



Final Examination
2018/2019 Academic Session

June 2019

**JIM211 – Advanced Calculus
(Kalkulus Lanjutan)**

Duration : 3 hours
(Masa: 3 jam)

Please check that this examination paper consists of **EIGHT (8)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN (8)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **ALL** questions.

Arahan : Jawab **SEMUA** soalan].

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

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1. (a). Given

$$x^2 + y^2 + 2u^2 + 3v^2 - 7 = 0$$

$$2x^2 - 3y^2 + 3uv - 2 = 0$$

Find

(i). $\frac{\partial u}{\partial x}$

(ii). $\frac{\partial v}{\partial y}$.

(40 marks)

(b). Find the local extreme values and saddle points of the function $f(x, y)$, if there is any, given that

$$f(x, y) = x^2 - 2xy + \frac{y^3}{3} - 3y.$$

(30 marks)

(c). Use the method of Lagrange multipliers to find the extreme values of the function $f(x, y) = 3x^2 + 2y^2 - 4y + 1$ on the region $x^2 + y^2 \leq 16$.

(30 marks)

2. (a). Evaluate

(i). $\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{x}{\sqrt{x^2 + y^2}} dy dx,$

(ii). $\iint_D 2x dx dy,$

over the region D enclosed by the curve $y = e^x$, the lines $x = 0$, $x = 2$ and $y = 0$.

(50 marks)

...3/-

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(b). Evaluate

(i).
$$\int_1^e \int_{\frac{\pi}{2}}^{\pi} \int_{\ln y}^{\ln 2y} \frac{e^x \sin y}{y} dx dy dz.$$

(ii).
$$\iiint_V (x^2 + y^2 + z^2)^{\frac{1}{2}} dx dy dz$$

$$V = \{(x, y, z) \in \mathbb{R}^3 \mid 1 \leq x^2 + y^2 + z^2 \leq 4\}$$

(50 marks)

3. (a). State the definition for each of the following:

- (i). increasing sequence,
- (ii). bounded sequence
- (iii). convergence of a sequence,
- (iv). Cauchy sequence.

(40 marks)

(b). The sequence $\{a_n\}$ is defined as

$$a_1 = 1, \quad a_{n+1} = 3 - \frac{1}{a_n}, \quad n \geq 1$$

Show that

- (i). $\{a_n\}$ is increasing
- (ii). $a_n < 3, \forall n \in \mathbb{N}$
- (iii). $\{a_n\}$ is convergence

(40 marks)

(c). Show that the sequence $\left\{\frac{1}{n}\right\}$ is Cauchy.

(20 marks)

...4/-

4. (a). State the Squeezing Theorem for sequences. Hence, use the theorem to evaluate

$$\lim_{x \rightarrow 0} x \sin \left(\frac{1}{x} \right).$$

(30 marks)

- (b). Calculate the following limits:

(i). $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x + \ln x - 1}$

(ii). $\lim_{x \rightarrow 0} \left(\frac{1}{\ln(x+1)} - \frac{1}{x} \right)$

(iii). $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{\tan x}$

(iv). $\lim_{x \rightarrow 0} \frac{3^x - 1}{x}$

(70 marks)

5. (a). Determine whether each of the following series converges or diverges:

(i). $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

(ii). $\sum_{n=0}^{\infty} \frac{n^n}{n!}$

(iii). $\sum_{k=2}^{\infty} \frac{1}{\ln k}$

(iv). $\sum_{k=1}^{\infty} \frac{3^k}{4^k(k+1)}$

(60 marks)

- (b). Find the Taylor's polynomial of degree 3 about $x = 1$ and the remainder for the function f given by

$$f(x) = xe^x.$$

(40 marks)

...5/-

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1. (a). Diberi

$$x^2 + y^2 + 2u^2 + 3v^2 - 7 = 0$$

$$2x^2 - 3y^2 + 3uv - 2 = 0$$

Cari

(i). $\frac{\partial u}{\partial x}$

(ii). $\frac{\partial v}{\partial y}$.

(40 markah)

(b). Cari nilai ekstremum setempat dan titik pelana bagi fungsi $f(x, y)$, jika ada, diberi bahawa

$$f(x, y) = x^2 - 2xy + \frac{y^3}{3} - 3y.$$

(30 markah)

(c). Guna kaedah pendarab Lagrange untuk mencari nilai maksimum dan nilai minimum bagi fungsi $f(x, y) = 3x^2 + 2y^2 - 4y + 1$ pada rantau $x^2 + y^2 \leq 16$.

(30 markah)

2. (a). Hitung

(i). $\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{x}{\sqrt{x^2+y^2}} dy dx,$

(ii). $\iint_D 2x dx dy,$

pada rantau D dibatasi oleh lengkung $y = e^x$, garis

$$x = 0, x = 2 \text{ dan } y = 0.$$

(50 markah)

...6/-

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(b). Hitung

$$(i). \int_1^e \int_{\frac{\pi}{2}}^{\pi} \int_{\ln y}^{\ln 2y} \frac{e^x \sin y}{y} dx dy dz.$$

$$(ii). \iiint_V (x^2 + y^2 + z^2)^{\frac{1}{2}} dx dy dz$$

$$V = \{(x, y, z) \in \mathbb{R}^3 \mid 1 \leq x^2 + y^2 + z^2 \leq 4\}$$

(50 markah)

3. (a). Nyatakan definisi bagi setiap yang berikut:

- (i). jujukan menokok,
- (ii). jujukan terbatas,
- (iii). penumpuan suatu jujukan,
- (iv). jujukan Cauchy.

(40 markah)

(b). Jujukan $\{a_n\}$ ditakrifkan sebagai

$$a_1 = 1, \quad a_{n+1} = 3 - \frac{1}{a_n}, \quad n \geq 1$$

Buktikan bahawa

- (i). $\{a_n\}$ adalah menokok
- (ii). $a_n < 3, \forall n \in \mathbb{N}$
- (iii). $\{a_n\}$ adalah menumpu

(40 markah)

(c). Buktikan bahawa $\left\{\frac{1}{n}\right\}$ adalah jujukan Cauchy.

(20 markah)

4. (a). Nyatakan Teorem Menyempit bagi jujukan. Seterusnya, guna teorem ini untuk menghitung

$$\lim_{x \rightarrow 0} x \sin \left(\frac{1}{x} \right).$$

(30 markah)

- (b). Hitung had yang berikut:

(i). $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x + \ln x - 1}$

(ii). $\lim_{x \rightarrow 0} \left(\frac{1}{\ln(x+1)} - \frac{1}{x} \right)$

(iii). $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{\tan x}$

(iv). $\lim_{x \rightarrow 0} \frac{3^x - 1}{x}$

(70 markah)

5. (a). Tentukan sama ada setiap siri berikut menumpu atau mencapah:

(i). $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

(ii). $\sum_{n=0}^{\infty} \frac{n^n}{n!}$

(iii). $\sum_{k=2}^{\infty} \frac{1}{\ln k}$

(iv). $\sum_{k=1}^{\infty} \frac{3^k}{4^k(k+1)}$

(60 markah)

- (b). Cari polinomial Taylor darjah 3 sekitar $x=1$ dan baki bagi fungsi f diberi oleh

$$f(x) = xe^x.$$

(40 markah)

...8/-

APPENDIX**List of formula:**

$$1. \quad A = \frac{\partial^2 f}{\partial x^2}(x_0, y_0), \quad B = \frac{\partial^2 f}{\partial y \partial x}(x_0, y_0), \quad C = \frac{\partial^2 f}{\partial y^2}(x_0, y_0), \quad D = AC - B^2$$

$$2. \quad \nabla f(x, y) = \lambda \nabla g(x, y)$$

$$3. \quad x = r \cos \theta, \quad y = r \sin \theta, \quad z = z$$

$$4. \quad f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x)$$

$$\text{with } R_n(x) = \frac{1}{n!} \int_a^x f^{(n+1)}(t)(x-t)^n dt$$