NUMERICAL ANALYSIS STUDY OF SARAWAK BARRAGE RIVER BED EROSION AND SCOURING BY USING SMOOTH PARTICLE HYDRODYNAMIC (SPH)

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By

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I hereby declare that all corrections and comments made by the supervisor(s) and examiner have been taken into consideration and rectified accordingly.

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Name of Examiner :

Date :

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ABSTRAK

Smooth Particle Hydrodynamic (SPH) adalah model simulasi tiga dimensi. Dalam kajian yang dijalankan ini, tiga situasi dan satu situasi pengesahan telah disimulasikan menggunakan DualSPHysics. Kawasan kajian yang dijalankan adalah Sarawak baraj. Situasi yang dipilih untuk menjalankan kajian adalah dengan pembezaan paras air pada hilir sungai. Kajian ini sebenarnya bertujuan untuk mensimulasikan hakisan dasar sungai dan rupa bentuk kerukan yang terjadi dengan menggunakan kes dua fasa dimana menggunakan pasir sebagai sedimen dan air. Halaju dan rupa bentuk kerukan telah direkodkan dan dipaparkan dalam bab hasil keputusan. Keputusan yang diperolehi dari kajian pengesahan boleh diterima pakai dimana profil kerukan dan halaju mempunyai sedikit perbezaan antara ujikaji makmal dan simulasi. Oleh itu, kesimpulan yang boleh dibuat adalah simulasi dengan menggunakan SPH boleh digunakan sebagai alternatif untuk mensimulasikan situasi sebenar.

ABSTRACT

Smooth Particle Hydrodynamic is the three-dimensional (3D) model. In this research work, three cases and one validation have been simulate using DualSPHysics. Study area of this research work was at Sarawak Barrage. The cases have different water level at the downstream. This study actually to simulate riverbed erosion and scouring properties by using multi-phases cases which use sand as sediment and water. The velocity and the scouring profile have been recorded as the result and shown in the result chapter. The result of the validation is acceptable where the scouring profile and the velocity were slightly different between laboratory experiment and simulation. Hence, it can be conclude that the simulation by using SPH can be used as the alternative to simulate the real cases.

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LIST OF ABBREVIATIONS

3D	Three-Dimensional
2D	Two-Dimensional
1D	One-Dimensional
WC-MPS	Weekly Compressible MPS
I-SPH	Smooth Particle Hydrodynamic Incompressible Fluid
SERF	Simulator of Erosion Rate Function
BSTEM	Mechanistic Bank Erosion
PC	Personal Computer
GPU	Graphic Processing Unit
CPU	Computer Processing Unit

NOMENCLATURES

C	Cohesion
ϕ	Friction angle
S_u	Shear Strength
I_P	Plasticity Index
I_D	Relative Density

CHAPTER 1

INTRODUCTION

1.1 General

Water flow has their own energy in the river. By that energy, the river changes their physical characteristics such as the shape which can be wider, deeper and longer. The material that has been transported from places to another places is caused by erosion of the river bed and river banks. Focusing on the river bed erosion, the changes of the river bed level can happen in both upstream and downstream of the river through the different of the head pressure. According to Chu (2014), river channel-bed erosion and deposition not only affects river sediment flux to the sea but also influences the river regime and associated management practices like in-channel navigation, flood control, and levee maintenance. These have been longstanding issues for many large rivers around the world, such as the Mississippi, the Amazon, the Yellow, the Yangtze and the Pearl.

Generally at the barrage, the level of water at upstream is higher than the level of water at downstream. The greatest part of river bed to be eroded is at downstream of the river. When the barrage is opened, the highest head pressure of water will flow to the lower head pressure and scouring or river bed erosion will occurred at those location. Based on Stefan et al. (2016) the residual flow energy will produce scour in an alluvial riverbed if no mitigation structure (such as a dissipation basin) is installed downstream of a grade-control structure. In the absence of a technical dissipation structure or a downstream apron, scour can occur in a loose riverbed downstream. Bhuiyan et al. (2007) investigated the effect of a W-weir (without upstream or downstream apron) on the downstream sediment transport processes, focusing on the scour-hole formation. The maximum scour depth was observed a short distance downstream of the weir, independent of the boundary conditions.

Therefore, by undergo the numerical simulation method the river bed erosion and scouring can be easily identified. The location of the phenomenon and other data can be obtained by using Smooth Particle Hydrodynamics (SPH). SPH could play a very useful role in determine the condition of the river bed at the barrage. The gathered information could lead to a new discovery and helps to improve the river condition.

1.2 Problem Statement

The erosion of the river bed happens at downstream of the barrage when the gate is opened and water discharged through the gate. Because of that, transportation of the sediment material and channel degradation occur due to the flow of water. All of the sediment material that comes from the wash load activities by the river flow itself change the catchment hydrology and geomorphology. The erosion of the river bed need to be predicted because the excessive water erode the bed can cause the stability of the hydraulic structure to be significantly reduced. The problems are the river bed not in flat levelled and some places becomes shallow and another will deeper. River bank also risk with the bank failure that may damage the river corridor.

Prediction of the sediment transport movement from the river bed erosion process need to be studied by using physical modelling and numerical analysis simulation. The result of the two methods has been mentioned need to compare and validate between each other. In this research, numerical analysis simulation has been selected based on Smooth Particle Hydrodynamic (SPH) software. By using this software, random movement the particles of the sediment materials can be seen clearly through the 3 dimensional (3D) model where it is among the benefits of this software. Sediment transport in hydrodynamics is difficult to simulate due to random movement and complex boundary (Jilani & Hashemi 2013).

1.3 Objectives

The objectives of this study are:

a) To validate experimental model of bed erosion and scouring by using SPH model
b) To simulate bed erosion and scouring of Sarawak Barrage River by using SPH model

1.4 Scope of work

To achieve the objectives above, Sarawak Barrage River geometrical and properties need to be studied in this research. All the details such as drawing of the barrage need to be extracted. Besides that, the numerical analysis which is numerical modelling by using Smooth Particle Hydrodynamic (SPH) will be used in this research. The cases of multi-phase liquid-sediment is related to this study. In addition, from the SPH results, the sediment particle movement and scouring process due to the erosion by the flow of water will be evaluate. This study explores the pattern of the scouring profile and scouring depth of the sediment bed erosion.

1.5 Benefits of study

This research surely will provide some valuable amount of information as the outcomes. The benefit of this research are:

1. By using numerical method, sediment material that eroded from the river bed can be predict easily and civil engineer work becomes easier to make improvement of the river.
2. Give the real picture of the erosion process and scouring of the sediment particles.

1.6 Thesis outline

This dissertation has been separated into several chapters for better understanding of the study. The first chapter is the introduction which give overview of the thesis including problem statement, objective of the study, benefits of this study for civil engineering and scope of work for this thesis study. Chapter two is literature review where it provides theoretical and conceptual understanding of related information based on various researches and findings. Chapter three is the methodology of this study which explain the step and procedures of this study. In chapter three also introduction about the SPH software and its concept are given. Then, for the chapter four presents the results, analysis and discussion of this study. Finally in chapter five presents final conclusions and recommendations of this research.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

River bed erosion phenomenon will occur to any river that has a water flow through the channel. A lot of research has been done to determine the problem of erosion and to overcome the problem happen. This is contributed to the many of the techniques to determine the problem might happen. The research always be there and it’s also based on the cost and the way to make research either physical modelling or numerical modelling. This chapter will describes on the erosion, river modelling and review on SPH and its application in numerical simulation.

2.2 Erosion

Generally, erosion is the natural process that occur by which the surface of the earth is removed by the action of water, wind, glacier, waves and etc. Erosion involves the transportation of rock and soil found along the river bed and bank. It also involves in the break down process of the rock particles and carried to the downstream by the river flow. On the other hand, erosion also can be defined as the moving of a bits of rock or soil particles from one place to another (National Geographic Society, 2017).

The mechanism of erosion is when particles strike the surface at small impingement angle the material is removed by cutting mechanism. The abrasive grits roll or slide when they strike on the surface at small impingement angle and cause erosion by abrasion or cutting mechanism. Then, the material is removed by scouring or scrapping by sharp edges of the particles forming short track-length scars (Padhy & Saini 2012).

Erosion also known as degradation which can be seen in Figure 2.1, the evidence of bed degradation at Sanchez Cerro Bridge. The foundation of the bridge is exposed, as well as the vertical sheet piles in the bank protection.

Figure 2.1 : River bed degradation in Piura River

River bed erosion cause the riverbed to degrade and dump particles and sediments into receiving water body. The bed form particles, along with river bank would be outcast from their interlocking due to the action of water flow. The transportable particles would start to move and deposit at the downstream part of a river section. The process of erosion as shown in the Figure 2.2. This process would cause severe engineering and environmental problems if monitoring programs are not well-managed and practiced (Abidin et al., 2017). The bed surface of the river become progressively coarser and the fine particles are entrained more easily in the erosion process (Lee & Hsieh, 2003).

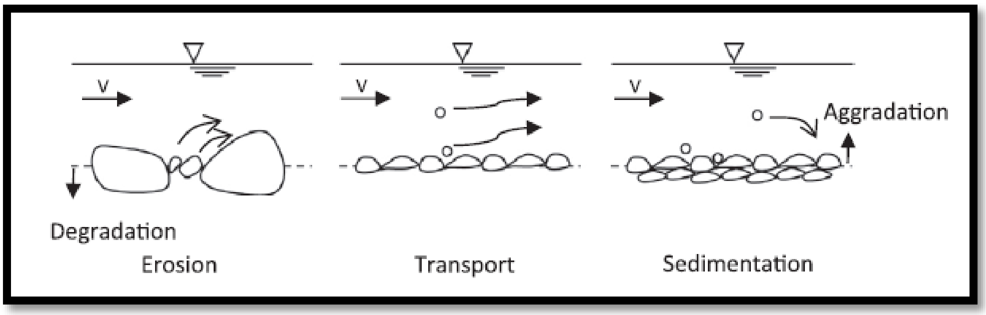

Figure 2.2 : Process of erosion

From the erosion process, sediment transport of the bed material will change the location from one place to another place whether in floatable or not floatable condition. In addition, sedimentation transport can cause major effect to the hydraulic structure and the river characteristics. Levy (2005) says, in historical of Yangtze River, the dikes has been build due to the flood mitigation project. But the dikes start to fail and weakened to some degree cause by the erosion of the bed. Flow underneath of the gates water opening have high amount of potential energy and through this energy can causes excessive scouring waterway bed due to the erosion process and also can cause the structure safety is endanger (Abdelhaleem, 2013). Chatterjee (1995) says that the hydraulic structure that has high discharge for a limited time cause the rate of erosion high. Because of that phenomena, the hydraulic structure cannot survive to the design life expectation.

The process erosion of the sand and gravel bed rivers can be simulated effectively with known resisting forces of non-cohesive material and the driving forces transmitted by the flow of water (Collison & Simon n.d.). The erosion rate is very important and it is generally assumed to be proportional to the availability of the sediment fraction. Fine sediments are partly shielded by high velocity flow while coarser sediments more exposed and they would be among other similar size through to the process of bed armouring (Sanyal, 2016). Smaga (2015) states that river erosion is difficult to predict and it is a complex process that depends on the hydraulic condition.

Nowadays scientists start to include other parameters such as the geotechnical condition of the river beside critical velocity and grain size. The erosion process is where the soil losses their shear strength and particle detach each other. Factors influencing the bed erosion are:

a) Mechanical – cohesion (c), friction angle (ϕ), shear strength (su)
b) Physical – plasticity index (Ip) , relative density (ID), grain size
c) Erosion resistance (cohesionless soils) – particle size and shape, porosity and density.

Vertical stress of the sediment particle not affect the erosion rate which means the erosion process only surface process. The initiation of sediment movement is highly anticipated by action of the velocity, bedform conditions and kinetic energies at the river bed (Yang, 2006). If the shear stress imposed by the flow exceeds the particle shear stress, then the particles start to move (Sulaiman et al., 2013). In addition, to get the stable riverbed slope and minimizing the influence of erosion, sediment transport and deposition, check dams can be constructed to control the problems (Hsieh et al., 2013). When the erosion process is fast and the change of tidal erosion force is slow, erosion only occurs when the tidal flow is accelerating, and the erosion process is always close to the equilibrium state (Maa, 2008).

2.3 Scouring

Scouring is the process where particles of the soil or gravel (sediment) around the hydraulic structure get eroded and removed over a certain depth which called as scour depth. It is usually happen due to the increasing of the water flow or velocity and related to the erosion processes. Scouring also known as the water flow transport the sediment from one place to another place. Figure 2.3 shows the figure of the scouring phenomenon. The scouring depth is very important in designing the hydraulic structure foundation. By neglecting this phenomena, the hydraulic structure cannot stand within the design life and it can give major impact to river bed changes.

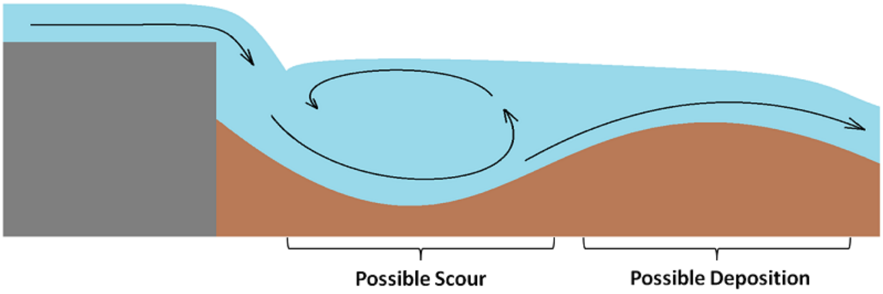

Figure 2.3 : The scouring phenomenon

Scouring occur on downstream of the river that has loose riverbed. Scour depth and characteristics mostly depends on the sediment characteristic, the discharge, the head difference and the tail water depth (Jüstrich et al., 2012). The scouring will be formed through the erosive action of the flows on the sediment beds in front of the outlet during drawdown flushing. The scouring process dampened the flow energy and reduce the flow intensity. The scour hole reduces the flow velocity and thus increase the flow area. Vertical velocity of the water flow makes the scouring hole developed very fast during the early stage (Xue et al., 2013).

The sediment scouring phenomena is basically induced by rapid flows that creating shear forces at the surface of the sediment which can cause the surface to yield and produce a shear layer of suspended particles at the interface and finally sediment suspension in the fluid (Fourtakas & Rogers, 2016). Scouring process will continue until an equilibrium scour depth is reached in the water flow, which corresponds to a situation where increased water depth in the scour hole that cause further bed erosion or to a condition where the rate of bed erosion is balance by the rate of deposition of material.

Many factors governing the depth of scour which are hydraulic, morphologic, and geotechnical (Azmathullah et al. 2005). The scouring profile and characteristics in the water body are very important to protect hydraulic structure. The survey has been made in Taiwan, one fifth of the failures of hydraulic structures (such as check dams etc.) resulted from downstream scour. The flow shear stress acting upon the bed is greater than the resistance of the bed material, a scour hole will be formed and most of the energy will be dissipated (Chen & Hong, 2001). Abdelhaleem, (2013) used baffle block arrangements to reduce the maximum scour depth as well as the maximum scour length and move the position of the maximum scour depth closer to the floor in his experiment. Figure 2.4 shows the layout of the baffles block.

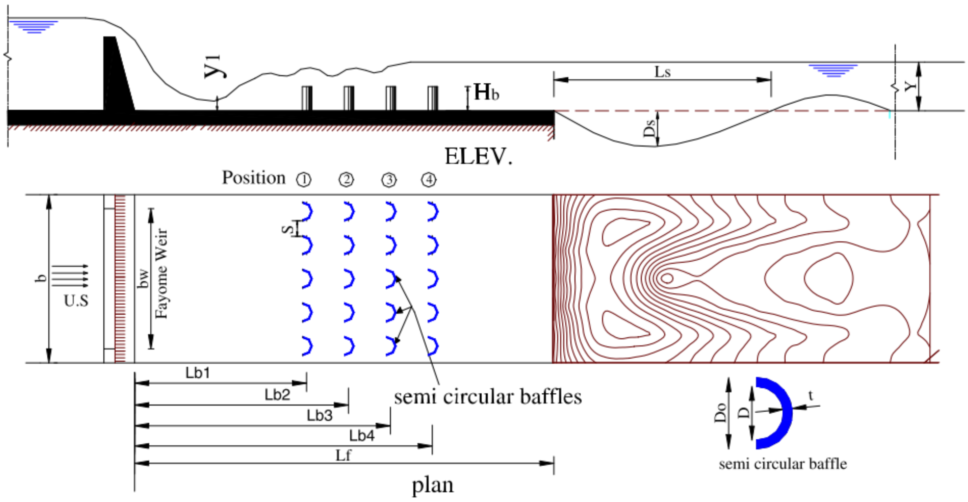

Figure 2.4 : Layout of the baffle block (Abdelhaleem 2013)

The characteristics of the scouring can be described by the time to reach the equilibrium stage, the locations of the maximum scour depth, and the locations of the peak scour from the end of the rigid apron (Chatterjee, 1995).

2.4 Modelling technique

River modelling involves the interaction of the channel, external controls and internal controls. River modelling mostly been made by scale which is reducing the size from the original sizes based on the scale factor. It is more or less same as the origin on site, but it have little different with the model and it also been run indoor. The river modelling can be operated within two methods which are physical model and numerical model. There is a different between the two methods. Mostly the researcher in the field of hydraulics will use the modelling to find the problem happen and to solve the problem. It is quite easy because the modelling method easy to handle and less dangerous in context of safety as compare to the real site.

2.4.1 Numerical modelling

NETSTARS is used to simulate bed evolutions of channel network. It simulate bed load and suspended load separately which simulating scouring and deposition behaviours under an unsteady flow and it is in 1-D (Lee & Hsieh, 2003). Iwasaki et al., (2016) used numerical simulations of depth-average two-dimensional (2D) model. It used real scale simulation of bed and bank erosion during flood event and the model could be potential tool to analyse the complex and mutual interaction and feedbacks of physical and ecological processes occurring. Figure 2.5 shows the result by using NETSTARS numerical simulation model.

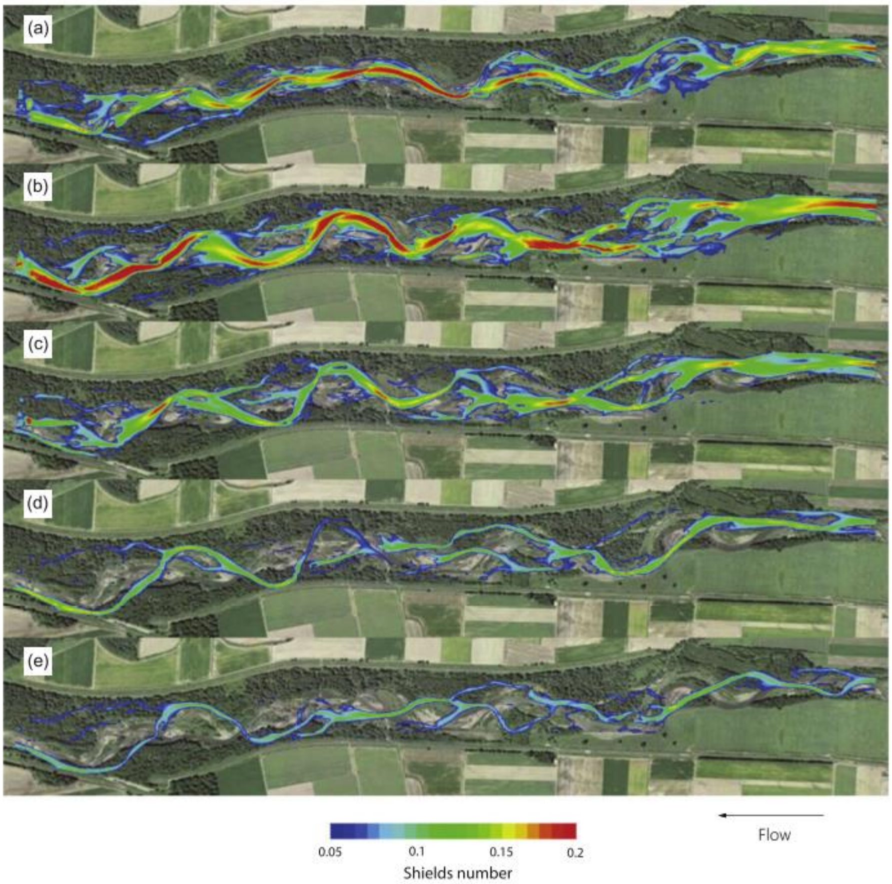

Figure 2.5 : Result of the analysis by depth-average two-dimensional (2D) model (Iwasaki et al., 2016)

Other method of numerical simulation is by using Eulerian two-phase model which consists sand and water in three-dimensional (3D). The model was used to simulate scouring by considering both the flow particle and particle interaction where the results agree with the experimental results (Xue et al., 2013). Besides that, Shakibaeinia & Jin, (2011) uses weekly compressible MPS (WC-MPS) as their numerical simulation to simulate mobile bed dam break problem which consists of two phases. The method is suitable for highly erosive and transient flow for two-dimensional (2D) model. The result is acceptable and has been compared to the experiment. Figure 2.6 shows that the comparison between numerical simulation and experiment results by using MPS model.

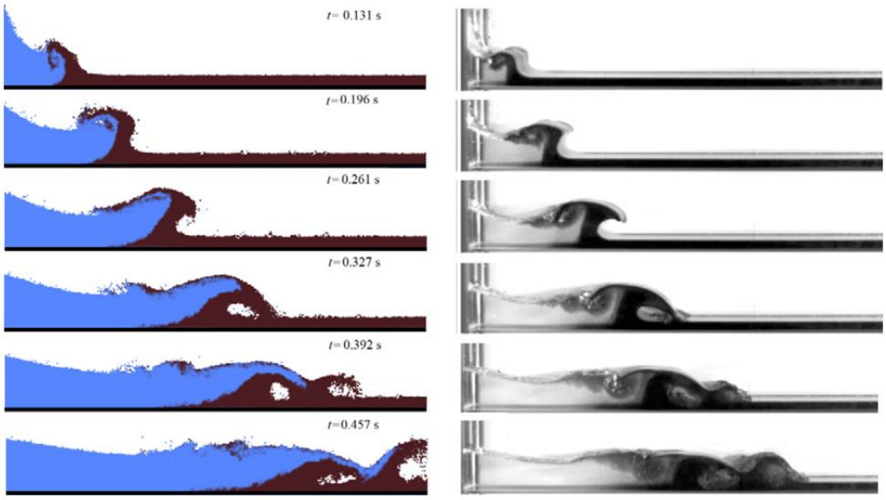

Figure 2.6 : Comparison numerical simulation and experimental results

By using Smooth Particle Hydrodynamic incompressible fluid (I-SPH), the numerical simulation on bed load sediment transportation with uniform steady flow in two-dimensional (2D), the developed model is reliable in prediction of moving patterns of sediment, velocity and location near to the bed. The results of comparing with laboratory experiment were acceptable (Jilani & Hashemi, 2013). The other method was used to estimate contraction scour in riverbed by using Simulator of Erosion Rate Function (SERF). SERF is useful tool for rapid measurement of erosion of stiff clay beds. It also can simulate the hole as a clear water scour process (Jiang et al., 2004).

To simulate sediment transport and bank stability model to predict bank retreat in alluvial streams by the development of a general framework and the associated numerical procedure that allow a seamless integration by using mechanistic bank erosion model (BSTEM) and a widely used 2D mobile-bed model (SRH-2D). The bank stability model handles the mechanistic basal erosion and mass failure of a bank, while the 2D model simulates the fluvial processes near the bank toe and in the stream. But, the model is limited to river bank erosion only (Lai et al., 2014).

For three dimensional (3D) numerical model, the simulation of seabed scour potential has been simulated by using FLOW-3D program. The flow-3D model considers tidal level, current direction and also the velocity of the flow (Burkow & Griebel, 2016). At last but not least, by using Smooth Particle Hydrodynamic (SPH) multi-phase model, SPH focuses on the scouring and resuspension of the solid sediment induced by rapid flows of the liquid. It was well predicted the sediment scouring profile across the bed (Fourtakas & Rogers, 2016).

2.5 Gap of knowledge

This study actually is about to determine the scouring depth and velocity of the water during certain level of water at upstream and downstream of the Sarawak Barrage. Besides that, the contour of the river bed after erosion and scouring take place also need to determine. It is due to mitigate and make the hydraulic structure in safely condition and also to make sure the geological, morphological and ecosystem of the river in good condition. As known, Sarawak Barrage has been constructed to control the flood and also to prevent the sea water intrusion with the river water during the season. The annual rainfall is higher in Sarawak. Sarawak generally flat and low lying. The area are influenced by tidal effect. Flood risk not only include loss of human lives and properties, but also disruption to the transportation and communication. Besides that, Sarawak Barrage also act as the mini dam which supply the water for treatment plant.

Due to the tidal effect, a lot of major problems should be aware to avoid from the unnecessary events going to happen. Because of that, in this dissertation is to simulate the Sarawak Barrage gate operation during tidal water by using numerical simulation to determine the scouring depth and the erosion with related to the water level at upstream and downstream.

2.6 Summary

In this chapter, numerical models have been used to simulate numerical analysis problem. There are in 1D, 2D model and 3D model. From the previous research, most of the researcher discussed 2D model which the result is more accurate than 1D and 3D simulations. This chapter also explaining about the scouring and erosion. The method has being use by other researcher to simulate hydraulics problem has been discussed in this chapter. A lot of method has been used and the validation of the software model also been made. The concept of scouring and erosion discussed in this chapter. The next chapter will discuss step and the methodology of this study by using 3D model SPH.

CHAPTER 3

METHODOLOGY

3.1 Introduction

Numerical analysis is the easier way to predict the problem that we can't see by naked eyes and it's the method that has been widely used in the world. It also reduce the cost as much as possible compared to physical modelling. Numerical analysis study is the simulation of the case by using software through the high performance personal computer (PC). By using the software, we can see clearly what is going to happen on that case before, during and after. There are a lot of advance software that is very useful for water resources and hydraulics related problems. Many software with different techniques have been used by the software maker to solve the problem that related to the hydraulics. The numerical model simulation process will be discussed in this chapter. The computer software that will be used in this research is SPH. It has several important steps that needs to run the simulation by using extraction data from the drawing and site data.

3.2 Study area

This study actually is about the river bed erosion at downstream of the Sarawak Barrage which is over the Sarawak River connected to the sea. Sarawak River mouth is located near to the Kuching city. There is a barrage that also known as Kuching Barrage which has been constructed and completed in October 1997 which is located at the mouth of the Sarawak River (Figure 3.1). The operation of the barrage started on January 1998 and the barrage has been built for the flood mitigation project. The barrage also act as a dam by providing fresh water for the water treatment plant. It also prevent or minimize saline intrusion to the water treatment plant.

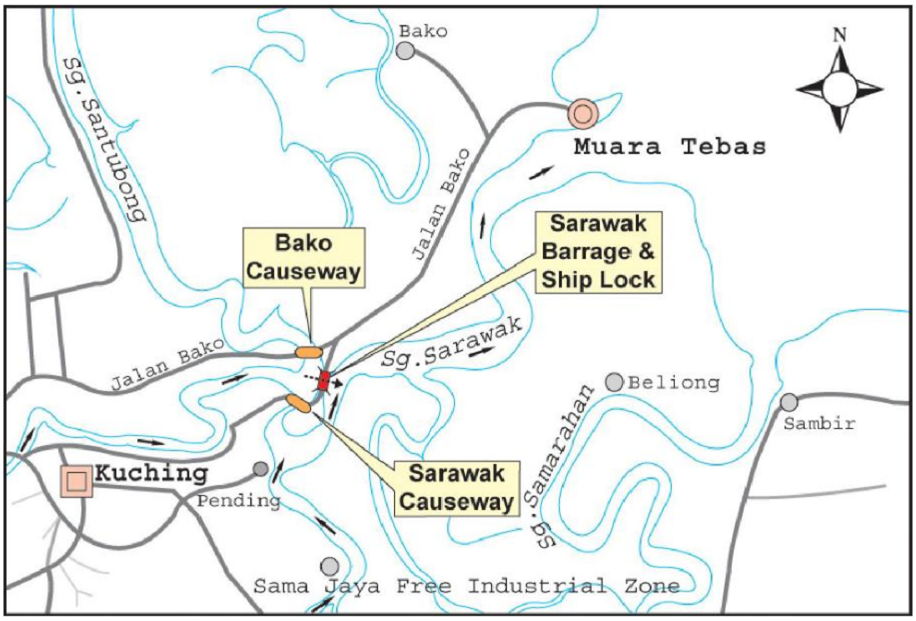

Figure 3.1 : Sarawak River

The barrage has 5 movable gates that consist of radial and tidal gates and 1 ship lock with 2 radial gate which all the gates operated hydraulically. The barrage gates are 25 meters in width of each and a ship lock alongside of the barrage as shown in the Figure 3.2. The barrage also has a flyover on top of it with two causeways across Sarawak River at Pending and Santubong River at Bako. The photos of the barrage are as shown in the Figure 3.3 and Figure 3.4.

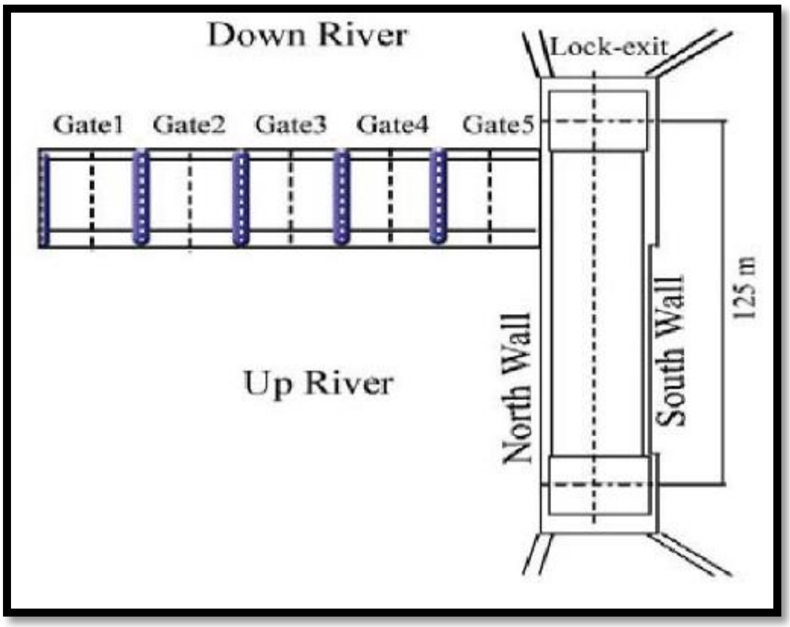

Figure 3.2 : Layout of Sarawak Barrage

Figure 3.3 : Side view of Sarawak Barrage

Figure 3.4 : Top view of Sarawak Barrage

For this study, the simulation of the numerical analysis about river bed erosion and scouring is at downstream of the Sarawak Barrage where is the water discharged from the Sarawak River to the sea.

3.3 Numerical experiment

In this dissertation, numerical software that will be used to simulate the case is DualSPHysics. It is Lagrangian meshless method software that can simulate using high resolution and wide range formulation. For this study, the case that will be chosen in the software is Multi-phase case where it's liquid-sediment model solver.

3.3.1 Integral Representation of a Function

In SPH, the initial integral representation of a function can be expressed as follow

$$f(\boldsymbol{x}) = \int_{\Omega} f(\boldsymbol{x}')\delta(\boldsymbol{x}-\boldsymbol{x}')dx' \tag{1}$$

whereby f is a function of the three-dimensional position vector x , and $\delta(x - x')$ is the Dirac delta function given by

$$\delta(\boldsymbol{x} - \boldsymbol{x}') = \begin{cases} 1 & \boldsymbol{x} = \boldsymbol{x}^2 \\ 0 & \boldsymbol{x} \neq \boldsymbol{x}^2 \end{cases} \tag{2}$$

From equation (1), Ω represents the volume of the integral that contains x (integration domain). It is also noted that equation (1) is exact and rigorous as long as $f(x)$ is defined and continuous due to the presence of Dirac delta function. The Delta function kernel $\delta(x - x')$ can be replaced by a smoothing function, the resulting integral function is now become

$$f(\boldsymbol{x}) \doteq \int_{\Omega} f(\boldsymbol{x}')W(\boldsymbol{x} - \boldsymbol{x}')d\boldsymbol{x}' \tag{3}$$

Where W is the smoothing kernel function and h is the smoothing length defining the influence area of the kernel function. In the SPH convention, < > is used as the kernel approximation operator and equation (3) can be now written as

$$< f(\boldsymbol{x}) >= \int_{\Omega} f(\boldsymbol{x}')W(\boldsymbol{x} - \boldsymbol{x}')dx' \tag{4}$$

The kernel function that is adopted in fluid dynamic computations usually inherent some important features or satisfy the following conditions. The first condition is normalization condition which states that

$$\int_{\Omega} W(\boldsymbol{x} - \boldsymbol{x}', h)d\boldsymbol{x}' = 1 \tag{5}$$

The above condition is also referred to as unity condition as the value of the integration of the smoothing function is equal to one. The following condition is named Delta function property where the smoothing length is approaching zero given as:

$$\lim_{h \to 0} W(\boldsymbol{x} - \boldsymbol{x}', h) = \delta(\boldsymbol{x} - \boldsymbol{x}') \tag{6}$$

The last one is termed as compact condition in which

$$W(\boldsymbol{x}-\boldsymbol{x}', h) = 0 \;\; if \; |\boldsymbol{x}-\boldsymbol{x}'| > \kappa h \tag{7}$$

where κ is a constant and it is multiplied by the smoothing length, and it describes the effective (non-zero) area of the smoothing function. It is also known as the support domain for the smoothing function of point x. Under the compact domain, integration over the whole problem domain can be localized as integration over the support domain of smoothing function. Hence, the integration domain Ω is the same as the support domain.

3.3.2 Particle approximation

In this operation, the whole system is represented by a finite number of particles that carry individual mass and occupy individual space. Particle approximation aims to discretize the kernel approximation as expressed in Equation (4) within the support domain. Figure 3.5 provides a clearer picture on how the kernel function can be used to estimate a field variable of particle i within a radius h .

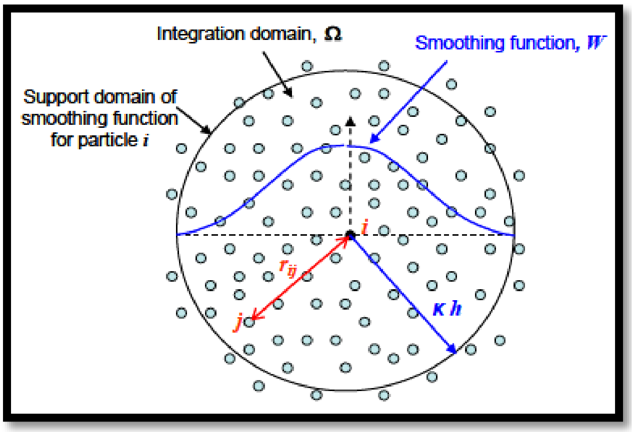

Figure 3.5 : Support domain with a radius of κh and smoothing kernel function, W for particle i

The mass of the particles m_j is related to the finite volume of the particle, ΔV_j and its density of ρ_j, as follows:

$$m_j = \Delta V_j \rho_j \tag{8}$$

The integral form of the interpolation can be discretized as the summation of N neighboring particles (number of particles that interact with particle i and can be written as

$$f(\boldsymbol{x}) = \int_{\Omega} f(\boldsymbol{x}')W(\boldsymbol{x}-\boldsymbol{x}')d\boldsymbol{x}' \cong \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(\boldsymbol{x}_j)W(\boldsymbol{x}-\boldsymbol{x}_j, h) \tag{9}$$

It is noted that infinitesimal volume dx' is replaced by the finite volume of the particle ΔV_j . The final discretized form for the particle approximation function of particle i is

$$< f(\boldsymbol{x}_i) >= \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(\boldsymbol{x}_j) \cdot W_{ij} \tag{10}$$

where

$$W_{ij} = W(\boldsymbol{x}_i - \boldsymbol{x}_j, h) \tag{11}$$

Thus, the value of a variable at particle i is estimated as the sum of the values it assumes on the neighboring points within the support domain weighted by the smoothing kernel function.

3.3.3 SPHysics formulation

As a review, SPH involves the computation of discretized fluid into a set of particles, each particle is referred to a nodal point where physical quantities are computed as an interpolation of the values of the nearest particles. The integral approximation of any function $A(\vec{r})$ is

$$A(\vec{r}) = \int_{\Omega} A(\overrightarrow{r'})W(\vec{r} - \vec{r}', h)dr' \tag{12}$$

where h is the smoothing length and $W(r - r', h)$ is the weighting function or smoothing kernel.

The function $A(\vec{r})$ can be expressed in the discretized form and the approximation of the function at particle a is given by

$$A(\vec{r}) = \sum_b m_b \frac{A_b}{\rho_b} W_{ab} \tag{13}$$

where $m_b$and $\rho_b$being the mass and density associated to the neighboring particle b, respectively. Besides, the term $W_{ab} = W(\overrightarrow{r_a} - \overrightarrow{r_b}, h)$ describes the weight function.

3.3.4 Weighting function or smoothing Kernel

The smoothing kernel functions must fulfil several properties such as normalization, compact support and monotonically decreasing with distance from point a when smoothing length, h approaches zero. SPHysics provides four different kind of kernel definitions. For this particular research, the cubic spline kernel is selected and showed as follows:

$$W(r, h) = \propto_D \begin{cases} 1 - \frac{3}{2}q^2 + \frac{3}{4}q^3 & 0 \leq q \leq 1 \\ \frac{1}{4}(2 - q)^3 & 1 \leq q \leq 2 \\ 0 & q \geq 2 \end{cases} \tag{14}$$

where $\propto_D = 1/(\pi h^3)$ in three-dimensional flow domain.

The momentum conservation equation in a continuum Eulerian field is shown as follows:

$$\frac{D\vec{v}}{Dt} = -\frac{1}{\rho}\vec{\nabla}P + \vec{g} + \vec{\Theta} \tag{15}$$

where $\vec{\Theta}$ denotes the diffusion term.

The momentum equation in SPH method can be described by different approaches based on various existing formulations of the diffusive terms. The momentum equation [39] is implemented to determine the acceleration of a particle a as a result of the particle interaction with its neighbors, particles b:

$$\frac{d\vec{v}_a}{dt} = -\sum_b m_b \left(\frac{P_b}{{\rho_b}^2} + \frac{P_a}{{\rho_a}^2} + \mathrm{II}_{ab}\right)\nabla_a W_{ab} + \vec{g} \tag{16}$$

such that v being the velocity, P is the pressure, ρ is the density, m is the mass of particle, $\vec{g} = (0, 0, -9.81)\ \mathrm{m/s^2}$ is the three-dimensional gravitational acceleration vector and W_{ab} denotes the kernel function that dependent on the distance between particles a and b.

Generally in SPH notation, the pressure gradient term in symmetrical form is expressed as

$$\left(-\frac{1}{\rho}\nabla P\right)_a = -\sum_b m_b \left(\frac{P_b}{{\rho_b}^2} + \frac{P_a}{{\rho_a}^2} + \mathrm{II}_{ab}\right)\nabla_a W_{ab} \tag{17}$$

where II_{ab} is the viscosity term which is defined as:

$$\mathrm{II}_{ab} = \begin{cases} \frac{-\alpha\overline{c_{ab}}\mu_{ab}}{\overline{\rho_{ab}}} & \vec{v}_{ab}\vec{r}_{ab} < 0 \\ 0 & \vec{v}_{ab}\vec{r}_{ab} > 0 \end{cases} \tag{18}$$