

**GAME THEORY-BASED DISCRETIONARY LANE  
CHANGING CONTROLLING COMPULSORY  
BEHAVIOR IN CONFLICTING SITUATION**

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**GAME THEORY-BASED DISCRETIONARY LANE  
CHANGING CONTROLLING COMPULSORY  
BEHAVIOR IN CONFLICTING SITUATION**

by

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## LIST OF ABBREVIATIONS

<b>BLP</b>	Bi-Level Programming
<b>CAG</b>	Collision Avoidance Gap
<b>CF</b>	Car Following
<b>CFM</b>	Car Following Model
<b>DLC</b>	Discretionary Lane Changing
<b>DLCD</b>	Discretionary Lane Changing Decision
<b>DS</b>	Direct Search
<b>FV</b>	Front Vehicle
<b>GA</b>	Genetic Algorithm
<b>GD</b>	Gradient Descent
<b>GTM</b>	Game Theory Model
<b>HTC</b>	Hyperbolic Tangent Curve
<b>HTF</b>	Hyperbolic Tangent Function
<b>IDM</b>	Intelligent Driver Model
<b>LC</b>	Lane Changing
<b>LCTC</b>	Lane Changing Trajectory Curve
<b>LCTPM</b>	Lane Changing Trajectory Planning Model
<b>LCM</b>	Lane Changing Model
<b>LCDM</b>	Lane Changing Decision Model

<b>LLC</b>	Left Lane Changing
<b>MAE</b>	Mean Absolute Error
<b>MLC</b>	Mandatory Lane Changing
<b>MOPC</b>	Multi-Order Polynomial Curve
<b>NE</b>	Nash Equilibrium
<b>NGSIM</b>	Next Generation Simulation
<b>NLC</b>	Non-Lane Changing
<b>PC</b>	Polynomial Curve
<b>QBC</b>	Quintic Bezier Curves
<b>QPC</b>	Quantic Polynomial Curve
<b>QRE</b>	Quantal Response Equilibrium
<b>RLC</b>	Right Lane Changing
<b>RV</b>	Rear Vehicle
<b>RMSE</b>	Root Mean Square Error
<b>SBC</b>	Spline Based Curve
<b>SPSA</b>	Simultaneous Perturbation Stochastic Approximation
<b>SQP</b>	Sequential Quadratic Programming
<b>SSE</b>	Sum of Square Error
<b>STR</b>	Subject And Target Rear Vehicles
<b>SV</b>	Subject Vehicle
<b>TFTR</b>	Target Front and Target Rear Vehicles



**TFV** Target Front Vehicle

**TRV** Target Rear Vehicle

**TTC** Time To Collision

## LIST OF SYMBOLS

$a$	Acceleration
$  $	Absolute value of the number
$\theta_{T_{end}}$	Angle of trajectory curve at ending position
$\theta_{T_{origin}}$	Angle of trajectory curve at starting position
$\vartheta$	Average angle of trajectory curve of starting and ending position
$v_d$	Average LC velocity
$S_d$	Collision avoidance gap (rear safety distance)
$\sigma$	Driver lateral aggressiveness as LC parameter
$b$	Deceleration
$v_0$	Desired speed
$\Delta$	Gradient operator
$\delta_1$	First weighted parameter
$\delta_2$	Second weighted parameter
$e$	Exponential function
$\dot{\theta}$	First differentiation of $\theta$
$\forall$	For all
$\gamma$	SPSA hidden parameter
$\hat{\theta}_k$	Improved parameter of k-th iteration
$\dot{x}_0$	Initial velocity

$u_0$	Initial LC velocity
$\infty$	Infinity
$\iota$	Probability of head of the first player
$\kappa$	Probability of head of the second player
$v_0^r$	Initial velocity of target rear vehicle
$l_n$	Length of following vehicle
$\lambda$	Driver bounded rational behavior
$a_Y$	Lateral acceleration along trajectory curve
$v_y$	Lateral velocity
$p_{lc}$	Lane-changing probability of SV
lim	Limit
$a_X$	Longitudinal acceleration during LC
$v_{x_1}$	Longitudinal velocity before middle of the two lanes
$v_{x_2}$	Longitudinal velocity after middle of the two lanes
$s_0$	Minimum jam distance
$x_n$	Position of following vehicle
$x_{n-1}$	Position of front vehicle
$\xi(t)$	Path position of trajectory curve at time $t$
$p$	Probability of player 1
$q$	Probability of player 2
$\dot{r}$	Radial component of velocity

$x_r^r$  Rear vehicle position  
 $\gamma_c$  Reference angle of LC trajectory curve  
 $\rightarrow$  Right arrow  
 $S_f$  Scaling factor  
 $\Omega$  Set of optimization value of parameters  
 $\sum_{i=1}^N$  Sum of all elements from 1 to N  
 $\alpha_i$  Utility parameter of its associated factor  
 $EU_{lc}$  Utility for LC decision  
 $EU_{nlc}$  Utility for non-LC decision  
 $\tanh$  Tan hyperbolic function  
 $T_h$  Time head way  
 $T$  Total lane changing time  
 $T_f$  Translation factor  
 $l_d$  Total LC longitudinal distance  
 $\beta_i$  Utility parameter of its associated factor  
 $EU_{ny}$  Utility for forbidding decision  
 $EU_y$  Utility for yielding decision  
 $\hat{e}_r$  Unit direction vector  
 $\epsilon_k^{(\mp)}$  Noise terms  
 $\vartheta$  Average reference angle  
 $\vec{r}$  Polar form of trajectory curve

$q_y$  Yielding probability of TRV

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- APPENDIX A    LATERAL TRAJECTORY MODEL
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# **KELAKUAN PENGENDALIAN PERTUKARAN LORONG SECARA WAJIB BERASASKAN TEORI PERMAINAN DALAM SITUASI KONFLIK**

## **ABSTRAK**

Keputusan budi bicara pertukaran lorong (DLC) di jalan raya bandar adalah tindak-an yang sangat mencabar dan bercanggah. Selama dua dekad yang lalu, banyak jenis penyelidikan telah berusaha menyelesaikan masalah ini dengan menggunakan model pertukaran lorong berdasarkan keputusan binar. Walau bagaimanapun, sangat sedikit kajian yang dilakukan untuk menangani masalah jenis ini dengan menggunakan model teori permainan berasaskan Keseimbangan Nash sebagai sekurang-kurangnya empat model berdasarkan keputusan. Walaupun tingkah laku pemain (pemandu kenderaan yang menukar lorong dan pemandu kenderaan belakang yang disasarkan dalam sistem lalu lintas bandar) dibatasi secara rasional dalam situasi yang berinteraksi, penyelidikan tersebut menerapkan model teori permainan Keseimbangan Nash yang menggunakan tingkah laku pemandu yang rasional sepenuhnya. Tugas yang mencabar ini akan diatasi dengan menerapkan teori permainan Keseimbangan Sambutan Kuantal (QRE) yang menerapkan pemain rasional terikat. Model QRE memberikan keputusan per-tukaran lorong interaktif dengan menggunakan faktor lintasan yang berbeza. Faktor-faktor ini dinyatakan dalam penyelidikan lalu lintas pertukaran lorong dan kenderaan yang mengikut. Model Pemandu Pintar (IDM) sebagai model kereta yang mengikut menggunakan faktor kelajuan yang diinginkan, dan model perancangan lintasan pertu-karan lorong memberikan faktor jurang keselamatan. Dengan mengelakkan kedua-dua faktor keputusan pertukaran lorong ini, penyelesaian berdasarkan penyelidikan yang disebutkan di atas mungkin tidak mungkin dilakukan, sedangkan literatur menyarankan untuk memasukkan faktor-faktor tersebut dalam mendorong penyelidikan lalu lin-

tas berdasarkan tingkah laku. Penyelidikan ini mengumpulkan faktor kelajuan yang diinginkan dari model simulasi IDM yang dikalibrasi, dan faktor jurang keselamatan dari model lintasan pertukaran lorong yang dicadangkan, serta mencadangkan model keputusan pertukaran lorong berdasarkan QRE untuk kawasan lalu lintas sesak di bandar. Kaedah kalibrasi menggunakan set data Algorithma Ginetik (GA) dan Simulasi Generasi Hadapan (NGSIM). Algoritma genetik juga digunakan untuk mengkalibrasi model lintasan pertukaran lorong yang diubah, di mana parameter parameter fungsi yang digunakan, ditentukan. Selanjutnya, masalah pengaturcaraan dua peringkat menerapkan model teori permainan berasaskan QRE. Masalah pengaturcaraan dwi-tahap mengkalibrasi faktor-faktor yang berkaitan dengan parameter utiliti teori permainan dan kebarangkalian keputusan pemandu dengan menggunakan GA. Oleh itu, model teori permainan ini dikalibrasi untuk mencadangkan tingkah laku pemandu DLC, dan disahkan dengan menggunakan 30% (37 contoh pertukaran lorong) set data. Penemuan kajian ini mencadangkan model empat keputusan sehingga kadar penggera palsu model adalah 9.38% (keputusan menukar lorong kenderaan subjek), 28.13% (menghasilkan keputusan sasatan kenderaan belakang ) dan 20% (keputusan melarang sasaran kenderaan belakang ) dengan menggunakan ujian pengesahan. Selanjutnya, penyelidikan ini juga menyarankan untuk mengawal faktor lintasan dinamika yang digunakan dalam situasi yang bertentangan. Perisian simulasi lalu lintas berasaskan prestasi tinggi pada masa akan datang dapat mengembangkan model ini untuk mengurangkan kemalangan jalan raya, kesesakan dan isyarat panjang di persimpangan.



# GAME THEORY-BASED DISCRETIONARY LANE CHANGING CONTROLLING COMPULSORY BEHAVIOR IN CONFLICTING SITUATION

## ABSTRACT

The challenging and contradictory Discretionary Lane Changing (DLC) is to happen for comfortable or safe journey in urban roadway. For the last two decades, many studies have been trying to solve this problem by using the binary decisions-based lane changing model. However, very few researches were conducted to handle this type of problem by using the Nash equilibrium-based game theory model as an at-least four decisions-based model. Despite bounded rational behavior of the game theory players (lane changing vehicle driver and target rear vehicle driver in urban traffic system), existing researches apply the Nash-equilibrium game theory model including the full rational behavior. This challenging task needs to be overcome by applying the Quantal Response Equilibrium (QRE) game theory including the bounded rational players. The QRE model provides the interactive lane changing decision by using different trajectory factors. These factors are found in car-following and lane-changing traffic researches. The Intelligent Driver Model (IDM) as a car-following model incorporates the desired speed factor, and the lane-changing trajectory planning model provides a safety gap factor. By avoiding these two factors, the above-mentioned research-based solution may not be possible, whereas literature suggested to include such factors in driving behavior-based traffic research. This research collects the desired speed factor from calibrated IDM, and safety gap factor from lane changing trajectory model, to propose the QRE-based lane changing decision model for urban congested traffic areas. The calibration method uses Genetic Algorithm (GA) against the real dataset, Next Generation Simulation (NGSIM). GA is also used to calibrate the modified lane chang-

ing trajectory model, and determine efficient model parameters. Further, a bi-level programming problem includes the QRE-based game theory model in this research. The bi-level programming calibrates parameters of game theory utilities (factors) and driver decision probabilities by using GA. Therefore, this game theory model employs the calibration by using 70% (92 lane-changing instances) of dataset to propose the DLC driver behavior. Further, this model check the validation by using 30% (40 lane-changing instances) of dataset. This research finds false alarm rates of the model, 10.81% (lane changing decision of subject vehicle), 0.00% (non-lane changing decision of subject vehicle), 36.36% (yielding decision of target rear vehicle), and 42.86% (forbidding decision of target rear vehicle) by using validation test. Further, finding results suggest overcoming conflicts in this dataset by controlling the used dynamic factors. High performance-based traffic simulation software in the future can use the further developed model to decrease traffic crashes, bottlenecks, and long signals in the intersection.

# CHAPTER 1

## INTRODUCTION

### 1.1 General Introduction

Unplanned Lane Changing (LC) behavior of vehicle drivers in urban roadway influences traffic bottleneck and increases traffic crash. Generally, a driver decision in various hazard situations is the leading cause of congestion (Malikopoulos & Aguilar, 2013), and merging roadway is the most reliable source of high congestion (Margiotta & Snyder, 2011). The time–cost of people produced in congested urban traffic areas is more than 6.9 billion hours on the road, the purchasing cost of fuel is additional 3.1 billion gallons, and the average resulting total cost is \$160 billion in 2014 (Afrin & Yodo, 2020; Rahaman et al., 2019; Schrank, Eisele, Lomax, & Bak, 2015). Moreover, the driver distraction, discomfort and frustration are twisted from traffic congestion and may result in fierce driving behavior (Malikopoulos & Aguilar, 2012)

Planned driver behavior is anticipated to resolve many transport problems and provide comfort, safety, productivity and flexibility during travel. These planned vehicle movements are contained either in macroscopic or microscopic factor analysis. The macroscopic factor addresses traffic flow, density and average traffic speed and the microscopic factor differentiates the vehicle trajectory based movements, such as position, velocity, acceleration, gap and time headway (Treiber & Kesting, 2013b). The analysis of microscopic factor suggests that the driving system is flexible and safe.

Car following is a microscopic-based vehicle movement analysis in multilane roads.

When a vehicle driver continues his movement in the current lane, he includes the car following behavior, and his vehicle is known as the following or subject vehicle (SV). In the last four decades, the limited number of Car Following Models (CFMs) have been developed to control the driver trajectory-based behavior, wherein Intelligent Driver Model (IDM) is the best Car Following Model (CFM) for the comfortable journey because of the model parameter (desired speed) (Treiber & Kesting, 2013b). The desired speed of a driver corresponds to his highest expected speed in the current lane. This parameter depends on the real trajectories of Subject Vehicle (SV) and Front Vehicle (FV) (C. Chen, Li, Hu, & Geng, 2010).

A vehicle driver who changes the current lane is called the LC driver. This type of driver needs high or controlling speed and tries to overcome any obstacle in this lane. When a driver must change the current-lane, this changing is a Mandatory Lane Changing (MLC). In the last two decades, many researchers have developed MLC models to overcome traffic obstacles. Furthermore, when a driver changes the lane for either comfortable or safe journey, this changing is Discretionary Lane Changing (DLC). The DLC action provides more relaxation and safety to drivers in congested traffic areas, thereby bringing more comfort when the driver needs more speed in the freeway road (M. Yang, Wang, & Quddus, 2019). However, DLC action is not compulsory. Thus, the safety factor of DLC action is more significant than that of MLC action.

The safety factor is determined using the trajectories of Target Rear Vehicle (TRV) and SV after LC. In recent years, researchers developed a gap acceptance model that included safety factor and proposed the distribution of trajectories (Balal, Cheu, &

Sarkodie-Gyan, 2016). For DLC decision, a lane-changing vehicle (SV) driver tends to identify the gap between the FV and the TRV at the target-lane after LC. When an SV driver identifies the gap at the target-lane, he applies binary decision (e.g. LC and Non-Lane Changing (NLC) decisions). TRV may also apply another binary decision (e.g. yielding or forbidding decisions) immediately. The SV changes lane when the gap is accepted; otherwise, SV does not change lane. Also, TRV either gives permission or forbids to change lane.

The binary decision model can provide the decision to either SV or TRV by using their trajectories. When this model applies SV trajectories, SV can make LC decision. However, when this model applies TRV trajectories, TRV can make the decisions. As a result, the binary decision model priorities persons/drivers separately. To date, the binary decision model is used to determine the decisions for individual person/driver (Arbis, 2017; Arbis & Dixit, 2019).

All driver decisions are more important for DLC action. When the current safety gap is less than the minimum safety gap, the TRV driver may decelerate his vehicle to create a huge safety gap through interaction, and the SV driver may change his current lane without the rear crash. Thus, the LC decision of SV may depend on binary decision of TRV to avoid the rear crash. In this environment, driver interaction is generated between two drivers. However, the binary decision-based gap acceptance model could not combine interacted decisions for DLC action. The reason is that the binary model can only suggest the decision to a single driver (Balal et al., 2016). Therefore, a modification approach is demanded in decision-based research.

Game theory-based decision model evolved for more than one driver to make their own decision. This model combines the decisions in an interacted driving environment. The Game Theory Model (GTM) is used to determine the probabilities of SV (e.g. LC and NLC decisions) and TRV driver decisions (e.g. yielding or forbidding decisions) of DLC in the interacted driving environment. Thus, during DLC, these two vehicles decide according to the GTM. Here SV is considered as a first player, whereas TRV is considered as an opposition player. This competitive game may be cooperative or non-cooperative (M. Wang, Hoogendoorn, Daamen, van Arem, & Happee, 2015).

Nash Equilibrium (NE) is a pioneer research tool in game theory-based decision model because it can provide the best strategy to decision-makers. If interacting drivers choose a strategy from Nash equilibria (solution points), this strategy is the best strategy in competing environment. Thus, decision-makers can implement the Nash equilibria in conflicting scenarios because many solutions belong to this strategy set. Moreover, the conflict probability is likely to increase when SV takes the LC decision, and TRV forbids the LC decision of SV. Hence, the probability of LC may be changed by controlling the dynamic factors because the SV driver intention occurs the DLC action. In addition, given that TRV intention occurs a forbidding decision, the forbidding probability may also be changed by controlling its dynamic factors in conflicting time. Therefore, the SV and TRV drivers may control their dynamic factors using the GTM in conflicting scenarios to reduce the crashing probability amongst these vehicles.

## 1.2 Problem Statement

CFM was developed to control the driver longitudinal movements in the current lane, wherein the desired speed is an IDM parameter that provides the best CFM regarding comfortable journey. The desired speed parameter influences the DLC because DLC depends on driver intention. Balal et al. (2016) found that the desired speed factor collection is problematic because it may significantly influence the binary decision model. Furthermore, according to M. Yang et al. (2019), the desired speed factor creates the DLC intention in binary decision-intended gap acceptance model. However, they avoided the collection of this factor. The main issue in collecting the desired speed factor is calibration, especially when microscopic-based big data is implemented. Kang and Rakha (2017, 2018) found that the desired speed factor highly affected the MLC decision using GTM. However, none of these studies developed the game theory-based decision model including the desired speed factor in DLC decision model.

Safety gap factor is a significant component for driver safety in Lane Changing Trajectory Planning Model (LCTPM). As a result, this factor also affects driver decision significantly as a safety measurement. The safety gap factor could be determined by using the LC trajectory model. However, a few studies used the lateral trajectory models: Quintic Bezier Curves (QBC) (Shen, Zhang, & Fang, 2017), Multi-Order Polynomial Curve (MOPC) (D. Yang, Zheng, Wen, Jin, & Ran, 2018) and Hyperbolic Tangent Curve (HTC) (B. Zhou, Wang, Yu, & Wu, 2017). The QBC was only applied for robotic planning vehicle. Most of the MOPCs was derived based on velocity and acceleration. Velocity and acceleration are assumed to be zero in starting and ending

points of LC, except in D. Yang et al. (2018). D. Yang et al. argued that this assumption was unrealistic for congested traffic scenarios. HTC parameters were determined from LC reference angles by real views. The assumption of HTC is more realistic than MOPC because realistic parameters are used. B. Zhou et al. (2017) used the parameters and regression coefficient to fit with microscopic-based real data remarkably. However, these parameters and regression coefficients were calibrated against a very few real data. Further, most studies used a straight line to represent the longitudinal movements in safety gap measurement. Eventually, D. Yang et al. showed that the longitudinal direction does not follow always a straight line to adjust the vehicle in the target-lane. That directional straight line includes a weighted parameter after crossing the middle line that was not calibrated. Therefore, the longitudinal trajectory line and lateral trajectory HTC parameters were not properly calibrated to determine the safety gap.

DLC decision provides a comfortable and time-saving journey and releases the frustration of drivers. However, the DLC decision model can suggest some crash-avoiding LC decisions to the driver (Arbis & Dixit, 2019; H. Zhou, Sun, Qin, Xu, & Yao, 2020). Studies regarding DLC actions are extremely limited. However, as mentioned previously in Section 1.1, the binary model can only suggest the decision to a single decision-maker (Balal et al., 2016), whereas GTM can suggest to more than one decision-maker (Ali, Zheng, Haque, & Wang, 2019). In H. Zhou et al. (2020), a binary decision model explored the conflict probabilities for LC and NLC decisions in DLC action. The NE-based GTM on LC decision is gradually improving for MLC, wherein the driver behavior is fully rational. However, some research proved that a driver behavior in congested traffic area is bounded rational (Barmounakis, Vlahogianni, & Golias, 2016; H. Zhou et al., 2020). A seminal work (Arbis & Dixit, 2019) included



bounded rational behavior-based QRE model and proposed the MLC decision. Moreover, no study used the bounded rational behavior-based Quantal Response Equilibrium (QRE) model for DLC decision.

### **1.3 Objectives of the Research**

This research has four objectives:

- To collect the realistic desired speed from a CFM (IDM) using microscopic data.
- To determine the safety gap factor from LCTPM using microscopic data.
- To propose a bounded rational behavior-based QRE model of DLC action by using the aforementioned car following and lane changing trajectory factors.
- To suggest the controlling driving behavior in DLC conflicting situation.

### **1.4 Scopes and Limitations of the Study**

This research collects the desired speed factor as an IDM parameter, and safety gap factor as a factor of modified parametric trajectory models. This study also proposes the QRE-based LC decision model fitting NGSIM dataset. To collect the desired speed factor, this research considers IDM as the best CFM for a comfortable journey. The IDM parameters are collected by comparing calibration methods (e.g. SPSA and GA).

The existing lateral trajectory movements along the Hyperbolic Tangent Curve (HTC), and the longitudinal trajectory movements along the straight lines are modified by using GA. The Next Generation Simulation (NGSIM) dataset (US-101) has three parts (e.g. 7.50 am to 8.05 am; 8.05 am to 8.20 am; and 8.20 am to 8.35 am). This

research uses the 7.50 am to 8.05 am dataset, wherein this macroscopic and microscopic dataset includes trajectory-based 1,118,059 8-row vectors and 18-column vectors. Therefore, 123 LC groups and 9 NLC groups are collected by using MATLAB coding, wherein every group has four vehicles.

To fit the model against the collected vehicle groups, the decision model includes the bi-level programming; and the bi-level programming provides the probabilistic fitting value of driver decisions, such as LC and NLC decisions for SV, and yielding and forbidding decisions for TRV. Using genetic algorithm and Sum of Square Error (SSE) function, the bi-level programming also provides the realistic GTM parameters, where the used MATLAB coding achieves an outstanding result in this research.

### **1.5 Significance of the Study**

This thesis focuses on scheming the lateral control decision that the desired speed of calibrated car-following parameters and safety gap factor of LCTPM influences this decision. The SV and TRV drivers control the dynamic factors used in this model and decrease the rear crash by applying the proposed QRE-based GTM during DLC. In the future, high performance-based traffic simulation software can develop this model further to reduce traffic crash, bottlenecks and long signal in the intersection. The proposed model very effectively fits in human-based real trajectory NGSIM (US-101) dataset. As such, the proposed model application can promote next-generation automated driving systems.

## 1.6 Organization of the Thesis

The presentation of the thesis is organized as follows:

Chapter 2 provides the comprehensive literature of calibrated algorithms of CFM (IDM), and research opportunity in the LCTPM. This chapter also specially reviews the literature of the QRE-based GTM as an emerging tool to solve the bounded rational behavior-based driver interacted decision.

Chapter 3 designs the methodological approaches of the calibration method enhanced parameters of IDM, and modification of the LCTPM. The desired speed belongs to CFM parameters, and safety gap belongs to LCTPM. Incorporated above-mentioned factors in GTM are theoretically designed to propose the DLC decision model. This decision model can be applied to decrease the rear crash by controlling vehicle trajectories.

Chapter 4 delivers the information about the data collection site and data processing style. This data improves the model, tests the accuracy, and provides the GTM influenced factors. This chapter also shows the statistical figure of the extracted-data and collected-factors.

Chapter 5 discusses the comparative best calibration method of CFM, calibration of modified LCTPM, and figure of collected factors. Finally, this chapter presents the proposed solution of the research problem by testing the validation against real trajectory data, where the proposed solution suggests that the controlling of driver dynamic factors is able to avoid the rear crash in urban roadway.

Chapter 6 summarizes the proposed suggestions, concludes the hypothetical solution of the problem in this chapter, and provides the future research direction for development of the research in this thesis.

## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 Introduction

This chapter briefly reviews the literature of Car Following Model (CFM) and Lane Changing Trajectory Planning Model (LCTPM). CFM provides the desired speed factor, and LCTPM provides the safety gap factor. In the Section 2.3, the Intelligent Driver Model (IDM) as the best CFM for a comfortable journey, and calibration approaches of CFM are reviewed. In the third section, reviewed LCTPM contains lateral and longitudinal trajectory models for a safe and comfortable journey. Different lateral trajectory curves and a longitudinal trajectory line are discussed. Moreover, related studies on the safety gap factor are addressed in the same section. Above-mentioned two factors included in Game Theory Model (GTM) are discussed in the Section 2.4. Besides, this section includes the discussion of fully rational-based Nash Equilibrium (NE) model and bounded rational-based Quantal Response Equilibrium (QRE) model.

#### 2.2 Traffic Flow Analysis

A complex traffic network behavior has been used to analyze the macroscopic and microscopic traffic flow models. The macroscopic simulation model is a procedure that executes the density, flow, and average speed of steady-state traffic flow (Treiber & Kesting, 2013b). Besides, the microscopic simulation model plays a significant role in driver to driver interactions and allows a decision by exploring and planning traffic network facilities (Koutsopoulos & Farah, 2012). Generally, a traffic simulation

platform consists of many models as discussed in different traffic flow models (C. Chen et al., 2010; Kesting & Treiber, 2008; Ossen & Hoogendoorn, 2009; Punzo, Ciuffo, & Montanino, 2012).

The macroscopic-based traffic model is essential for controlling the traffic area. However, this model parameter improves the microscopic-based traffic flow model (Rakha & Gao, 2010). The microscopic-based driver behavior is given more priority when exploring safe and comfortable journeys and solving many traffic problems in congested areas (Islam, 2014) because dynamic parameters involve in this model. Besides, the traffic network flow parameters of macroscopic structures, such as critical density, free flow velocity and jam density, are significant in various models, and these parameters can be directly projected to loop indicator traffic flows (D. Chen, Laval, Zheng, & Ahn, 2012). Nevertheless, other parameters (gap, desired speed, maximum acceleration and maximum deceleration) cannot be derived from macroscopic capacities in different CFMs (Kesting & Treiber, 2008; Ossen & Hoogendoorn, 2009; Paz, Molano, Martinez, Gaviria, & Arteaga, 2015; Wagner, Buisson, & Nippold, 2016). Therefore, the Subsection 2.2.1 briefly explains the microscopic-based traffic flow model, where driver comfortable level is prioritized.

### **2.2.1 Car following model**

The longitudinal and lateral movements of the vehicle are called car following and lane changing (LC) trajectories, respectively. The longitudinal movement has a one-dimensional direction, and the lateral movement has multidimensional directions. By analyzing the microscopic factors, one-dimensional CFM controls the vehicle. The

current research trend for CFM uses real trajectory data as the exhibition of driving behavior in real environment (Ciuffo, Punzo, & Montanino, 2012a; Kesting & Treiber, 2008; Ossen & Hoogendoorn, 2009; Punzo & Simonelli, 2005).

The widely used Gipps CFM prioritizes maximum velocity and deceleration by hard-brake for safety (Ciuffo, Punzo, & Montanino, 2012b; M. M. Rahman, Ismail, & Ali, 2019b; Spyropoulou, 2007). However, these actions are often unnecessary in highway scenarios. For instance, hard braking deceleration is unnecessary because the rear vehicle cannot perform hard-brake instantly (Treiber & Kesting, 2013a). High braking deceleration is a threat in the comfortable journey (Treiber, Hennecke, & Helbing, 2000; Treiber & Kesting, 2013a). IDM is a comfortable CFM that can explain the complexity better than Optimum Velocity Model (OVM) L. Liu, Zhu, and Yang (2016); M. Rahman, Chowdhury, Dey, Islam, and Khan (2017); Treiber and Kesting (2013b) .

Learning-based CFMs, such as Gaussian mixture and hidden Markov model (W. Wang, Xi, & Zhao, 2018; W. Wang, Zhao, Xi, LeBlanc, & Hedrick, 2017), fuzzy logic model (Wu, Brackstone, & McDonald, 2003) and artificial neural network model (Panwai & Dia, 2007), are artificial intelligence systems for microscopic traffic analysis. However, all these models highly depend on real trajectory datasets. Moreover, the data collection is not an easy procedure (Coifman, Wu, Redmill, & Thornton, 2016; Dogru & Subasi, 2018; Leduc et al., 2008; Peng, Abdel-Aty, Shi, & Yu, 2017; Punzo & Simonelli, 2005), thereby making the research on learning-based methods less interesting. Furthermore, if the dataset has a low frequency (few data with respect to time) or many outliers, CFMs behave unrealistically. The under-fitted and over-fitted

datasets also produce another problem as a data-driven CFM (Aghabayk, Sarvi, & Young, 2015). As such, the research on parametric-based CFM becomes highly interesting for a comfortable journey.

After the exploration of IDM (Treiber et al., 2000), research has found that the most reliable and efficient parametric-based CFM expands the microscopic traffic research area and makes significant rules for traffic network (Brockfeld, Kühne, & Wagner, 2005; C. Chen et al., 2010). M. Rahman et al. (2017) evaluated the effectiveness of IDM based on an ordinary differential equation for  $n$  number of a vehicle platoon. CFM simulation results provide a better suggestion for comfortable driving behavior.

The IDM is a car following dynamic system that includes parameters, such as maximum acceleration, maximum deceleration, desired speed, time headway and minimum jam distance. Using these parameters, the IDM system provides simulation data, such as the position, velocity, acceleration or deceleration and gap of Subject Vehicle (SV). Gap refers to the distance between the Front Vehicle (FV) rear bumper position and the SV front bumper position. Time headway of SV corresponds to the total time to touch the FV (C. Chen et al., 2010). Treiber and Kesting (2013b) proposed the improved IDM parameters where the desired speed relates to maximum deceleration as a parameter of comfortable journey. In Table 2.1, the different CFMs analysis the macroscopic and microscopic environments.



Table 2.1: Related works that have employed the CFM.

No.	Author	Traffic analysis		Car following model				Note
		Mac	Mic	Gi	ID	OV	Oth	
1	C. Chen et al. (2010)		✓		✓		✓	The desired speed factor is not included in comparison analysis.
2	Treiber and Kesting (2013b)	✓	✓		✓	✓		The IDM performs better than OVM as comfortable journey.
3	Treiber and Kesting (2013a)		✓	✓	✓			The IDM performs better than Gipps in desired speed as comfortable journey.
4	Punzo, Montanino, and Ciuffo (2014)		✓		✓			FV trajectories influence more in simulation model.
5	Aghabayk et al. (2015)		✓	✓	✓		✓	The research prioritized more in IDM.
6	Wagner et al. (2016)		✓				✓	Only stochastic value is employed in this model.
7	L. Liu et al. (2016)		✓		✓	✓		IDM can explain the complexity better than OVM.
8	Kurtc and Treiber (2016)		✓		✓		✓	Required IDM parameters varied from full velocity driver model.
9	L. Li, Chen, and Zhang (2016)		✓		✓	✓		The IDM is best to explain the real trajectory by improving parameters.
10	M. Rahman et al. (2017)		✓		✓	✓		The research focused simulation trajectories.
11	Zhu, Wang, Tarko, et al. (2018)	✓	✓	✓	✓		✓	IDM is better performed than other models for this dataset.
12	M. M. Rahman et al. (2019b)		✓	✓				The research only compared the simulation data and real data.

Mac- Macroscopic analysis; Mic- Microscopic analysis; Gi-Gipps model; ID-IDM; OV-OVM; oth- Other models

### 2.2.2 Calibration methods

Parameter estimation against real data has become a familiar and important phenomenon in CFM-based traffic research because earlier researchers could not fit perfectly CFM simulation parameters with real traffic system (Brockfeld et al., 2005; Koutsopoulos & Farah, 2012). Ordinary differential equation-based microscopic simulation models are categorized by some parameters to explain the CFM and fitted by different types of calibration methods. The fitted microscopic simulation models can reproduce the traffic scenarios (Bevrani & Chung, 2011; da Rocha et al., 2015; Kesting & Treiber, 2008; Ossen & Hoogendoorn, 2009; Punzo et al., 2012; M. M. Rahman, Ismail, & Ali, 2019a).

Gipps CFM has unrealistic factors; for example, velocity factor rarely has a physical link to the traffic scenarios, (Kesting & Treiber, 2008; Koutsopoulos & Farah, 2012). However, parameter calibration methods can improve the CFM performance in real traffic. Calibration parameters represent real traffic driver behavior after improving the model performance (Punzo et al., 2014; Wagner et al., 2016; Zhu et al., 2018). Therefore, the parameter calibration method depends on the objective function of the optimization approach (Kesting & Treiber, 2008).

Many optimization functions, such as Genetic Algorithm (GA), Sequential Quadratic Programming (SQP), Simultaneous Perturbation Stochastic Approximation (SPSA), and Nelder–Mead-algorithm (L. Li et al., 2016; Nelder & Mead, 1965), can be used for calibration methods. GA is an efficient calibration method that produces nearly accurate value (Rakha & Gao, 2010). Furthermore, for the calibration of CFM simulation parameters, a stochastically global search GA is the most broadly used system

for unconstrained and constrained objective functions (Ciuffo et al., 2012a). L. Li et al. (2016) explained that the GA, Sequential Quadratic Programming (SQP) and Simultaneous Perturbation Stochastic Approximation (SPSA) calibration methods produce realistic IDM parameters. Besides, L. Li et al. (2016) suggested that the driver decision depends on calibration parameters. Zhu et al. (2018) explored that using calibration method (GA) in the IDM better fits with datasets (Shanghai expressways in China). Zhu et al. (2018) only compared CFMs for best fitting with real datasets. However, they avoided the comparison of calibrated models.

Most of the researches used small-sized data to calibrate the simulation model as shown in Table 2.2. A few numbers of vehicle trajectories are used in the small-sized data, whereas many vehicle trajectories are used in big-sized data. Therefore, big data provide a more realistic explanation and more opportunities to explain the driving behavior and improve the simulated model parameters. Furthermore, the big data that includes dynamic trajectories could supply the error of simulated driving performance and real driving performance more realistically. Different calibration approaches used real data as shown in Table 2.2.

Table 2.2: Related works that have used the calibration approach.

No.	Author	Data size		Calibration method			Note
		Sm	Big	GA	SA	Oth	
1	C. Chen et al. (2010)	✓		✓			The desired speed factor was not calibrated.
2	Rakha and Gao (2010)		✓			✓	The research compared the different CFMs.
3	Bevrani and Chung (2011)	✓					That research disclosed the improved model for only safety factor.
4	Punzo et al. (2012)		✓	✓		✓	The Gipps model parameters are calibrated by synthetic data.
5	Treiber and Kesting (2013b)	✓				✓	The research provides the all calibrated parameters.
6	Punzo et al. (2014)		✓			✓	A few parameters were calibrated.
7	Wagner et al. (2016)		✓				Only stochastic value is employed in this model.
8	Kurtc and Treiber (2016)		✓			✓	The research calibrated the IDM parameters.
9	L. Liu et al. (2016)		✓	✓			GA calibrated the IDM parameters.
10	L. Li et al. (2016)	✓		✓	✓	✓	The research tested the convergence speed of calibration methods.
11	Zhu et al. (2018)		✓	✓			IDM parameters have good fitting capability by using GA.
12	M. M. Rahman et al. (2019a)	✓		✓	✓		Calibrated methods are applied to the simulation data for more fitting with real data.

Sm-Small; Oth- Other calibration approaches; SA- SPSA

## **2.3 Lane Changing Trajectories Planning Model**

Car following and LC are two vehicle movements on multilane roads. When a vehicle driver needs high speed or controlling speed and tries to overcome any obstacle in the current lane, he/she changes the current-lane (M. Rahman, Chowdhury, Xie, & He, 2013). This action refers to the LC behavior of the vehicle driver. The LC is categorized into Mandatory Lane Changing (MLC) and DLC based on the driver intentions. The driver must change his current-lane for MLC. In the last few years, many researchers have developed the MLC model to overcome traffic obstacles (e.g. rear crash, stop-and-go oscillation and working zone signal).

DLC action provides relaxation and comfort in congested road and freeway road, respectively (M. Yang et al., 2019). The safety factor in DLC model is used more significantly than those of MLC model because of safety priority. In recent years, using safety factors and trajectory distribution, a few studies have developed the gap acceptance model. The proposed driver behavior is a planned action before the execution of LC. The planned action depends on two factors, namely, lateral and longitudinal directions. In the coordinate system, both lateral and longitudinal movements can arrive in a planned position and identify the gap in the target lane. Therefore, these direction-based LC trajectory models need to develop and determine the safety gap in the target lane. Subsections 2.3.1 and 2.3.2 review models of these two movements.

### **2.3.1 Lateral lane changing trajectory**

DLC trajectory planning model is important for identifying and ensuring safety in any traffic system. The model helps predict the accepted gap and dynamic trajectories

of the lateral movement. Trajectory planning model has been developing for more than two decades. A few simulation models were developed for DLC trajectory planning by using Quintic Bezier Curves (QBC) (González, Pérez, Milanés, & Nashashibi, 2015; Kawabata, Ma, Xue, & Zheng, 2013; Shen et al., 2017), Hyperbolic Tangent Curve (HTC) (W. Li, Gao, & Duan, 2010; B. Zhou et al., 2017) and Multi-Order Polynomial Curve (MOPC) (C. Wang & Zheng, 2013; D. Yang et al., 2018; You et al., 2015) for urban and freeway roads. Since a sine function-based trajectory curve was adapted to generate the safety factor (J. Wang, Zhang, Zhang, & Yan, 2016; Y. Y. Wang, Pan, Liu, & Feng, 2018), so maximum acceleration was used to derive the unrealistic curve. Used curves in some studies provide the LC trajectory planning to determine the safety gap factor, as shown in Tables 2.3 to 2.5. In addition, three types of purpose for LC trajectory planning still have research gap (Katrakazas, Quddus, Chen, & Deka, 2015):

- Best geometric trajectory is necessary for the SV. That is, the curvature at every point on the curve needs to be as small as possible, and the curvatures at the starting and ending points needs to be nearly zero.
- Realistic dynamic system is important for path planning. The vehicle LC-time versus position should be validated by real trajectory.
- By using the geometric curve, LC-time should be safe.

### **2.3.1(a) Quintic bezier curve**

The QBC is used in LC trajectory planning for shortest-distance and smoothness path, time-optimal and comfortable journey. Shen et al. (2017) addressed the trajectory planning based on the fifth-order QBC for a comfortable journey. They used this

curve for a few tiny vehicle LC scenarios to test comfort measurement. They avoided the error testing between the proposed path planning and real trajectory planning for longitudinal and lateral path positions. Meanwhile, González et al. (2015) found that the fifth-degree QBC was very smooth, however it was only applied on unicycle trajectory. They agreed that the high-degree QBC lost flexibility of trajectory. Further, Kawabata et al. (2013) explored that only a small robotic wheel could use the QBC for smoothness in LC trajectory planning. Therefore, the QBC avoids curvature and smoothness for real vehicle trajectory, whereas the shortest distance of the path was prioritized significantly.

### **2.3.1(b) Multi-order polynomial curve**

The parameters of MOPC are described by acceleration, speed and position constraints. Sometimes, inexperienced driving causes an uncomfortable journey during LC. Resende and Nashashibi (2010) used fifth-order Polynomial Curve (PC) for dynamic longitudinal and lateral trajectory planning. This Polynomial Curve (PC)-based dynamic model is suitable for freeway traffic system for autonomous vehicles, but this curve has limitations for the urban traffic system. C. Wang and Zheng (2013) provided a simulation model for LC trajectory planning by using seven-order PC and assumed that the initial velocity and acceleration are zero. They did not test the validation with real vehicle trajectory.

Yao, Zhao, Bonnifait, and Zha (2013) proposed a data-driven LC path planning model and fifth-order PC by using 223 LC observed data. However, this data-driven model is still a problem due to data collection limitations. You et al. (2015) realized

the problem of the path planning algorithm and provided a proper solution by drawing six-order PC for the longitudinal position and fifth-order PC for the lateral position. To derive the PC model, You et al. (2015) assumed zero-based acceleration and velocity at the LC starting and ending points. Ntousakis, Nikolos, and Papageorgiou (2016) also assumed that the acceleration and velocity at the starting and ending points are zero to generate the lateral trajectory curve.

Heil, Lange, and Cramer (2016) developed the PC-based LC trajectory planning and found the computational cost using maximum acceleration and overshooting behavior. D. Yang et al. (2018) proposed the trajectory planning curve using PC, where the reference angle at the starting and ending points were used to derive the trajectory curve. Therefore, the LC trajectory model is developed by using the MOPC. Most of the LC trajectory model used zero-based velocity and acceleration at the starting and ending points to derive the PC model, in which these assumptions are unrealistic (D. Yang et al., 2018).

### **2.3.1(c) Hyperbolic tangent curve**

A reference angle-based trajectory planning model was modified by using HTC; curvatures and trajectories of HTC are compared with curvatures and trajectories of PC, in which HTC performed better than PC (B. Zhou et al., 2017). W. Li et al. (2010) created another trajectory planning model by combining sine function and HTC and by comparing with the Spline Based Curve (SBC) model to avoid the high curvature at the starting and ending points. B. Zhou et al. (2017) modified HTC for trajectory planning. Thus, they suggested to use the HTC in MLC and DLC actions in future



research. Therefore, this research is for the DLC driver behavior to determine the safety factor using this HTC trajectory planning.

### **2.3.2 Longitudinal lane changing trajectory**

Without the longitudinal movements of the vehicle, the target gap point is unrealistic, or the safe distance cannot be identified by using the lateral trajectory curve. A very few articles use longitudinal movements in the trajectory planning curve to determine the safety factor. Some studies proposed a straight line for longitudinal movements (C. Wang & Zheng, 2013; Y. Y. Wang et al., 2018). However, this straight line may not fit with real trajectory vehicle movements during LC. Driver has assumed points that he/she may want to achieve after LC, and the longitudinal trajectory line direction may change. The longitudinal movement line in previous research can not be fitted the longitudinal positions during LC. However, they avoided the proposal of the planning of longitudinal movements to better fit. The existing longitudinal trajectory planning can not fitly determine the safety factor during LC due to model accuracy. So, the literature has a huge gap. The studies that used longitudinal trajectory line with lateral trajectory curve as shown in Tables 2.3 to 2.5.

### **2.3.3 Calibration and validation approaches of trajectory model**

Literature suggests that the simulation model should be improved by using the calibration method against real trajectory data. Otherwise, the model may not be applicable to the real field. Again, this research explores the literature gap, in which B. Zhou et al. (2017) proposed a trajectory model as a lateral direction curve that is more effective than other trajectory curves for a comfortable journey. The safety gap factor

Table 2.3: Some early literature of LC trajectory model.

SI	Author	Trajectory curve				Position			Safety		Note
		QB	HT	PC	Oth	Lo	La	Cv	Sg		
1	Resende and Nashashibi (2010)			✓		✓	✓				That research was suitable for only freeway traffic system.
2	W. Li et al. (2010)		✓		✓	✓		✓			This combination model gives good performance about the continuity of the curve and curvature.
3	C. Wang and Zheng (2013)				✓						The research derived the trajectory curve, and assumed the initial velocity and acceleration are zero.
4	Yao et al. (2013)				✓				✓		The research proposed a data-driven model to generate the fifth order polynomial trajectory curve.
5	Katrakazas et al. (2015)	✓				✓			✓		The research explored in literature that only tiny vehicle could use the QBC for smoothness.
6	You et al. (2015)				✓	✓			✓		These PCs were very simple and continuous curvature, but it is not optimal shortest trajectory planning.

Sg-Safety gap; Cv- Curvature; QB-QBC; HT-HTC; Oth-Other curves; Lo-Longitudinal trajectory; La-Lateral trajectory;

Table 2.4: More literature of LC trajectory model.

SI	Author	Trajectory curve					Position			Safety			Note	
		QB	HT	PC	Oth	Lo	La	Cv	Sg					
7	González et al. (2015)	✓		✓										They found in literature that QBC was applied on only unicycle trajectory. High degree QBC lost the malleability at trajectory.
8	Luo, Xiang, Cao, and Li (2016)			✓		✓		✓						This research simulated the trajectory curve considering surrounding vehicles.
9	Ntousakis et al. (2016)			✓		✓		✓						By using maximum acceleration, the research proposed simulation of LC trajectory.
10	Heil et al. (2016)			✓										By using maximum acceleration, the research proposed simulation of LC trajectory.
11	J. Wang et al. (2016)						✓	✓		✓	✓	✓		The research assumed unrealistic acceleration for LC trajectory by using sine function.

Sg-Safety gap; Cv- Curvature; QB-QBC; HT-HTC; Oth-Other curves; Lo-Longitudinal trajectory; La-Lateral trajectory;