex latent variable cut across a few domains of mathematics, IRT will be suitable to measure it (Gnaldi, 2017). In order to ascertain that the Mathematics ability among the participants in this study is diversified and contributes to the homogeneity of mastery of fundamental mathematics skills, this study measures the mathematics ability. In this study, Rasch Analysis is used to measure the Mathematics Ability through their performance in the Pre-Test assessment. The outcomes of the Rasch Analysis are further investigated through the Simpson Diversity Index to measure the diversity level of Mathematics Ability among the students.

This group of students with diversified fundamental prior knowledge creates heterogeneous groups which will be a challenge to education providers. This problem might have an impact to the teaching of numerical courses such as mathematics, statistics, and accounting (Du et al., 2019; Komara & Yulianto, 2018; Aida, et al., 2017) The poor mastery of mathematical fundamental skills leads to the under-performance achievement in higher-level mathematical courses (Sarrell, 2014). As for every newly introduced mathematics skill requires knowledge in a set of fundamental mathematics skills for students to acquire new skills. Institutions globally are addressing the problem by introducing preparatory, foundation, or similar mathematical courses. In the traditional teaching method, the instructor will not be able to address the poor mastery and homogeneity

of mastery due to various factors such as time constraints, individual unique understanding issues, curriculum requirements, and many more. In this study, BDMM has been constructed as an alternative teaching and learning method in a mathematics course that emphasises the mastery of fundamental mathematics skills. BDMM is also constructed as a method where the heterogeneous group (in terms of mastery level in fundamental mathematics skills) will be able to acquire new mathematics skills regardless of the individual prior mastery level. BDMM will ensure that students have mastered the required basic mathematics skills before the teaching of new skills. By using this method, teachers will be able to teach the students without worrying that students might not be able to follow or acquire the skill due to a lack of fundamental mathematics skills. Students will be well-equipped with the required fundamental skills to learn the new skill. This method will ensure that the curriculum standard is maintained, the time constraints to cover the syllabus is taken care of with students acquired the necessary fundamental skill prior, reduce the gap of fundamental mastery level among students, and finally improves the performance of the course. In this study, the effectiveness of the model will be tested based on the performance of students in formative and summative assessment.

This study intended to construct a new alternative method to overcome the issue of diversified mastery level of required fundamental skills in acquiring new mathematics skills. BDMM is developed as part of this study and its effectiveness to solve the above-mentioned issue is assessed as the research framework in Figure 1.3. The figure showed the method, grouping, treatment, and measurement of this study.

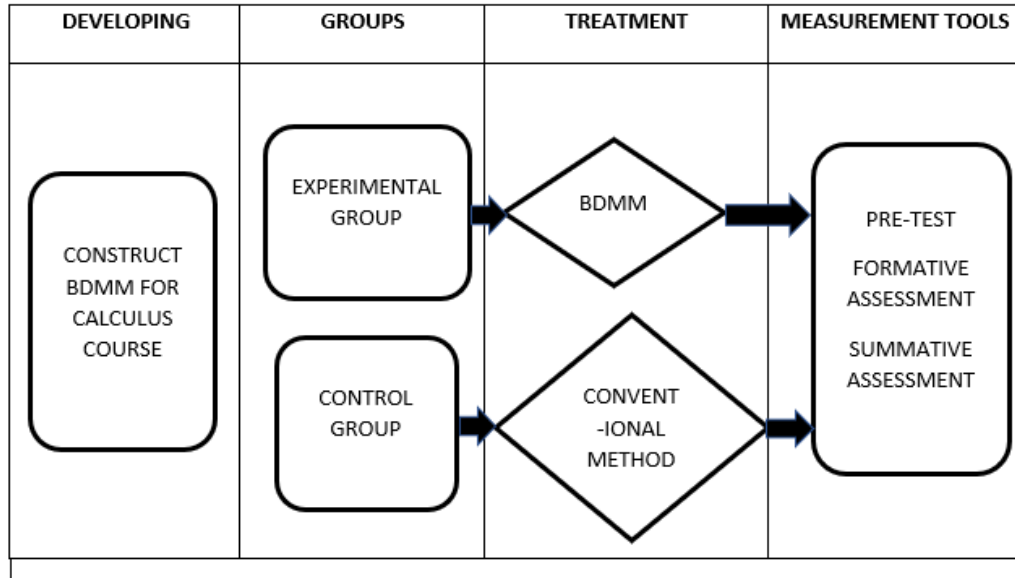


Figure 1.3 Research Framework

1.4 Objectives of the Study

The purpose of this study is to construct a method to overcome the diversity and poor grasp of fundamental mathematics skills in the process of learning new mathematics skills. The Blended Discrete Mathematical Model (BDMM) is constructed and its effectiveness is examined in the teaching and learning process of Calculus. The objectives of this study,

- (i) To measure the diversity of prior knowledge of the students
- (ii) To examine the effectiveness of the BDMM model by comparing the performance of the experimental group with the control group in Calculus

1.5 Research Questions

The following research questions are posed to this study:

- (i) Are the students diverse in terms of prior knowledge?
- (ii) Is there a significant difference in the students' performance in Calculus formative assessment between the students in the BDMM teaching model and conventional method?
- (iii) Is there a significant difference in the students' performance in Calculus summative assessment between the students in the BDMM teaching model and the students in the conventional method?

Based on the above questions, the following hypotheses were developed and tested at a 95 % confidence level

- H₁ There is a significant difference in the students' prior knowledge between the control group and experimental group.
- H₂ The student's prior knowledge measured from the performance in Pre-Test is significantly diverse.
- H₃ There is a significant difference in the students' performance in the formative assessment of Calculus between the control group and experimental group.

H₄ There is a significant difference in the performance in summative assessment of Calculus between the control group and experimental group.

Figure 1.4 shows the relation between the research objectives and their respective research questions and hypotheses. The hypotheses were mapped to the research questions and the research questions were mapped to the research objectives.

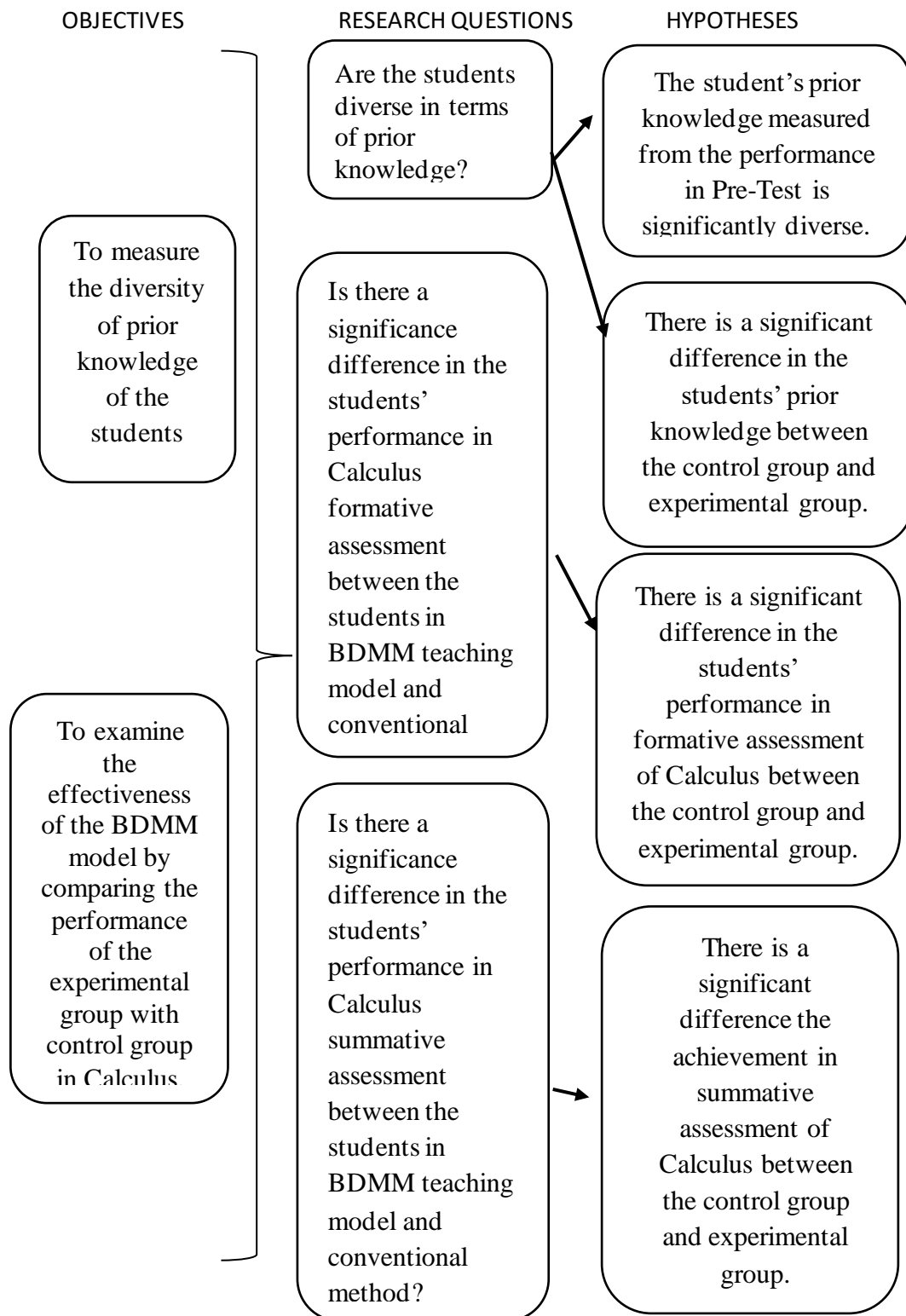


Figure 1.4 Relation between the study's objectives, research questions and hypotheses.

1.6 Theoretical & Conceptual Framework

The constructivist approach in mathematics education is well received by mathematics educators (Bhowmik, 2014). Bhowmik mentioned that many constructivist mathematics instructors maintain one of the main tenets in the constructivist approach is the existing knowledge in children should form the basis of a new skill. Cometto (2008) stated that constructivism dictates any new mathematics skill through existing knowledge structures. A person learns by constructing the existing knowledge he or she had based on his or her own experience (Glaserfeld, 1995). BDMM applies the constructivism approach where new mathematical skills are constructed from prior knowledge of basic mathematics skills with an additional feature which is to strengthen prior knowledge before the 'construction' process. The feature goes along with the concept of constructivism that instructors should not assume that students had mastered the prior knowledge but rather engage prior formal understandings of prior material (Cometto, 2008). Gagne's hierarchy theory of the learning process is based on associated learning where new knowledge is earned through prior knowledge being associated with a new experience. The accumulated new knowledge will be used to learn new skills (Gagne, 1962). Talbi (1990) suggested that Gagne's hierarchy learning theory begins from existing knowledge of student (prior knowledge) and the student is considered ready to learn new skills if he or she has mastered the prior knowledge. Connectivism is defined as an amalgamation between constructivism and cognitive (Conradie, 2014). Downes, 2007 said that knowledge is spread across various networks of connections and learning takes place with the ability to construct and traverse those networks. Connectivism is the

enhancement of a learning process with the knowledge and perception acquired through the addition of personal networks.

As connectivity is an indicator of access to other information, it indicates learning between the access to information and learning in line with connections. The study suggests that the ability to acquire new skills heavily correlates to the student's capability to associate prior knowledge and skill to the skill. The study concludes that the lack of competence in one or more of the prior skills will cause the failure to perform or acquire the new skill.

Based on the DCT, Constructivism theory, Connectivism theory, and Gagne's hierarchy learning theory which emphasises the great importance of the mastery of basic mathematical skills and connecting them in the process of learning higher-level mathematics, this study introduces BDMM as an alternative method in teaching and learning mathematics. BDMM will infuse the necessity of increasing mastery level and connectivity level of required fundamental skills for each new higher-level mathematical skill so that the skills will be successfully acquired. The following Figure 1.5 illustrates the Theoretical Framework of this study.

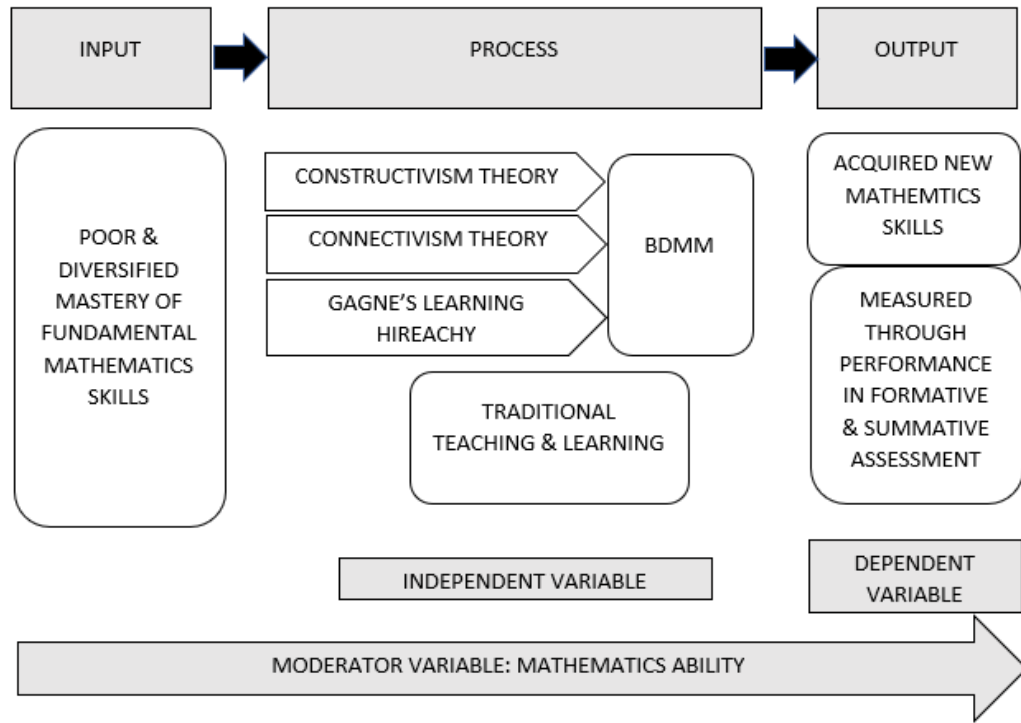


Figure 1.5 Theoretical Framework

The theoretical framework consists of three components. Students with poor mastery of and diversified level of prior knowledge in fundamental mathematics skills are the input in this framework. The BDMM model which has elements of Gagne's hierarchy learning theory, Connectivism theory, and Constructivism form the base of the framework. The inputs are the students who will go through the BDMM model that ensure the mastery of the required fundamental mathematics skills for any newly introduced mathematics skill. The output will be the students who possess an adequate level of mastery of fundamental mathematics skills to acquire new mathematics skills. The independent variables are the methods in the teaching and learning Calculus used in the teaching Calculus in the study. BDMM and conventional teaching methods are the independent variables used by the experimental and control group respectively. The performance of both groups is the dependent variable. The mathematics ability among

these two groups which can influence the dependent variables is the moderator variable. Figure 1.5 shows the cause-effect of the variables.

1.7 Significance of this Research

Mathematics educator has conducted numerous researches on the poor mastery of fundamental mathematics skills which is one of the factors that affected the performance of mathematics modules. This factor combined with the heterogeneity of mastery level of fundamental mathematics skills remains a challenge for mathematics instructors in providing effective teaching and learning methods. Thus, this study hopes to provide an alternative method to address this problem with the construction of BDMM.

This study proposed to evaluate the effectiveness of BDMM by implementing the model in teaching and learning on the Calculus course. The teaching of Calculus is conducted through two different methods, BDMM, and the traditional method. Besides that, this study is also measuring the mathematics ability of students to ascertain the diversity of the students.

The study on measuring the mathematics ability of students through Rasch Analysis which measures the probability of students giving a correct response based on the traits of difficulty of items can be a useful tool besides ascertain the diversity of students. This tool can be very useful to ascertain if a student is suitable to enroll for a certain program based on his mathematics ability. A program that consists of a higher number of numerical courses requires higher mathematics ability among the students.