AN INVESTIGATION ON CONSTRUCTION SEQUENCE OF IRREGULAR SINGLE LAYER PRISM TENSEGRITY

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AN INVESTIGATION ON CONSTRUCTION SEQUENCE OF IRREGULAR SINGLE LAYER PRISM TENSEGRITY

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I hereby declare that all corrections and comments made by the supervisor(s)and examiner have been taken into consideration and rectified accordingly.

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Date :

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ABSTRAK

Struktur *tensegrity* merupakan sistem yang boleh mengekal keseimbangan diri tanpa sokongan sistem lain. Ia terdiri dari unsur batang tertegas individu disambung dengan jaring kabel tarik di mana mereka boleh mencapai ketegangan dengan mengenakan pra-tekanan kepada sistem. Struktur *tensegrity* adalah struktur yang sangat sensitif dan ia hanya akan mencapai keseimbangan apabila semua kabel dikenakan tekanan. Pembinaan untuk struktur *tensegrity* adalah satu proses yang sangat sukar disebabkan ketidakstabilan struktur *tensegrity* sebelum pra-ketegangan dikenakan pada sistem. Namum, tiada dokumentasi yang membincangkan urutan pembinaan struktur *tensegrity* dan maklumat dari segi ini adalah terhad. Untuk mengaplikasikan kelebihan struktur *tensegrity* sepenuhnya dalam realiti, urutan pembinaan struktur *tensegrity* secara praktikal perlu dikenalpasti selapas bentuk dan pra-tekanan diperoleh dari analisis. Lapisan tunggal prisma *tensegrity* secara tidak teratur difokus dalam kajian ini. Kaedah pengiraan nisbah kuasa tekanan yang diperoleh dari analisis telah dikaji untuk tujuan pembinaan dengan menggunakan persamaan keanjalan dan keserasian. Kabel dipotong menjadi lebih pendek dari panjang yang didapati melalui analisis berdasarkan pemanjangan kabel yang dikira. Pra-tekanan diperkenalkan pada kabel boleh didapati melalui pemanjangan kabel. Urutan praktikal pembinaan tunggal prisma *tensegrity* secara tidak teratur telah dibentangkan dalam kajian ini. Model *tensegrity* skala besar telah dibina. Urutan pembinaan di mana kabel dan batang disambung langkah demi langkah dan pra-tekanan dikenakan untuk mencapai bentuk akhir telah dikenalpasti.

ABSTRACT

Tensegrity structure is a self-balancing system where the continuous tensional cables and compressional struts are located among the cables where they could achieve rigidity through the application of pre-stressing. Tensegrity structure is a super sensitive structure and it will only achieve self-equilibrium when all the cables and struts are stressed. Construction of tensegrity structure is a very difficult process due to the highly unstable condition of tensegrity structure before pre-tension is applied. There is however limited information about actual construction sequences of tensegrity structure. In this study, a detailed study on practical construction sequences of tensegrity structure was carried out. Irregular single layer of prism tensegrity was the focus in this study. Method of force ratio calculation based on results of form-finding for construction purpose was determined by adopting elasticity and compatibility equation. Cables were designed to be shorter than length found from form-finding based on elongation calculated and forces introduced on cables were observed from elongation of cables. A practical sequence of construction of irregular single layer prism tensegrity was presented. A large scale tensegrity model was erected. A sequence whereby cables and struts were connected step-by-step and then pre-stressed to achieve the final shape was identified.

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CHAPTER 1

INTRODUCTION

1.1 Background

Tensegrity is a self-balancing system where set of the continuous tensional members (cables) and compressional members (struts) are located among the tensional members. Rigidity of tensegrity is resulted by self- equilibrium between cables and struts through the introduction of pre-stress. Figure 1.1 shows some examples of tensegrity structures.

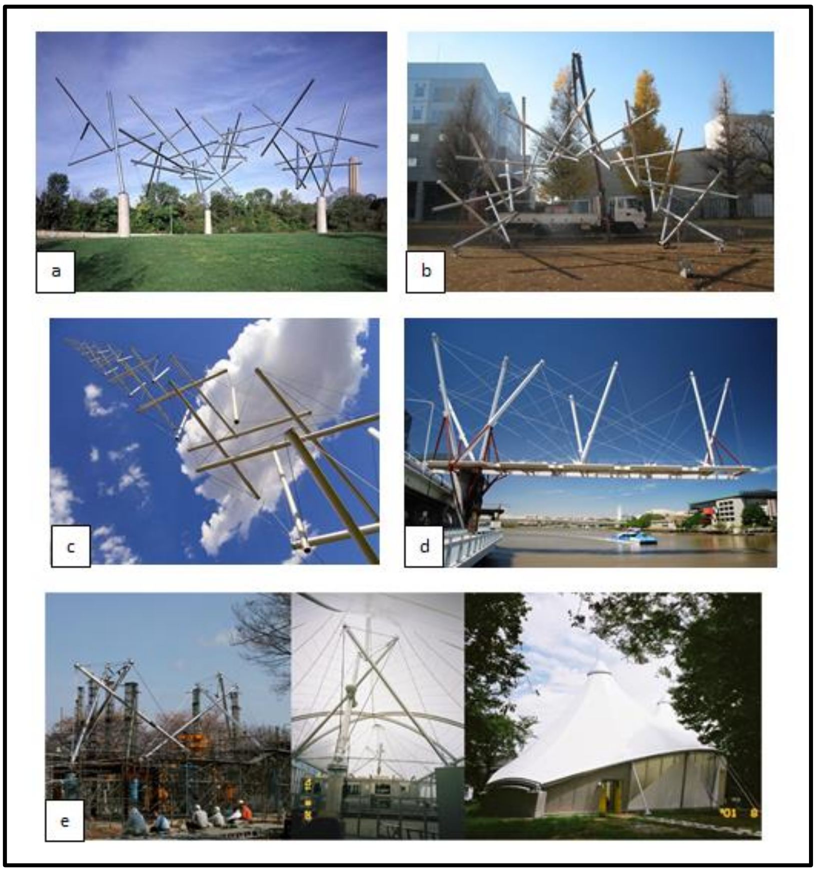

Figure 1.1: Examples of Tensegrity Structure (a) Tensegrity Sculpture- Triple Crown (http://kennethsnelson.net), (b) Tensegrity Arch (tensegritywiki.com/arch), (c) Needle Tower ll, Netherlands, (d) Kurilpa Bridge, Australia (https://www.flickr.com) and (e) White Rhino, Japan (Institute of industrial science, 2016)

The first tensegrity structure was invented and built by a young artist named Kenneth Snelson in 1947. The bars are seen to be floating in the air without visible supporting solid which seems like a magical phenomenon. R. Buckminster Fuller gives it a name *tensegrity* which generated from "tensional integrity". Figure 1.2 shows an example of tensegrity sculpture by Kenneth Snelson. Anthony Pugh gives definition of tensegrity as *"A tensegrity system is established when a set of discontinuous compressive components interacts with a set of continuous tensile components to define a stable volume in space"* (Pugh, 1967). R. Buckminster Fuller described a tensegrity structure as "Islands of compression in an ocean of tension." There are also other definitions given by different authors. In 2003, René Motro gave a complete definition about tensegrity structure, *"A systems in a stable self-equilibrium system comprising a discontinuous set of compressed components inside a continuum of tensioned components"* (Motro, 2003).

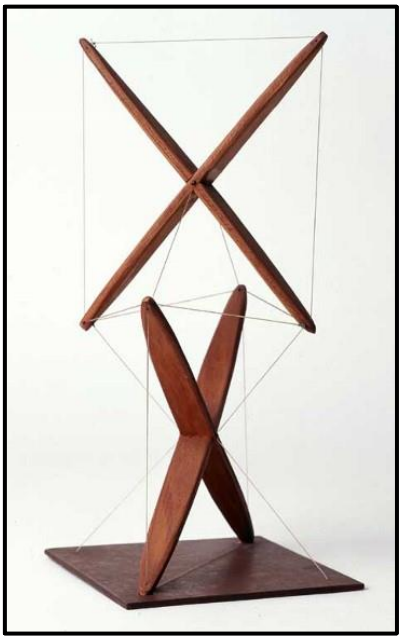

Figure 1.2: The "X-piece" made by Kenneth Snelson in the winter of 1948
(Source: http://www.tensegriteit.nl)

Tensegrity structures are considered to be important turning point in the evolution of special structure. Characteristics of tensegrity structure such as self-erection, lightweight, deployability and aesthetic elegance make tensegrity structure desirable for modern works. It presents possibility to use material in very economical way to provide maximum strength of structure.

Tensegrity structure can be applied to form a dome by arranging itself in spherical configuration with advantages such as use of equal-length struts and simple joints, improved rigidity, extreme resilience and high lightness (Burkhardt, 2004). Figure 1.3 shows an example of a tensegrity dome which is Georgia Dome stadium roof structures.

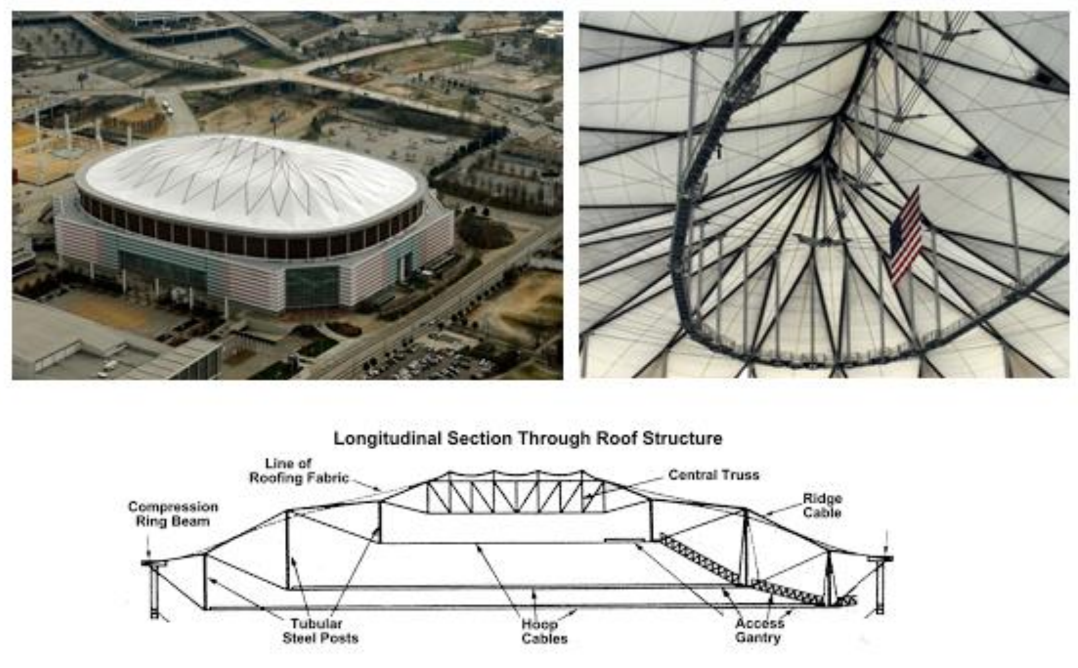

Figure 1.3: Georgia Dome stadium roof tensegrity structures, Atlanda, U.S
(Source: http://www.rockwellgroup.com and http://www.columbia.edu)

Tensegrity structure can also be constructed vertically to form tower structure. Jáuregui (2004) proposed tensegrity tower which can function as lightning tower, communication tower, wind parks or aesthetic elements. Double layer grid configuration by Laboratoire de Génie Civil in Montpellier led by René Motro has the potential in roof construction or covering for structure (Jáuregui, 2004). Properties of tensegrity structure enable it to be applied in sculpture and arch. Figure 1.4 shows examples of application of tensegrity structure.

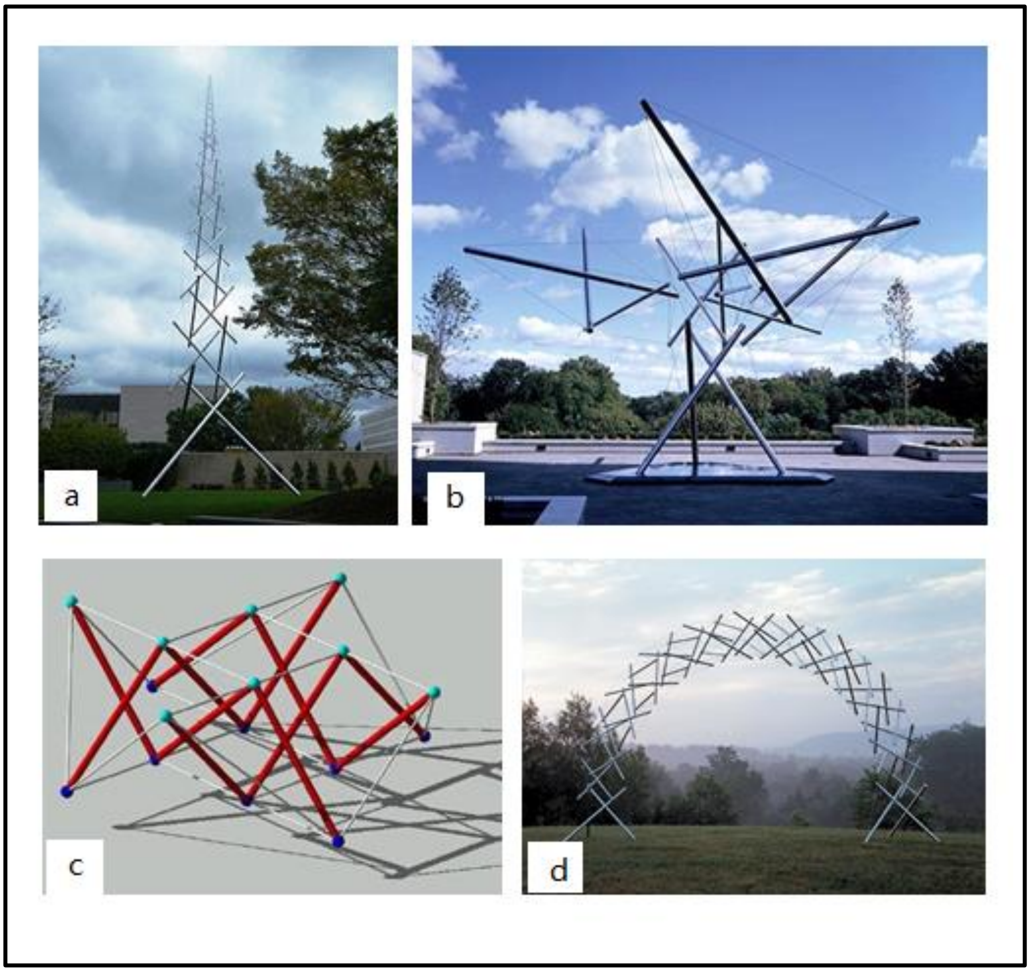

Figure 1.4: Examples of application of tensegrity structure
(a) Needle tower by Kenneth Snelson, (b) Bee-Tree by Kenneth Snelson (http://kennethsnelson.net),
(c) Double layer grid configuration by Laboratoire de Génie Civil (Illustrated by Jáuregui, 2004) and
(d) Rainbow Arch by Kenneth Snelson (http://kennethsnelson.net)

Other than application in the field of architectural engineering, attractive design of furniture can be found by applying concept of tensegrity structure. Figure 1.5 shows a few examples of tensegrity furniture.

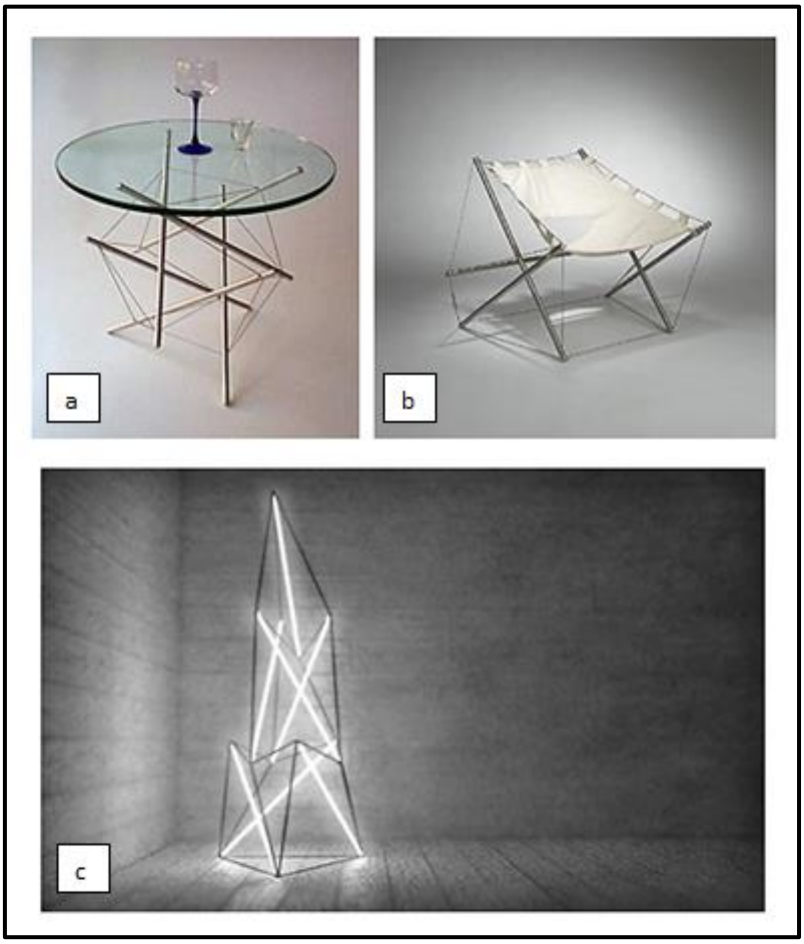

Figure 1.5: Examples of tensegrity furniture
(a) Six struts suspend accent coffee table by Koenig with wine glass,
(b) Tension lounge chair by J.H. Varichon, France
(https://www.flickr.com) and
(c) Lamp design by ARCHICHAOS (http://www.archichaos.be)

Tensegrity structure responses as a whole structure. Stresses are transmitted uniformly and absorbed throughout the structure. The basic of tensegrity structure is simple. It consists of compression element and a tension element. However, the manner they are assembled to form stable system is not intuitively obvious how a tensegrity system transfer loads. The structural property of tensegrity comes from its geometrical configuration and pre-stressing. Topology, geometry and the pre-stressing affect the stability and stiffness of tensegrity structure. Hence, form-finding is needed to determine its equilibrium configuration.

Complexity in the fabrication of tensegrity structure leads to production problem. Construction of tensegrity structure is a very difficult process due to the high sensitivity of tensegrity structure. Pre-stress in all members need to be ensured to achieve a stable self-equilibrium state. Pre-stress forces introduced should be high enough to support critical load, which is difficult in large construction (Jáuregui, 2004).

Tensegrity structure can be classified into three basic patterns that can be used to configure spherical or cylindrical tensegrity structure. They are diamond pattern (Rhombic configuration), Circuit pattern and Zigzag pattern (Pugh, 1976) as shown in Figure 1.6. This classification is based on relative position of struts. By applying different way of joining system, different shape can be achieved. In 2003, Motro (2003) gave a clearer classification to tensegrity system which also included other configuration and geometries where they are classified into spherical system, star system, cylindrical system and irregular system. In spherical system, there are three common configurations for forming this spherical system: Rhombic configuration, Circuit configuration, and Zig-zag configuration.

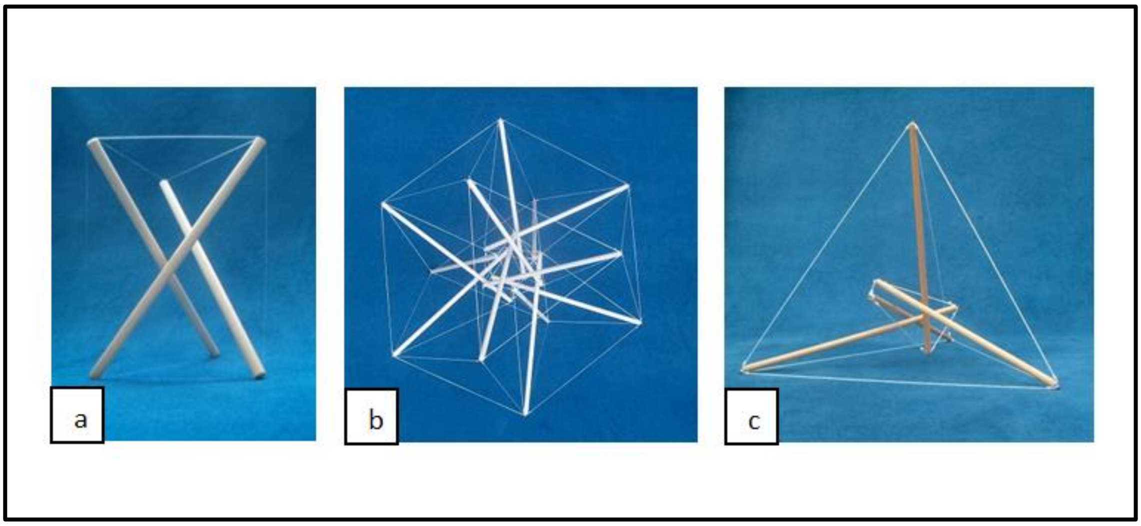

Figure 1.6: Basic pattern of tensegrity structure
(a) Rhombic pattern, (b) Circuit pattern and (c) Zigzag pattern
(Pugh, 1976)

In this study, the focus is on rhombic configuration or also known as prism tensegrity. Prism tensegrity is generated from a straight prism where the cables are horizontal or vertical and the struts are diagonal between the vertices of the two different levels. A relative rotation is introduced between the upper and lower polygons (Motro, 2003). Figure 1.7 shows generation of T-prism by introduced 30 ° rotation angle from a straight prism.

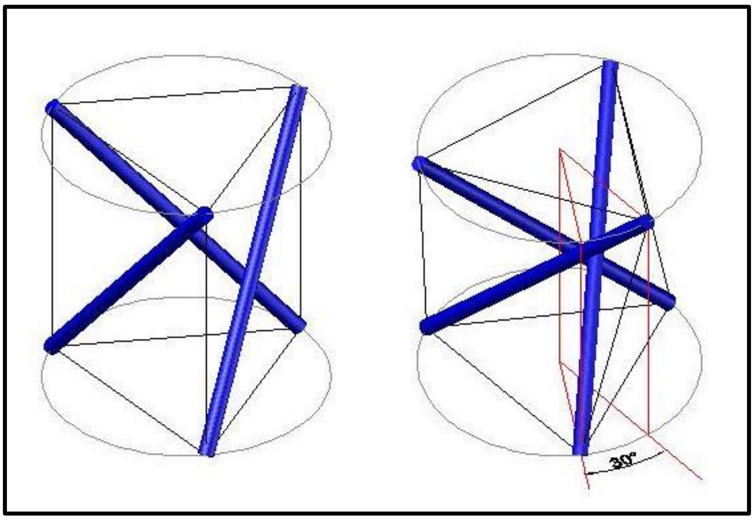

Figure 1.7: Generation of T-Prims

Prism tensegrity is the basic of tensegrity system as it can be modified into circuit configuration and zig-zag configuration by changing some cables orientation. The reasons why prism tensegrity was chosen in this research is due to its simple combining ability. It is easy to extend to form multilayer prism tensegrity by joining each other. Prism tensegrity is also flexible to be experimented in different shape. Irregular prism tensegrity means those configurations where top and bottom ring of prism tensegrity is not symmetrical. Figure 1.8 shows some example of irregular prism tensegrity structure. Irregular prism tensegrity gives more choices and freedom for designer as both symmetrical and unsymmetrical design of tensegrity structure can be considered.

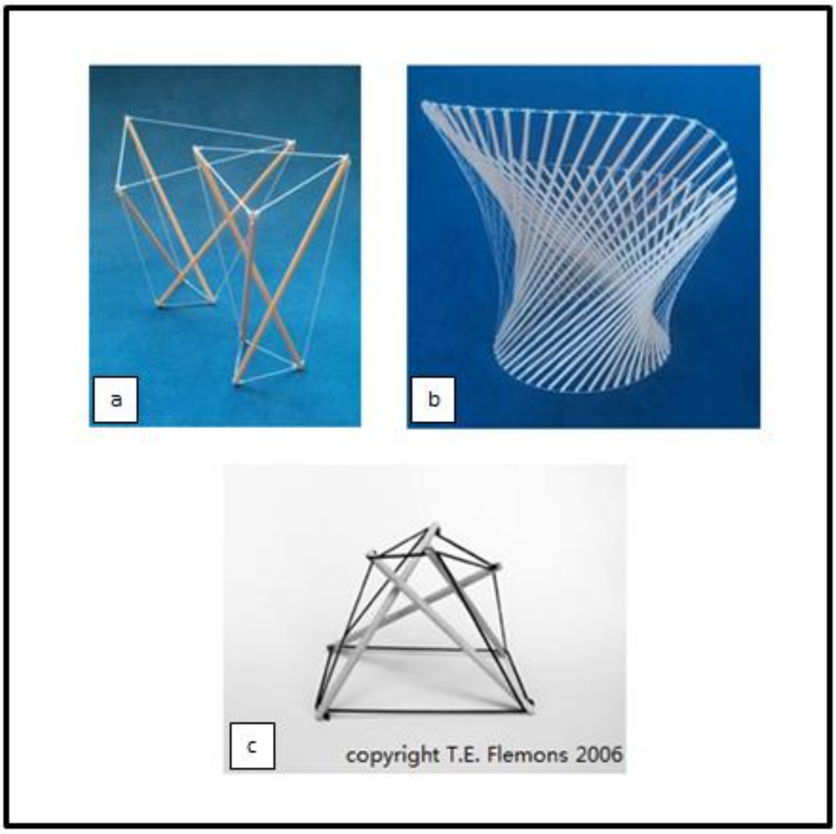

Figure 1.8: Example of irregular prism tensegrity
(a) triangular prism tensegrity (http://www.tensegriteit.nl),
(b) four-fold prism tensegrity (http://tensegritywiki.com) and
(c)tensegrity armchair (http://www.tensegriteit.nl)

1.2 Problem statement

Form-finding methods for tensegrity structures have been investigated by many researches. Many research studies have been carried out to overcome limitation of pervious method and introduce new form-finding method. However, most of the research studies are mainly focus on form-finding of tensegrity structure. There is however not enough information about detailed construction sequence of tensegrity structure. Even though tensegrity structure was introduced by 1947, but until now there are still very few full scale tensegrity structure constructed.

There are few existing tensegrities structures but there is no detailed documentation about construction process of these structures. Tensegrity structure is a super sensitive system and it will only achieve self- equilibrium when all cables and struts are stressed. It is a challenge to construct it in reality especially irregular tensegrity structure because the probability to change position and shape is very high. Thus, a practical sequence to build the irregular prism tensegrity needs to be identified. In order to construct a tensegrity structure, form-finding analysis is needed. A method to determine length of cables needed to be fabricated based on results of form-finding analysis is necessary to be formulated. Correct length of cable is essential to achieve the desired configuration of tensegrity structure.

1.3 Objectives

There are two objectives in this study:

i. To determine method of force ratio calculation from form-finding analysis for construction purpose.
ii. To identify practical sequence of construction of irregular prism tensegrity.

1.4 Layout of thesis

Chapter One describes the background, problem statement and objectives of this research.

Chapter Two describes the form finding method of tensegrity by previous researches. Existing tensegrity structure is also been studied.

Chapter Three is the summary of procedure from form-finding of tensegrity, selection of materials, and determination of construction sequence through physical modeling of irregular prism tensegrity.

Chapter Four shows results and discussion of physical model constructed. The results obtained from construction of physical models in small, medium and large scale are discussed.

Chapter Five presents the conclusion of the study.

CHAPTER 2

LITERATURE REVIEW

2.1 Form-finding of tensegrity structure

Form-finding for a tensegrity structure is needed before construction to obtain its self-equilibrium configuration. Different researchers have investigated different methods for form-finding of tensegrity. Tibert and Pellegrino (2011) have carried out a review on form-finding methods for tensegriry structures. They classified the form-finding methods into two categories, kinematic methods and static method. They reviewed on three kinematic methods: (i) analytical solution, (ii) non- linear programming method and (iii) dynamic relaxation; and four static methods which consist of (i) analytical solution, (ii) force density matrix method, (iii) energy minimization approach and (iv) reduce coordinates method. The first category which is kinematic methods were used to determine the geometry of given tensegrity structure by keeping the struts in constant length and maximizing the length of cables or either way. The second category contains static method which is used to determine the possible equilibrium configuration with a given topology and force in its members. In the review paper, they also discussed about some limitations in these pervious methods. Analytical solution is only applicable for simple and symmetrical structures; while non-linear programming and dynamic relaxation cannot be applied to problems that are not completely defined. For force density matrix and energy minimization method, it is easy to produce new configuration but difficult to control variation in length of elements as tension coefficient is varied. They also concluded that reduced coordinates provide a good control on configuration but it will involve many extensive symbolic manipulations for large system.

Many research studies about new methods for form finding were proposed. Li et al (2010) proposed Monte Carlo form-finding (MCFF) method that employs a stochastic procedure to determine equilibrium configurations of a tensegrity structure. This method does not involve complicated matrix manipulation. Thus, it is easy to implement and involves algebraic function. The method can be used not only for symmetrical structure but also irregular tensegrity structure in large scale. An approach to the analytical and numerical form-finding of tensegrity structures by combined formulation of the equilibrium and geometrical compatibility equations was introduced by Koohestani et al (2013). Matrix iteration was proposed by Lu et al (2015) as form-finding method for irregular structure. Self-stress of members and nodal coordinates can be obtained through singular value decomposition of equilibrium matrix and eigenvalue decomposition of force density matrix on the basis of two different forms of equilibrium equations. Zhang et al (2014) have developed stiffness matrix for form-finding which utilizes stiffness matrix and total potential energy of the structure to increase the efficiency of form-finding. Yamamoto et al (2011) presented genetic algorithm based form-finding method for tensegrity structures to assist designers to obtain tensegrity structures with less design variables. This form-finding method encoded connectivity matrix and tension coefficient into individual population and used in genetic algorithm searching problems. Harichandra et al (2016) reviewed various form-finding method based on force density and presented the techniques which require minimum initial parameters for form-finding of tensegrity structures which also helps in choice of methods based on known parameters of the regular tensegrity structure. Lee et al (2016) categorized form-finding into three main methods: stiffness matrix method, geometric stiffness method and dynamic equilibrium method. In their study, maximum status of tensegrity structure can be obtained by applying maximum natural frequency of structure. A form-finding method using frequency constraints with single self-stress states was proposed by combining stiffness matrix method and force density. Form-finding presented by Mohammad et al (2016) is an alternative linear approach by combining force equilibrium equation and length relation condition which can be used to determine self-equilibrium configuration of either symmetrical or unsymmetrical prism tensegrity structure. This method is fast and highly flexible to achieve the shape desired by designer.

2.2 Case study of existing tensegrity structure

In 2001, White Rhino (Figure 2.1) designed by Ken'ichi Kawaguchi was built in Chiba, Japan (Kawaguchi et al, 2011). It is the first tensegrity structure built in the world.

Figure 2.1: White Rhino, Japan
(Chiba Experimental Campus, Institute of Industrial Science, University of Tokyo, 2010)

White Rhino is a tension type spatial structure dome and it is a combination of membrance structure and tensegrity structure. Triangular prism tensegrity is used as a skeleton and support the membrane cover of the building to form a saddle-shaped curved membrane surface. However, there is insufficient detailed information about the construction sequence of White Rhino in documentation. Figure 2.2 shows design concept and planning outline of White Rhino.

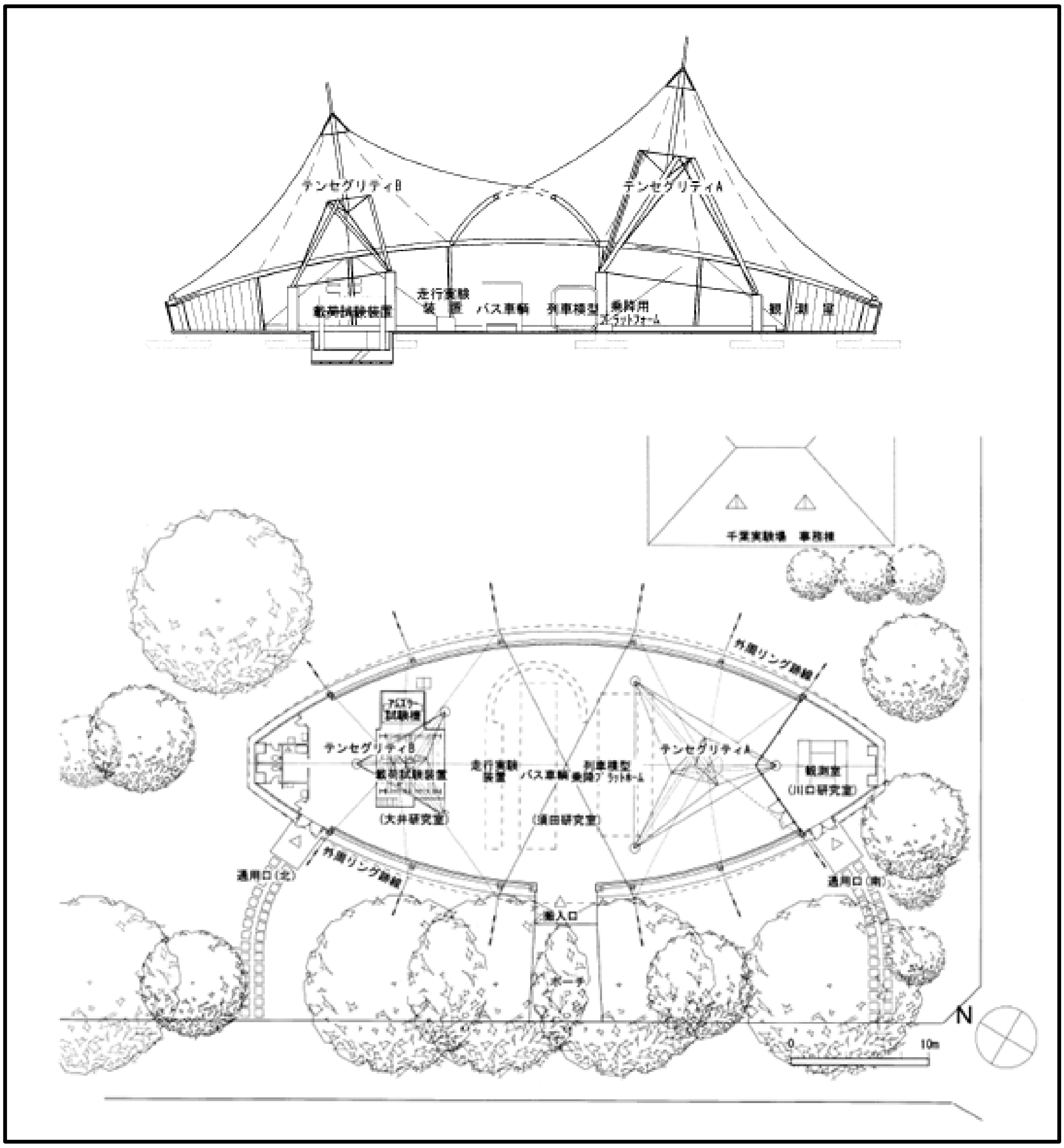

Figure 2.2: Design Concept and Planning Layout of White Rhino
(Fuji et al, 2001)

Kurilpa Bridges in Australia is a 470m long pedestrian and cycle way crossing over Brisbane River. Due to tight geometric restrictions of the site and requirement for clearance over river, level or underside and maximum deck level of bridge was limited to less than 1m (Oasys Software, 2011). Arup and Cox Royner conceived using tensegrity as a major structure to support the deck of this bridge. With the 6.5m clear width composite steel and concrete deck spans between the support piers and a tensegrity system of compression struts and tensional cables that laterally stabilize the structure, it provides torsional rigidity to the bridge. The structure needs to support itself at each stage of the erection without reliance upon temporary props and scaffolding due to site condition. Thus, the bridge was designed to be cantilevered out from each of the two major river piers, effectively using the permanent structure to support itself during construction. Figure 2.3 shows image of Kurilpa Bridge which is able to be self-supported during construction stage.

Figure 2.3: Kurilpa Bridge and self-supporting during construction stage
(Source: http://www.coxarchitecture.com.au)

Due to complexity of the structure and time constraints, all components of Kurilpa Bridge are accurately prefabricated and later connected them together without adjustment. Contractor needs to rely on accuracy of different components and construction process so that the main span will end up at complete correct position. To ensure the precision of all components during erection, Arup used GSA analysis models for each construction stages. GSA is software developed by Arup software house (Oasys). GSA is a highly intuitive nonlinear analysis software tool which enables engineers to realize the potential of their design. With staged analysis, Kurilpa Bridge at any stage in construction was examined. However, Kurilpa Bridge is not a pure tensegrity structure (Ali, 2010). Figure 2.4 shows construction stage modeling of

Kurilpa Bridge using GSA analysis. However, actual stage of construction was not presented.

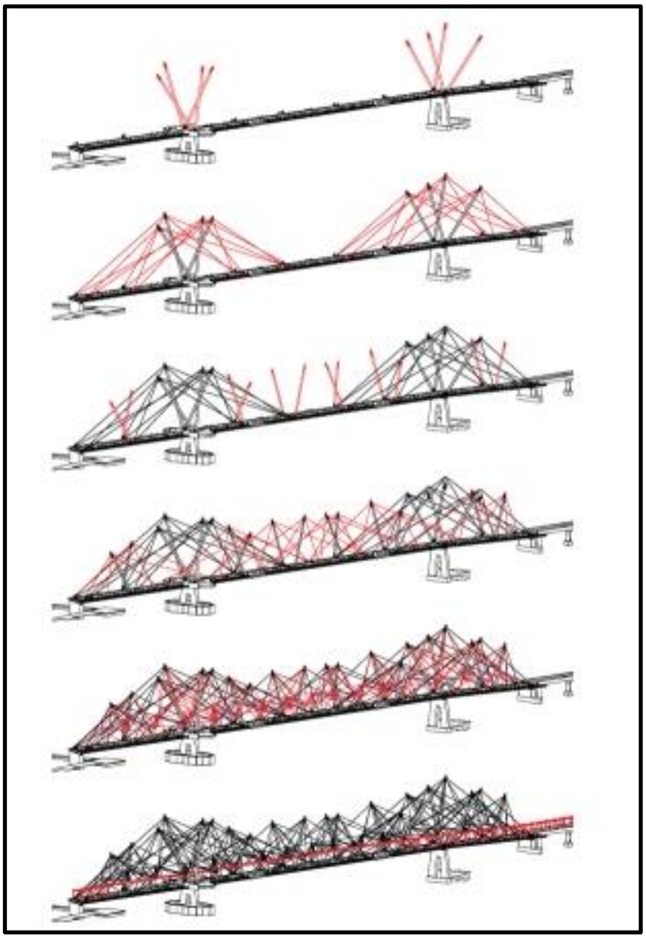

Figure 2.4: Construction stage modelling of Kurilpa Bridge using GSA analysis (http://tensegritywiki.com)

By considering case study of existing structures, it can be concluded that there is still lack of details documentation of construction sequence for tensegrity structure.

2.3 Modelling of tensegrity structure in video clip

Some video clips related to physical modeling of tensegrity structure can be found in internet (Figure 2.5).

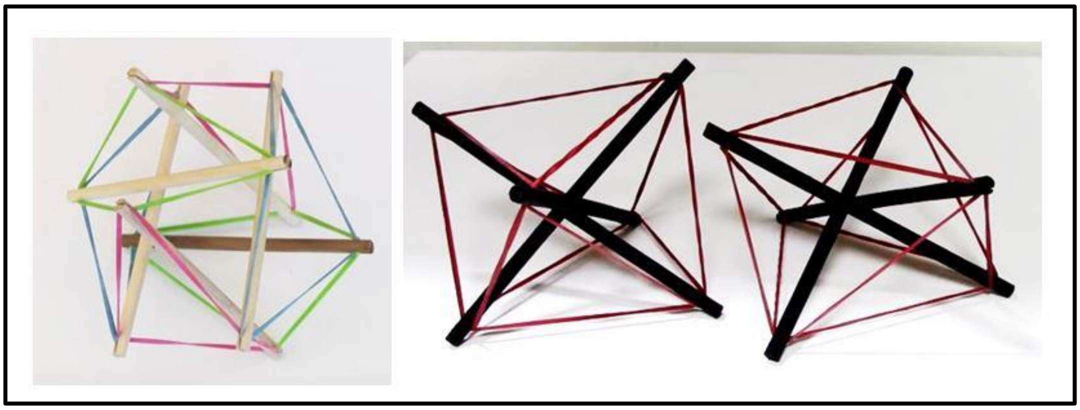

Figure 2.5: Physical Model found in Video Clips
(Sources: https://www.youtube.com/watch?v=wCBcwqY0laU
https://www.youtube.com/watch?v=TeMJ6Ny6dIc)

The steps to build the tensegrity model are clearly shown in the video. All the cables are connected to strut and the shape of the model is adjusted later. However, there are limitations where these models are only applicable to symmetrical shape and at small scale size. Besides that, material of cables used for the model in video clips is rubber band. Tension load applied to cables is based on natural elasticity of rubber band. This causes the structure to temporary deform when external load applied on the system. In real construction, the structure needs to be rigid and able to support design load. Other factors such as self-weight of structure needs be taken into consideration. Thus, the step in video clips may not be applicable to construction of large scale tensegrity structure.

2.4 Summary

Based on literature review, most of the research studies are mainly focused on form finding method of tensegrity structures. Some researchers worked on improvement of previous methods; while, there are also studies on new method for form-finding. However, information about construction sequences of an irregular prism tensegrity structure is found to be limited. Due to super sensitivity of tensegrity structure, it will only attain its rigidity until all cables and struts are pre-stressed. Thus, a sequence to practically build the irregular prism tensegrity needs to be identified.

CHAPTER 3

METHODOLOGY

3.1 Overview

This project is aimed at investigating the construction sequence of irregular single layer prism tensegrity structure. Two shapes of prism tensegrity are the focus of this study; they are triangular prism tensegrity and quadrilateral prism tensegrity.

Before construction of a tensegrity structure, self-equilibrium configuration needs to be found. Figure 3.1 shows component of triangular prism tensegrity. A computer program developed by Mohammad et al (2016) was used for form- finding in this study. It is a linear approach by combining force equilibrium with length relation condition which is applicable for both symmetrical and unsymmetrical prism tensegrity structure with high accuracy and fast computing time. Data specified by designer include coordinates of bottom ring of prism tensegrity, height of tensegrity structure, coordinate of first joint of upper ring of prism tensegrity, and azimuth angle of two members in upper ring. Through the computer program, force ratio of members and coordinates of upper ring were determined. These are the important input of mechanical and geometrical parameters necessary to achieve self-equilibrium of tensegrity structure.

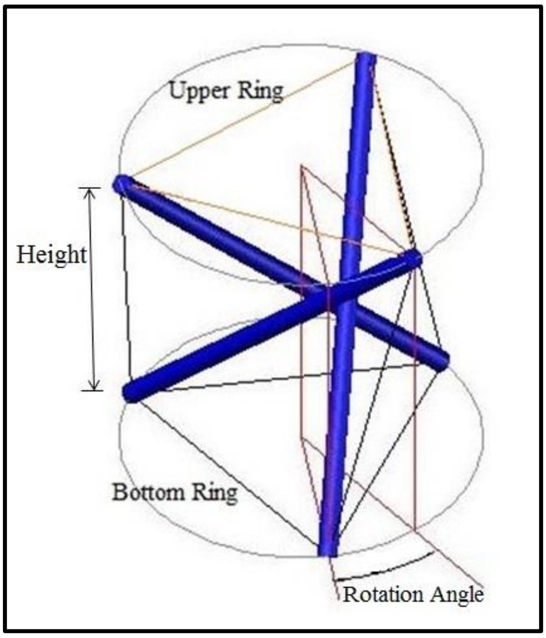

Figure 3.1: Triangular prism tensegrity

Physical models were built to identify practical sequence of construction of irregular prism tensegrity. First, material properties of tension and compression members need to be chosen. Different materials were used for different scale of models; 50mm height for small scale, 260mm height for medium scale and 1800mm height for large scale. Tensile test was carried out to determine the force and elongation properties and the maximum breaking load of cables for the construction of large scale tensegrity. A graph of force versus elongation is plotted and it becomes the reference for calculation of pre-stress to be applied to the members during erection of large scale model.

For small scale prototype, different shapes of prism tensegrity were used to compare the construction sequence. Once the sequence was found, large scale model with height of 1.8m was constructed. Figure 3.2 shows the flow chart of research procedure to determine the construction sequence.

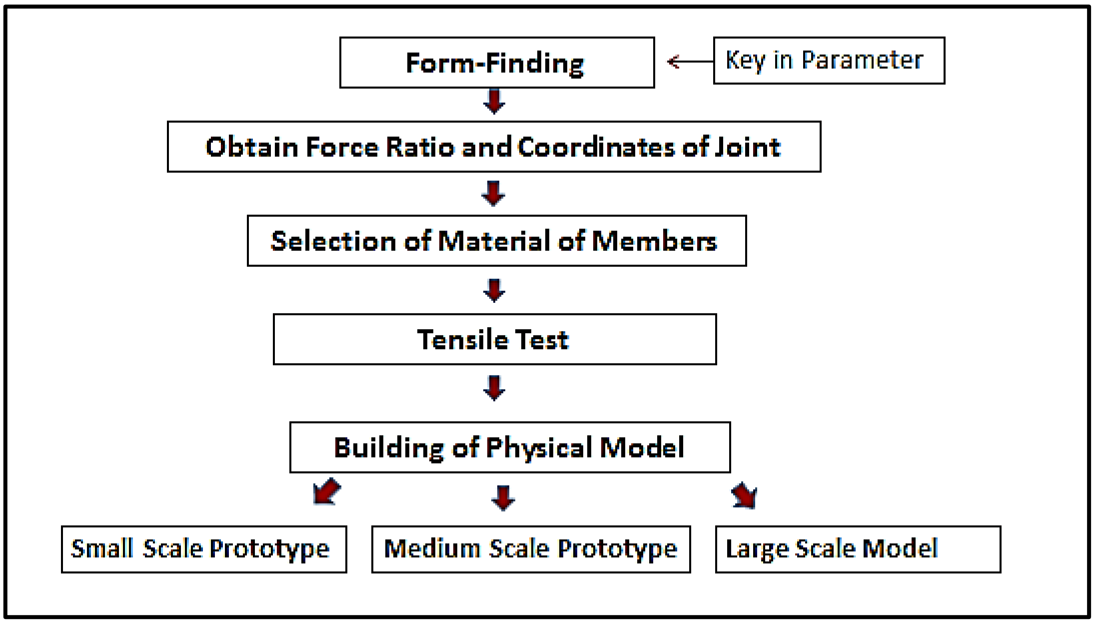

Figure 3.2: Flow chart of research procedures

3.2 Form-finding of Tensegrity

Computer program developed by Mohammad et al (2016) was used for form-finding in this study. In tensegity structure, shape and ratio of axial force in members need to be determined to achieve the self equilibrium configuration. In prism tensegrity, number of joints and members equal to the number of unknown parameter; while number of equilibirum equations is equal to number of joints. Therefore, only force equilibrium is not sufficient for form-finding. Additional conditions are needed which make the form-finding procedure becoming nonlinear. Nonlinear equation requires iteration calculation and it is only practical for tensegrity structure with few members. Mohammad et al (2016) presented a linear approach for form-finding of both symmetrical and unsymmetrical prism tensegrity by using force equilibrium condition and length relation condition simultaneously. This linear approach has high accuracy and is able to be controlled by designers for achieving tensegrities with different configuration including large numbers of members.

A prism tensegrity consists of two polygon in parallel plane, lower polygon is the first ring and the upper polygon is the second ring (Figure 3.1). Figure 3.3 shows a joint of first ring and second ring with their connceted members where, symbol '*n*' to represent numbers of joint, *i* for a particulr joint where $i\epsilon(1\text{-}n)$, while j_i and j'_i represent joints of first ring and second ring, respectively. The symbol for members at fist ring and second ring are donated as as l_i and d_i, respectively. Azimuth angle from x-axis of vertices in first ring is represented as α_i ; while azimuth angle of secong ring is shown as β_i. Diagonal tension members is labeled as t_i and compression members is denoted as c_i. Polar angle is the angle between diagonal members and xy plate. It is donated as δi for diagonal compression members and γ_i for diagonal tension members.

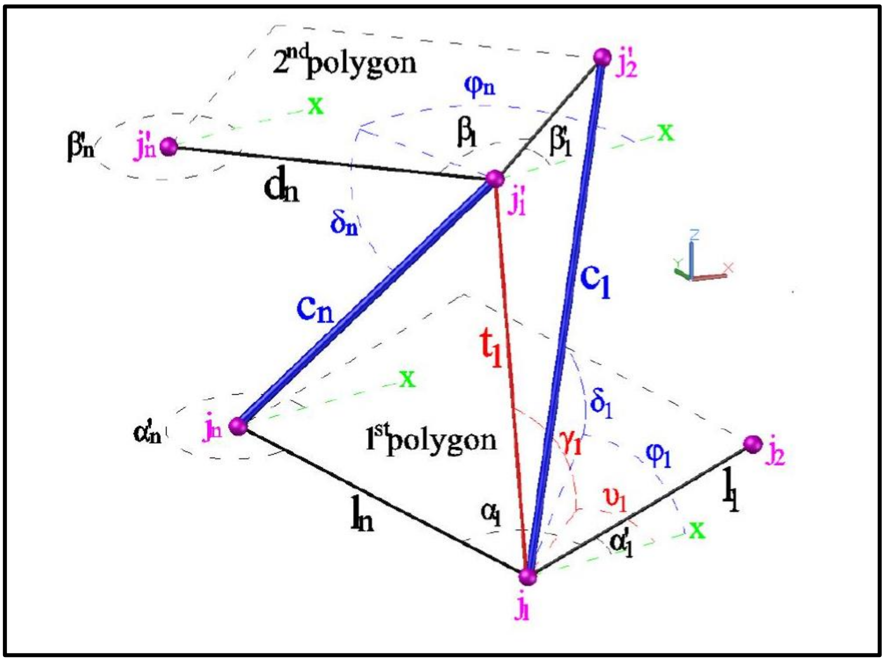

Figure 3.3: Joints of first ring and second ring with their connected members (Mohammad et al, 2016)

Based on condition of self-equilibrium for joints of prism tensegrity, force relation equation and length relation equation can be written. The following equations correspond to equilibrium condition of joint (Mohammad et al, 2016):

Self-equilibrium for joint is written as Equation (3.1).

$$\begin{cases} At\ joint\ j_i\ and\ j'_i \sum \bar{F}_{(x,y)} = 0 \\ At\ joint\ j_i\ and\ j'_i \sum \bar{F}_{(z)} = 0 \\ i\epsilon(1-n) \end{cases} \quad (3.1)$$

Self-equilibrium equilibrium equation corresponding to j_1 is written as shown in Equation (3.2).

$$\begin{cases} \bar{F}_{d_1} + \bar{F}_{d_n} + \bar{F}_{t_1}\cos\gamma_1 + \bar{F}_{c_n}\cos\delta_n = 0 \\ \bar{F}_{t_1}\sin\gamma_1 + \bar{F}_{c_n}\sin\delta_n = 0 \end{cases} \quad (3.2)$$

Length relation equation corresponding to a close polygon is written as shown in Equation (3.3).

$$\begin{cases} \sum \bar{L}_{(x,y)} = \bar{L}_{l_n} + \bar{L}_{t_1}\cos\gamma_1 + \bar{L}_{c_n}\cos\delta_n = 0 \\ \sum \bar{L}_{(z)} = \bar{L}_{t_1}\sin\gamma_1 + \bar{L}_{c_n}\sin\delta_n = 0 \end{cases} \quad (3.3)$$

Combining Equation (3.2) and Equation (3.3), the following Equation (3.4) is obtained where k is a constant coefficient.

$$(\bar{L}_{t_1}\sin\gamma_1 + \bar{L}_{c_n}\sin\delta_n)k = \bar{F}_{t_1}\sin\gamma_1 + \bar{F}_{c_n}\sin\delta_n \Rightarrow$$
$$(\bar{L}_{t_1}\cos\gamma_1 + \bar{L}_{c_n}\cos\delta_n)k = \bar{F}_{t_1}\cos\gamma_1 + \bar{F}_{c_n}\cos\delta_n = k\bar{L}_{l_n}$$
$$\begin{cases} \bar{F}_{d_1} + \bar{F}_{d_n} + k\bar{L}_{l_n} = 0 \\ k > 0 \end{cases} \quad (3.4)$$

Equation for joint, j_1 can also be written as follows, where k' is a constant coefficient.

$$\begin{cases} \bar{F}_{l_1} + \bar{F}_{l_n} + \bar{L}_{d_1} = 0 \\ \bar{L}_{d_1} = k'\bar{F}_{d_1} \end{cases} \quad (3.5)$$