Final Examination
2018/2019 Academic Session

June 2019

JIM319 – Vector Calculus
(Kalkulus Vektor)

Duration : 3 hours
(*Masa: 3 jam*)

Please check that this examination paper consists of **NINE (9)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEMBILAN (9)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **ALL** questions.

*[**Arahan** : Jawab **SEMUA** soalan].*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

1. (a). Given $\underline{a} = 3\underline{i} - \underline{j} + \underline{k}$, $\underline{b} = \underline{i} + 3\underline{j} + 2\underline{k}$ and $\underline{c} = 4\underline{i} - 3\underline{j} + 5\underline{k}$.

(i). Find $\underline{a} \cdot \underline{c}$ and $\| 3\underline{a} \times 2\underline{b} \|$.

(ii). Compute the volume of parallelepiped formed by $\underline{a}$, $\underline{b}$ and $\underline{c}$.

(iii). Calculate the work done in moving an object from origin to point $P(-1,2,5)$ by a force of 10 N that acts in the direction of $\underline{b}$.

(50 marks)

(b). Find Q and P if $(2\underline{i} + 6\underline{j} + 27\underline{k}) \times (\underline{i} + P\underline{j} + Q\underline{k}) = \underline{0}$. Hence, calculate the angle, θ between these two vectors. Describe the geometrical interpretation of these two vectors.

(30 marks)

(c). If the vector $\alpha\underline{i} + \alpha\underline{j} + c\underline{k}$, $\underline{i} + \underline{k}$ and $c\underline{i} + c\underline{j} + \beta\underline{k}$ be coplanar.

Show that

$$c^2 = \alpha\beta$$

where α, β and c are constants.

(20 marks)

2. (a). Give that $A(1,0,-3)$, $B(0,-2,-4)$ and $C(4,1,6)$.

(i). Calculate the area of the triangle with vertices A, B, and C.

(ii). Find the equation of the plane through the points A, B, and C.

(30 marks)

(b). The line L_1 and plane $\prod_1$ have equations

$$L_1 : \underline{r} = 2\underline{i} - 2\underline{j} + 3\underline{k} + \lambda(\underline{i} - \underline{j} + 4\underline{k})$$

$$\prod_1 : \underline{r}.(\underline{i} + 5\underline{j} + \underline{k}) = 5$$

(i). Write the equation of the line L_1 in parametric form and Cartesian form.

(ii). State the direction vector of the line L_1 and normal of the plane $\prod_1$.

(iii). Find the angle between the line L_1 and the plane $\prod_1$.

(iv). Compute the distance between the line L_1 and the plane $\prod_1$.

(70 marks)

3. (a). Compute the rate of change of the scalar field

$$\phi = 3x^2 - 5xy + xyz$$

at the point $(2,2,7)$ in the direction of $\underline{v} = \underline{i} + \underline{j} - \underline{k}$.

In which direction does ϕ change the most quickly? Therefore, calculate the maximum rate of change at $(2,2,7)$.

(50 marks)

(b). State the condition of a vector field to be solenoidal. Hence, determine the value of constant so that the vector

$$\underline{F} = (x+3y)\underline{i} + (y-2z)\underline{j} + (x+\alpha z)\underline{k}$$

is solenoidal.

(20 marks)

(c). The position of a particle in space at time *t* is given

$$\underline{r}(t) = \sqrt{3}\,\underline{i} + e^{t}\,\underline{j} + e^{-t}\,\underline{k}$$

Find the velocity vector, speed, and acceleration vector of the particle at time t.

(30 marks)

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4. (a). Given the scalar field ϕ and vector field $\underline{F}$, where

$$\phi(x,y,z) = 3x^2y - y^2z^2 \text{ and } \underline{F}(x,y,z) = xe^y\underline{i} - ze^{-y}\underline{j} + y\ln z\underline{k}.$$

(i). Compute the gradient of ϕ and divergence of $\underline{F}$.

(ii). Verify the identities

$$\underline{\nabla}\cdot(\underline{\nabla}\times\underline{F}) = 0 \text{ and } \underline{\nabla}\times(\underline{\nabla}\phi) = \underline{0}.$$

(iii). Is $\underline{F}$ rotational or irrotational? Justify your answer.

(40 marks)

(b). A vector field is given by

$$\underline{F}(x,y) = (4x^3y^2 - 2xy^3)\underline{i} + (2x^4y - 3x^2y^2 + 4y^3)\underline{j}$$

(i). Show that $\underline{F}$ is conservative field.

(ii). Find a potential function ϕ such that $\underline{F} = \underline{\nabla}\phi$.

(iii). Use the potential function to evaluate the line integral

$$\int_C \underline{F}\cdot d\underline{r},$$

where C is a curve with parametrization

$$\underline{r}(t) = (t + \sin\pi t)\underline{i} + (2t + \cos\pi t)\underline{j}, \qquad 0 \le t \le 1.$$

(60 marks)

5. (a). State clearly the Gauss theorem.

Use the Gauss theorem to evaluate the surface integral

$$\iint_S \underline{F}\cdot d\underline{S}$$

where $\underline{F}(x,y,z) = \sin(\pi x)\underline{i} + zy^3\underline{j} + (z^2 + 4x)\underline{k}$ and S is the surface of the box with $-1 \le x \le 2,\ 0 \le y \le 1$ and $1 \le z \le 4$.

Note that all six sides of the box are included in S.

(40 marks)

(b). State Stokes' theorem for a differentiable vector field $\underline{F}$ over a surface S bounded by a closed curve C.

Use Stokes's theorem to evaluate

$$\int_C \underline{F} \cdot d\underline{r} \, .$$

where $\underline{F}(x,y,z) = (x+y^2)\underline{i} + (y+z^2)\underline{j} + (z+x^2)\underline{k}$ and C is the triangle with vertices $(1,0,0),(0,1,0)$ and $(0,0,1)$.

Note that curve C is oriented counterclockwise as viewed from above.

(60 marks)

1. (a). Diberi $\underline{a} = 3\underline{i} - \underline{j} + \underline{k}$, $\underline{b} = \underline{i} + 3\underline{j} + 2\underline{k}$ dan $\underline{c} = 4\underline{i} - 3\underline{j} + 5\underline{k}$.

   (i). Cari $\underline{a} \cdot \underline{c}$ dan $\| 3\underline{a} \times 2\underline{b} \|$.

   (ii). Hitung isipadu parallelepiped yang dibentukkan oleh $\underline{a}$, $\underline{b}$ dan $\underline{c}$.

   (iii). Kira kerja bagi suatu objek bergerak dari asalan kepada titik $P(-1,2,5)$ dengan daya 10 N yang bertindak dalam arah vektor $\underline{b}$.

(50 markah)

(b). Cari nilai Q dan P jika $(2\underline{i} + 6\underline{j} + 27\underline{k}) \times (\underline{i} + P\underline{j} + Q\underline{k}) = \underline{0}$. Seterusnya, hitung sudut, θ di antara vektor berkenaan. Terangkan tafsiran geometri bagi kedua-dua vektor tersebut.

(30 markah)

(c). Jika vektor $\alpha\underline{i} + \alpha\underline{j} + c\underline{k}$, $\underline{i} + \underline{k}$ dan $c\underline{i} + c\underline{j} + \beta\underline{k}$ adalah sesatah.

Tunjukkan bahawa

$$c^2 = \alpha\beta$$

di mana α, β dan c ialah pemalar.

(20 markah)

2. (a). Diberi $A(1,0,-3)$, $B(0,-2,-4)$ dan $C(4,1,6)$.

   (i). Kira luas segitiga dengan bucu-bucu A, B, dan C.

   (ii). Cari persamaan satah yang melalui titik-titik A, B, dan C.

(30 markah)

(b). Suatu garis L_1 dan satah $\prod_1$ memiliki persamaan

$$L_1: \underline{r} = 2\underline{i} - 2\underline{j} + 3\underline{k} + \lambda(\underline{i} - \underline{j} + 4\underline{k})$$

$$\prod_1: \underline{r}.(\underline{i} + 5\underline{j} + \underline{k}) = 5$$

(i). Tulis persamaan garis L_1 dalam bentuk parametrik dan bentuk Cartesian.

(ii). Nyatakan vektor arah bagi garis L_1 dan normal kepada satah $\prod_1$.

(iii). Cari sudut di antara garis L_1 dan satah $\prod_1$.

(iv). Kira jarak di antara garis L_1 dan satah $\prod_1$.

(70 markah)

3. (a). Kira kadar perubahan medan skalar

$$\phi = 3x^2 - 5xy + xyz$$

pada titik $(2,2,7)$ dalam arah $\underline{v} = \underline{i} + \underline{j} - \underline{k}$.

Dalam arah manakah ϕ berubah dengan pantasnya? Oleh yang demikian, kira kadar perubahan maksimum pada titik $(2,2,7)$.

(50 markah)

(b). Nyatakan keadaan yang menyebabkan suatu medan vektor ialah dalam keadaan solenoidal. Maka, tentukan nilai pemalar supaya vektor

$$\underline{F} = (x + 3y)\underline{i} + (y - 2z)\underline{j} + (x + \alpha z)\underline{k}$$

ialah dalam keadaan solenoidal.

(20 markah)

(c). Kedudukan suatu zarah dalam ruang pada masa t diberikan seperti berikut

$$\underline{r}(t) = \sqrt{3}\,\underline{i} + e^{t}\,\underline{j} + e^{-t}\,\underline{k}$$

Cari vektor halaju, kelajuan, dan vektor pecutan bagi zarah tersebut pada masa t.

(30 markah)

4. (a). Diberi medan skalar ϕ dan medan vektor $\underline{F}$, dengan

$\phi(x,y,z) = 3x^2y - y^2z^2$ dan $\underline{F}(x,y,z) = xe^y\,\underline{i} - ze^{-y}\,\underline{j} + y\ln z\,\underline{k}$.

(i). Kira kecerunan bagi ϕ and kecapahan bagi $\underline{F}$.

(ii). Tentusahkan identiti

$\underline{\nabla}\cdot(\underline{\nabla}\times\underline{F}) = 0$ dan $\underline{\nabla}\times(\underline{\nabla}\phi) = \underline{0}$.

(iii). Adakah $\underline{F}$ putaran atau tak putaran? Berikan justifikasi bagi jawapan anda.

(40 markah)

(b). Suatu medan vektor diberi oleh

$\underline{F}(x,y) = (4x^3y^2 - 2xy^3)\underline{i} + (2x^4y - 3x^2y^2 + 4y^3)\,\underline{j}$

(i). Tunjukkan bahawa $\underline{F}$ ialah medan abadi.

(ii). Cari fungsi keupayaan ϕ supaya $\underline{F} = \underline{\nabla}\phi$.

(iii). Gunakan fungsi keupayaan untuk menilai kamiran garis

$$\int_C \underline{F}\cdot d\underline{r},$$

di mana C ialah suatu lengkung dengan persamaan parameter.

$\underline{r}(t) = (t + \sin \pi t)\,\underline{i} + (2t + \cos \pi t)\,\underline{j}, \qquad 0 \le t \le 1.$

(60 markah)

5. (a). Nyatakan dengan jelas teorem Gauss.

Gunakan teorem Gauss untuk menilai kamiran permukaan

$$\iint_S \underline{F}\cdot d\underline{S}$$

di mana $\underline{F}(x,y,z) = \sin(\pi x)\,\underline{i} + zy^3\,\underline{j} + (z^2 + 4x)\,\underline{k}$ dan S ialah permukaan kotak dengan $-1 \le x \le 2,\ 0 \le y \le 1$ dan $1 \le z \le 4$.

Kesemua 6 sisi kotak tersebut termasuk dalam S.

(40 markah)

(b). Nyatakan teorem Stokes untuk suatu medan terbeza $\underline{F}$ ke atas permukaan S yang dibatasi oleh lengkung tertutup C.

Gunakan teorem Stokes untuk menilai

$$\int_C \underline{F} \cdot d\underline{r} .$$

di mana $\underline{F}(x, y, z) = (x + y^2)\underline{i} + (y + z^2)\underline{j} + (z + x^2)\underline{k}$ dan C ialah segi tiga dengan bucu-bucu $(1,0,0), (0,1,0)$ dan $(0,0,1)$.

Lengkung C berorientasikan lawan arah jam jika dilihat dari pandangan atas.

(60 markah)

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